

COMPLETE SCHOOL ALGEBRA

RAWKES-LURY-TOUTON

TEACHERS' EDITION

IMPORTANT

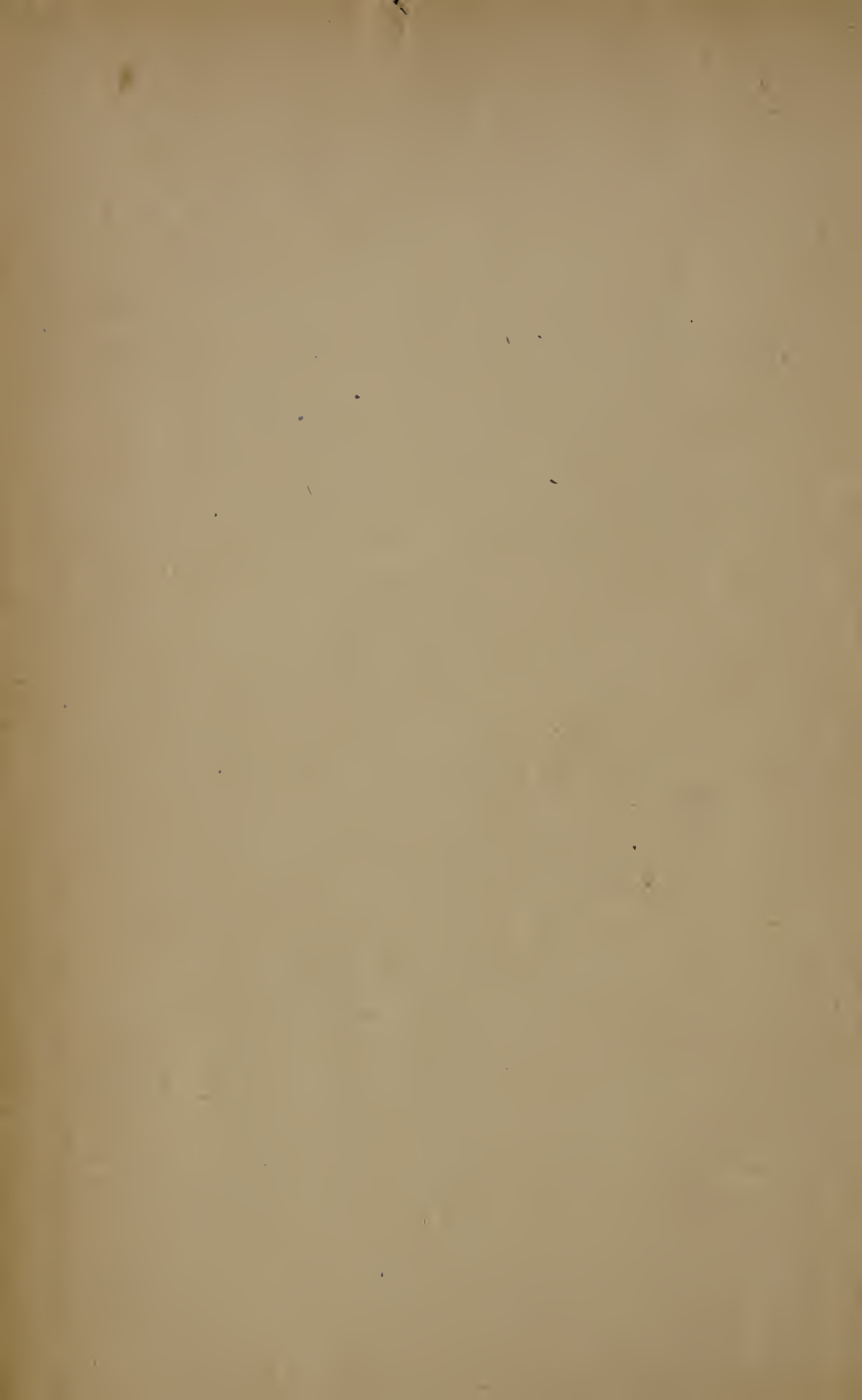
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TEACHERS' EDITION

BY

HERBERT E. HAWKES, Ph.D.

PROFESSOR OF MATHEMATICS IN COLUMBIA UNIVERSITY

WILLIAM A. LUBY, A.B.

HEAD OF THE DEPARTMENT OF MATHEMATICS
KANSAS CITY POLYTECHNIC INSTITUTE

AND

FRANK C. TOUTON, Ph.D.

SUPERVISOR OF HIGH SCHOOLS, STATE DEPARTMENT OF
PUBLIC INSTRUCTION, MADISON, WISCONSIN

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PREFACE

The busy teacher of high-school algebra often finds that limitations of time forbid the actual working out of all the exercises and problems in a text. This teachers' edition is designed to meet such a condition. In its plan it is brief and schematic rather than complete and detailed. Less time should be required to prepare a lesson from this outline than from a text giving the solutions in full.

For those exercises involving not more than one process in their solution the answers only are given. If two or more processes are required in a solution, an outline is given which covers each step of the work. Moreover, after each indicated step of the work, the result obtained by taking that step is set down. This scheme affords an easy method of tracing a solution.

In the solution of problems the meaning of each letter representing an unknown is clearly stated, the equations representing the conditions of the problems in terms of these letters are given, and the schematic solution of the equation or system always follows.

The authors and publishers desire the coöperation of teachers and school authorities in limiting the circulation of this book to actual teachers of algebra.

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1. Let l be the less number. Then $4l$ represents the greater. Therefore $l + 4l = 120$. Whence $l = 24$, and $4l = 96$.

2. Let n equal the number. Then $7n$ represents seven times the number. Therefore $n + 7n = 216$. Whence $n = 27$.

3. Let n equal the less number. Then $8n$ equals the greater. Therefore $n + 8n = 72$. Whence $n = 8$, and $8n = 64$.

4. Let x = the third number. Then $2x$ = the first, and $4x$ = the second. Therefore $x + 2x + 4x = 252$. Whence $x = 36$, $2x = 72$, and $4x = 144$.

5. Let x = the first number. Then $2x$ = the second, and $6x$ = the third. Therefore $x + 2x + 6x = 117$. Whence $x = 13$, $2x = 26$, and $6x = 78$.

6. Let x = the second number. Then $2x$ = the first number, and $3x$ = the third. Therefore $x + 2x + 3x = 192$. Whence $x = 32$, $2x = 64$, and $3x = 96$.

7. Let x = the first number. Then $5x$ = the second, and $20x$ = the third. Therefore $x + 5x + 20x = 312$. Whence $x = 12$, $5x = 60$, and $20x = 240$.

8. Let x = the first number. Then $3x$ = the second, and $4x$ = the third. Therefore $x + 3x + 4x = 208$. Whence $x = 26$, $3x = 78$, and $4x = 104$.

9. Let s = the son's age in years. Then $3s$ = the father's age in years. Therefore $s + 3s = 44$. Whence $s = 11$, and $3s = 33$.

10. Let s = the side of the square in feet. Then $4s = 160$. Whence $s = 40$.

11. Let w = the width of the rectangle in feet. Then $3w$ = the length. Therefore $2w + 6w = 216$. Whence $w = 27$, and $3w = 81$.

12. Let s = the side of the square in inches. Then the width of the rectangle is s , the length is $2s$, and the perimeter is $6s$. Therefore $6s = 258$. Whence $s = 43$, and the perimeter of the square = 172.

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- | | | |
|-------------------|---------------------------------|------------------------------------|
| 1. $3a + 4b.$ | 7. $4a^2b.$ | 13. $(x - y)\sqrt{7x}.$ |
| 2. $4b - 3a.$ | 8. $\frac{a + b}{ab}.$ | 14. $(a + b)^2.$ |
| 3. $b^2 - a^2.$ | 9. $a(2b - c).$ | 15. $a - b^2.$ |
| 4. $a^2 - b^3.$ | 10. $a(b + c).$ | 16. $\frac{3ab^2}{4a^3c}.$ |
| 5. $3a^2 - 2a^2.$ | 11. $7x - (a - b).$ | 17. $\frac{a}{3x} + \frac{4y}{c}.$ |
| 6. $\frac{a}{b}.$ | 12. $\sqrt{5a} + \sqrt[3]{7b}.$ | |

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|---|---|
| 1. $20 - 5 + 6 - 10 = 11.$ | 12. $(18 - 2) - (4 + 2 \cdot 8 - 18 \div 9) \div 6$
$= 16 - (4 + 16 - 2) \div 6 = 13.$ |
| 2. $16 - (8 - 2) = 16 - 6 = 10.$ | 13. $(16 - 6)(18 - 8) \div 100 \cdot 5 - 5$
$= 10 \cdot 10 \div 100 \cdot 5 - 5$
$= 1 \cdot 5 - 5 = 0.$ |
| 3. $14 - (16 - 8) + (12 - 4)$
$= 14 - 8 + 8 = 14.$ | 14. $(5 + 3)(5 - 3) \div 4 - 3$
$= 8 \cdot 2 \div 4 - 3 = 1.$ |
| 4. $6 \div 3 - 2 = 2 - 2 = 0.$ | 15. $3^2 - 2 \cdot 3 \cdot 1 + 1^2 = 9 - 6 + 1 = 4.$ |
| 5. $8 \cdot 6 \div 3 - 10 = 16 - 10 = 6.$ | 16. $8^2 - 2 \cdot 8 \cdot 3 + 3^2$
$= 64 - 48 + 9 = 25.$ |
| 6. $18 \div (2 \cdot 3) = 18 \div 6 = 3.$ | 17. $4 \cdot 3 - 2(6 - 2 \cdot 3) + 8$
$= 12 - 2 \cdot 0 + 8 = 20.$ |
| 7. $(6 - 3)(17 - 2 \cdot 5)$
$= 3(17 - 10) = 3 \cdot 7 = 21.$ | 18. $8(12 + 4 + 3 - 1) = 8 \cdot 18 = 144.$ |
| 8. $23 - 2 \cdot 6 - 4 \div 2 + 16$
$= 23 - 12 - 2 + 16 = 25.$ | 19. $3 + 2 \cdot 4 + (3 + 2)4$
$= 3 + 8 + 20 = 31.$ |
| 9. $18 \div (9 - 3) = 18 \div 6 = 3.$ | 20. $(8 - 2)3 + 8 - 2 \cdot 3$
$= 18 + 8 - 6 = 20.$ |
| 10. $(10 - 3)(16 - 3 \cdot 2 + 8 \div 4)$
$= 7(16 - 6 + 2) = 7 \cdot 12 = 84.$ | |
| 11. $14 - 3(16 - 2 \cdot 5) \div 6 + 8 \cdot 2$
$= 14 - 3(6) \div 6 + 16$
$= 14 - 3 + 16 = 27.$ | |
| 21. $3 \cdot 4 - 6 \cdot 0 + 2 \cdot 5 + 2 \cdot 7^2 - 2 \cdot 8 = 12 - 0 + 10 + 98 - 16 = 104.$ | |
| 22. $(12 + 24 \times 18 \div 3 + 6)(24 \div 4 + 3 - 2) = (12 + 144 + 6)(7) = 1134.$ | |

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- | | |
|---|---|
| 2. $4a + 3d = 4 \cdot 3 + 3 \cdot 7$
$= 12 + 21$
$= 33.$ | 5. $abcd - 5f^2 = 3 \cdot 1 \cdot 5 \cdot 7 - 5 \cdot 2^2$
$= 105 - 20$
$= 85.$ |
| 3. $ab + cd = 3 \cdot 1 + 5 \cdot 7$
$= 3 + 35$
$= 38.$ | 6. $\frac{d + c}{f} = \frac{7 + 5}{2} = 6.$ |
| 4. $c^2 - 3ab = 5^2 - 3 \cdot 3 \cdot 1$
$= 25 - 9$
$= 16.$ | 7. $\frac{cdf}{3f} + \frac{acf}{2a} = \frac{5 \cdot 7 \cdot 2}{3 \cdot 2} + \frac{3 \cdot 5 \cdot 2}{2 \cdot 3} = 16\frac{2}{3}.$ |
| | 8. $\frac{12b}{f} - \frac{c}{b} = \frac{12 \cdot 1}{2} - \frac{5}{1} = 6 - 5 = 1.$ |

9. $\frac{1}{c} + \frac{1}{d} + \frac{1}{f} = \frac{1}{5} + \frac{1}{7} + \frac{1}{2} = \frac{14 + 10 + 35}{70} = \frac{59}{70}.$
10. $\frac{f^2}{ac} + \frac{6cd}{af} = \frac{2^2}{3 \cdot 5} + \frac{6 \cdot 5 \cdot 7}{3 \cdot 2} = 35\frac{4}{5}.$
11. $\frac{a^2 + b^2 + c^2 + d^2}{a + b + c + d} = \frac{3^2 + 1^2 + 5^2 + 7^2}{3 + 1 + 5 + 7} = \frac{9 + 1 + 25 + 49}{16} = 5\frac{1}{4}.$
12. $5f^3 + 4f^2 - 4f - 5 = 5 \cdot 2^3 + 4 \cdot 2^2 - 4 \cdot 2 - 5 = 40 + 16 - 8 - 5 = 43.$
13. $3f^5 - 9f^4 + 11f^3 - 11f^2 + 13f - 20$
 $= 3 \cdot 2^5 - 9 \cdot 2^4 + 11 \cdot 2^3 - 11 \cdot 2^2 + 13 \cdot 2 - 20$
 $= 96 - 144 + 88 - 44 + 26 - 20 = 2.$
14. $af = 3^2 = 9.$
15. $df + fa = 7^2 + 2^3 = 49 + 8 = 57.$
16. $\frac{d + fc}{3c + a} = \frac{7 + 2^5}{3 \cdot 5 + 3} = \frac{7 + 32}{15 + 3} = \frac{39}{18} = 2\frac{1}{6}.$
17. $\frac{4a + 3b + 2d}{c + f + 2} = \frac{4 \cdot 4 + 3 \cdot 0 + 2 \cdot 7}{5 + 8 + 2} = \frac{30}{15} = 2.$
18. $\frac{b}{a + c + d} = \frac{0}{4 + 5 + 7} = 0.$
19. $\frac{ab}{c} + \frac{bd}{a} + \frac{bf}{cd} = \frac{4 \cdot 0}{5} + \frac{0 \cdot 7}{4} + \frac{0 \cdot 8}{5 \cdot 7} = 0.$
20. $\sqrt{a} + \sqrt{2f}$
 $= \sqrt{4} + \sqrt{2 \cdot 8}$
 $= 2 + 4 = 6.$
21. $c\sqrt{2af} = 5\sqrt{2 \cdot 4 \cdot 8}$
 $= 40.$
22. $\sqrt{c^2 - a^2} = \sqrt{5^2 - 4^2}$
 $= \sqrt{25 - 16} = 3.$
23. $(a + f)(c + d)$
 $= (4 + 8)(5 + 7) = 144.$
24. $ab(a + b)$
 $= 4 \cdot 0(4 + 0) = 0.$
25. $(a + c)(a + c)$
 $= (4 + 5)(4 + 5) = 81.$
26. $a + c(a + c)$
 $= 4 + 5(4 + 5)$
 $= 4 + 45 = 49.$
27. $(a + c)a + c$
 $= (4 + 5)4 + 5$
 $= 36 + 5 = 41.$
28. $a + ca + c$
 $= 4 + 5 \cdot 4 + 5$
 $= 4 + 20 + 5 = 29.$
29. $(f - a)^2$
 $= (8 - 4)^2 = 4^2 = 16.$
30. $(d - a)^3$
 $= (7 - 4)^3 = 3^3 = 27.$
31. $a^2 - 2ac + c^2$
 $= 4^2 - 2 \cdot 4 \cdot 5 + 5^2$
 $= 16 - 40 + 25 = 1.$
32. $a^2 + 2ab + b^2$
 $= 4^2 + 2 \cdot 4 \cdot 0 + 0^2$
 $= 16 + 0 + 0 = 16.$
33. $c^3 - 3c^2a + 3ca^2 - a^3$
 $= 5^3 - 3 \cdot 5^2 \cdot 4 + 3 \cdot 5 \cdot 4^2 - 4^3$
 $= 125 - 300 + 240 - 64 = 1.$
34. $d\sqrt[3]{2af}$
 $= 7\sqrt[3]{2 \cdot 4 \cdot 8}$
 $= 7 \cdot 4 = 28.$

35. $\sqrt{a^2 + ac}$
 $= \sqrt{4^2 + 4 \cdot 5}$
 $= \sqrt{36} = 6.$
36. $\sqrt[3]{ad + 36}$
 $= \sqrt[3]{4 \cdot 7 + 36}$
 $= \sqrt[3]{64} = 4.$
37. $13x - 5y = 11.$
 $13 \cdot 2 - 5 \cdot 3$
 $= 26 - 15 = 11. \text{ Yes.}$
38. $7x - 9 = 3x + 25.$
 $7 \cdot 8 - 9 = 3 \cdot 8 + 25.$
 $56 - 9 = 24 + 25.$
 $47 = 49. \text{ No.}$
39. $x^2 - 5x + 6 = 0.$
 $2^2 - 5 \cdot 2 + 6 = 0. \text{ Yes.}$
 $3^2 - 5 \cdot 3 + 6 = 0. \text{ Yes.}$
 $4^2 - 5 \cdot 4 + 6 = 2. \text{ No.}$
40. $x^2 - 7x + 12 = 0.$
 $2^2 - 7 \cdot 2 + 12 = 2. \text{ No.}$
 $3^2 - 7 \cdot 3 + 12 = 0. \text{ Yes.}$
 $4^2 - 7 \cdot 4 + 12 = 0. \text{ Yes.}$
41. $2x^2 - 5x - 3 = 0.$
 $2 \cdot 4^2 - 5 \cdot 4 - 3 = 9. \text{ No.}$
 $2 \cdot 3^2 - 5 \cdot 3 - 3 = 0. \text{ Yes.}$
 $2(\frac{1}{2})^2 - 5(\frac{1}{2}) - 3$
 $= \frac{1}{2} - \frac{5}{2} - 3 = -5. \text{ No.}$
42. $A = \frac{ab}{2} = \frac{11 \cdot 14}{2} = 77.$
 $\therefore \text{the area} = 77 \text{ square inches.}$
43. (a) $A = \pi r^2 = \frac{22}{7} \cdot 7^2 = 154.$
 $\therefore \text{the area} = 154 \text{ square inches.}$
(b) $\pi r^2 = \frac{22}{7} \cdot (\frac{1}{2})^2 = \frac{11}{14}.$
 $\therefore \text{the area} = \frac{11}{14} \text{ square inches.}$
44. (a) $V = \pi r^2 h = \frac{22}{7} \cdot 3^2 \cdot 5$
 $= \frac{22 \cdot 9 \cdot 5}{7} = 141\frac{3}{7}.$
 $\therefore \text{the volume} = 141\frac{3}{7} \text{ cubic inches.}$
(b) $\pi r^2 h = \frac{22}{7} \cdot 4^2 \cdot 7 = 352.$
 $\therefore \text{the volume} = 352 \text{ cubic inches.}$
45. $S = \frac{at^2}{2} = \frac{32 \cdot 5^2}{2} = 400.$
 $\therefore \text{the body falls 400 feet in 5 seconds.}$
46. $S = \frac{at^2}{2} = \frac{32 \cdot (16)^2}{2} = 4096.$
 $\therefore \text{the height of the balloon} = 4096 \text{ feet.}$
47. $H = \frac{d^2 n}{6} = \frac{4^2 \cdot 4}{6} = 10\frac{2}{3}.$
 $\therefore \text{the engine is } 10\frac{2}{3} \text{ horse power.}$

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1. (a) $14^\circ - 5^\circ = +9^\circ.$
(b) $14^\circ - 10^\circ = +4^\circ.$
(c) $14^\circ - 18^\circ = -4^\circ.$
2. (a) $-16^\circ + 7^\circ = -9^\circ.$
(b) $-16^\circ + 12^\circ = -4^\circ.$
(c) $-16^\circ + 25^\circ = +9^\circ.$
3. (a) $-16^\circ - 7^\circ = -23^\circ.$
(b) $-16^\circ - 12^\circ = -28^\circ.$
(c) $-16^\circ - 25^\circ = -41^\circ.$
4. $13^\circ + 7^\circ = 20^\circ.$
 $20 \cdot 69 \text{ miles} = 1380 \text{ miles.}$
5. $20^\circ - 5^\circ = 15^\circ.$
 $20^\circ - 10^\circ = 10^\circ.$
 $20^\circ - 15^\circ = 5^\circ.$
 $20^\circ - 20^\circ = 0^\circ.$
 $20^\circ - 25^\circ = -5^\circ.$
 $20^\circ - 30^\circ = -10^\circ.$
 $20 + 15 = 35.$
 $35 \div 5 = 7; \text{ after seven days.}$
6. (a) $\begin{cases} + \$4200 = \text{value of property.} \\ - \$2300 = \text{amount of debts.} \end{cases}$
(b) $+ \$4200 + (- \$2300)$
 $= + \$1900.$

$$7. (a) \begin{cases} - \$4200 = \text{amount of debts.} \\ + \$2300 = \text{value of property.} \end{cases}$$

$$(b) - \$4200 + \$2300 = - \$1900.$$

$$8. - 12^\circ + 9^\circ = - 3^\circ.$$

$$- 12^\circ + 12^\circ = 0^\circ.$$

$$- 12^\circ + 18^\circ = + 6^\circ.$$

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1. $4^2 - (-2)^2 = 16 - 4 = 12.$
2. $3^3 - (-2)^3 = 27 + 8 = 35.$
3. $(5 - 3)(3 + 2) = 2 \cdot 5 = 10.$
4. $(6 - 2)(7 - 3) \div (4 - 9) = 4 \cdot 4 \div (-5) = -3\frac{1}{5}.$
5. $(6 - \frac{1}{2})(5 + \frac{1}{3}) = \frac{11}{2} \cdot \frac{16}{3} = 29\frac{1}{3}.$
6. $(-1)^2 + (-1)^3 + (-2)^2 + (-2)^3 = 1 - 1 + 4 - 8 = -4.$
7. $9 + 3 \cdot 2 + 18 \div (-3) = 9 + 6 - 6 = 9.$
8. $5^2 - 4 \div (-2) \div 6(-3) = 25 - 1 = 24.$
9. $3 \cdot 6 \div 9 - 2 \cdot 6 \div 4 + (-3)^2 = 2 - 3 + 9 = 8.$
10. $3^2 + 2 \cdot 3(-4) \div (-4)^2 = 9 - 24 \div (16) = 7\frac{1}{2}.$
11. $(+2)^2 - 2(2)(-3) + (-3)^2 = 4 + 12 + 9 = 25.$
12. $3(4)^2(-5) - (-5)^3 = 3 \cdot 16(-5) + 125 = -115.$
13. $3^3 + 3(3)^2(-2) + 3(3)(-2)^2 + (-2)^3 = 27 - 54 + 36 - 8 = 1.$
14. $4^3 - 3(4)^2(-3) + 3(4)(-3)^2 - (-3)^3 = 64 + 144 + 108 + 27 = 343.$
15. $3^3 - 3(3)^2(0) + 3(3)(0)^2 - 0^3 = 27 - 0 + 0 - 0 = 27.$
16. $5^2 + 3(5)^2(-4) + 3(5)(-4)^2 - (-4)^3 = 25 - 300 + 240 + 64 = 29.$
17. $y^2 = (-2)^2 = 4.$
18. $y^3 = (-2)^3 = -8.$
19. $y^4 = (-2)^4 = 16.$
20. $y^5 = (-2)^5 = -32.$
21. $2y^2 = 2(-2)^2 = 8.$
22. $2y^3 = 2(-2)^3 = -16.$
23. $5x^2y^2 = 5 \cdot (3)^2(-2)^2 = 180.$
24. $4x^2y^4 = 4(3)^2(-2)^4 = 576.$
25. $x^2 - y^2 = 3^2 - (-2)^2$
 $= 9 - 4 = 5.$
26. $x^3 - y^3 = 3^3 - (-2)^3$
 $= 27 + 8 = 35.$
27. $x^2 + 2xy + y^2$
 $= 3^2 + 2 \cdot 3(-2) + (-2)^2$
 $= 9 - 12 + 4 = 1.$
28. $x^2 - 2xy + y^2$
 $= 3^2 - 2 \cdot 3(-2) + (-2)^2$
 $= 9 + 12 + 4 = 25.$
29. $(x + y)(x - y)$
 $= (3 - 2)(3 + 2)$
 $= 1 \cdot 5 = 5.$
30. $x^3 + 3x^2y + 3xy^2 + y^3$
 $= 3^3 + 3 \cdot 3^2(-2) + 3 \cdot 3(-2)$
 $+ (-2)^3$
 $= 27 - 54 + 36 - 8 = 1.$
31. $4x - 2 = 2x + 8.$
 $4 \cdot 5 - 2 = 2 \cdot 5 + 8.$
 $18 = 18. \text{ Yes.}$
32. $3x - 5 = 2x + 8.$
 $3(-9) - 5 = 2(-9) + 8.$
 $-32 = -10. \text{ No.}$

33. $x^2 - x - 12 = 0$.
 $4^2 - 4 - 12 = 0$. Yes.
 $(-8)^2 - (-8) - 12$
 $= 64 + 8 - 12 = 60$. No.
 $(-4)^2 - (-4) - 12$
 $= 16 + 4 - 12 = 8$. No.
34. $3x^2 + 19x = 14$.
 $3(\frac{2}{3})^2 + 19(\frac{2}{3})$
 $= \frac{4}{3} + \frac{38}{3} = 14$. Yes.
 $3(2)^2 + 19 \cdot 2$
 $= 12 + 38 = 50$. No.
 $3(-7)^2 + 19 \cdot (-7) = 14$. Yes.
35. $27^\circ - 15^\circ = 12^\circ$, noon.
 $27^\circ - 27^\circ = 0^\circ$, 4.00 P.M.
 $27^\circ - 30^\circ = -3^\circ$, 5.00 P.M.
 $27^\circ + 12^\circ = 39^\circ$.
 $39 \div 3 = 13$.
13 hours later than 7.00 A.M is
8.00 P.M.
36. $1800 - 1200 + 1750 - 400 + 820$
 $= 2770$, number of feet higher
than at first.
37. 300 B.C. = - 300.
1727 A.D. = + 1727.
38. - 450 means 450 B.C.
+ 1917 means 1917 A.D.
1917 years + 450 years
= 2367 years = difference.
39. $+12 + (+3) = 15$, number of
miles per hour toward bow.
 $12 + (-3) = 9$, number of miles
per hour toward stern.
40. - 500 = weight of balloon.
 $10(+150) = +1500$ = weight
of men.
+ 1500 pounds - 500 pounds
= + 1000 pounds, weight of
men and balloon.

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1. $a + 4b - c$.
2. $4x - 2b + 5y + 10$.
3. $3ab^2 + 2bx - cy + 4a^2b$.
4. $5x^3y - 5xy^3 + c^3y - 2cy^3$.
5. $6x - 3a + 2b - 5x + 3y = x - 3a + 2b + 3y$.
6. $5a - 4b + 3c + 7b - 2c^2 = 5a + 3b + 3c - 2c^2$.
7. $3a^2 + 2b^2 - 7c^2 - 4b^2 + 5a^2 = 8a^2 - 2b^2 - 7c^2$.
8. $5a^3b - 5ab^3 - 3a^2b + 3ab^2 + 4a^3b + 2ab^3$
 $= 9a^3b - 3ab^3 - 3a^2b + 3ab^2$.
9. $11 + 5 - 9 - 3 + 16 - 25 = 32 - 37 = -5$.
10. $16ac - 9xy + 5ac - 2xy - 6ac + 11xy = 15ac$.
11. $7x + 6a - 15a - 8x + 3x = 2x - 9a$.
12. $12y - 17y + 10b + 20y - 25b = 15y - 15b$.
13. $4ab - 8xy - 12ab + 15ab - xy = 7ab - 9xy$.
14. $14x^2 - 13y^2 + x^2 - 5x^2 + y^2 = 10x^2 - 12y^2$.
15. $5z^2 - 4y^2 - 11y^2 + z^2 - 7y^2 = 6z^2 - 22y^2$.
16. $7c^2d - 5a^2b + 6c^2d - a^2b + 9a^2b = 13c^2d + 3a^2b$.
17. $-4a^2b^2 + 6x^2y^2 - 15a^2b^2 + 3x^2y^2 + 0a^2b^2 = -19a^2b^2 + 9x^2y^2$.
18. $-b^3 - 19a + 17b^3 + b^3 - 0b^3 + 13a = 17b^3 - 6a$.
19. $12a^3b + 6c^2d - a^3b + 16a^3b - 13a^3b - 25c^2d = 14a^3b - 19c^2d$.

$$20. 11\sqrt{a} - 16\sqrt{a} + 21\sqrt{a} - \sqrt{a} = 15\sqrt{a}.$$

$$21. 3\sqrt{x-y} - \sqrt{x-y} + 8\sqrt{x-y} - 7\sqrt{x-y} = 3\sqrt{x-y}.$$

$$22. 3(a+b) - 5(a+b) + 8(a+b) = 6(a+b).$$

$$23. -7(a-2b) + (a-2b) + 12(a-2b) = 6(a-2b).$$

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$$\begin{array}{r} 1. \quad x + 3 \\ 2x - 6 \\ 3x - 5 \\ \hline 6x - 8 \end{array}$$

$$\begin{array}{r} 2. \quad x - y - 3 \\ 2x + y - 8 \\ 7x - 4y + 10 \\ \hline 10x - 4y - 1 \end{array}$$

$$\begin{array}{r} 3. \quad x + y + z \\ x - 3y + 4z \\ 3x + 4y - 7z \\ \hline 5x + 2y - 2z \end{array}$$

$$\begin{array}{r} 4. \quad 2x + 5y - z \\ 3x - 8y + 6z \\ x - y - z \\ \hline 6x - 4y + 4z \end{array}$$

$$\begin{array}{r} 5. \quad 3x - 1 \\ -2x + 4 \\ 6x - 8 \\ 3x + 3 \\ \hline 10x - 2 \end{array}$$

$$\begin{array}{r} 6. \quad x^2 - 9 \\ -4x^2 + 16 \\ 8x^2 + 4 \\ -2x^2 + 3 \\ \hline 3x^2 + 14 \end{array}$$

$$\begin{array}{r} 19. \quad a - x - 3c \\ 3a - 7x + 4 \\ -2a + 7c + 10 \\ a - 2x - c - 11 \\ \hline 3a - 10x + 3c + 3 \end{array}$$

$$\begin{array}{r} 7. \quad 5x + 5y \\ 4x - 9y + 6z \\ 3x - 3y - 3z \\ \hline 12x - 7y + 3z \end{array}$$

$$\begin{array}{r} 8. \quad 7x + 3z - 6y \\ 5x - 4z \\ 2x - 5z + 6y \\ \hline 14x - 6z \end{array}$$

$$\begin{array}{r} 9. \quad x^2 - x - 2 \\ 3x^2 - 4x + 5 \\ 5x^2 + 8x - 7 \\ \hline 9x^2 + 3x - 4 \end{array}$$

$$\begin{array}{r} 10. \quad x^2 - 4x + 1 \\ -2x^2 + 3x + 8 \\ -11x^2 + 10x - 18 \\ \hline -12x^2 + 9x - 9 \end{array}$$

$$\begin{array}{r} 11. \quad -4c^2 + 3c - 5 \\ c^2 + c + 1 \\ -2c^2 + 4c + 3 \\ -5c^2 - 2c + 5 \\ \hline -10c^2 + 6c + 4 \end{array}$$

$$\begin{array}{r} 12. \quad 4x - 5y - 5z \\ 6y - 2z \\ 7x - 6y - 9z \\ \hline 11x - 5y - 16z \end{array}$$

$$\begin{array}{r} 13. \quad x + 2z + 3y \\ x - 3z + y \\ -2x + z - 4y \\ \hline 0 \end{array}$$

$$\begin{array}{r} 14. \quad 5x + 7z - 6y \\ x - 11z + 2y \\ 9z - 5y \\ \hline 6x + 5z - 9y \end{array}$$

$$\begin{array}{r} 15. \quad 8a - 7b - 6c \\ -4a - 3b + 5c \\ 3b + 7c \\ \hline 4a - 7b + 6c \end{array}$$

$$\begin{array}{r} 16. \quad 9ac - bc \\ -3ac + 7ab \\ -ac - 12ab \\ \hline 5ac - bc - 5ab \end{array}$$

$$\begin{array}{r} 17. \quad a^2 - 4a + 10 \\ -6a^2 + 5a + 4 \\ 2a^2 + 3a - 16 \\ \hline -3a^2 + 4a - 2 \end{array}$$

$$\begin{array}{r} 18. \quad -a^2 + 3a + 9 \\ 6a^2 - 4a - 13 \\ 3a^2 - a \\ \hline 8a^2 - 2a - 4 \end{array}$$

$$\begin{array}{r} 20. \quad a + x + c + 5 \\ 2a + 2x + 2c + 10 \\ a - x - c - 7 \\ -2a - 3x + 4c + 7 \\ \hline 2a - x + 6c + 15 \end{array}$$

$$\begin{array}{r}
 21. \quad x - 1 - 3c \\
 \quad \quad x \quad - 2c - \quad a \\
 \quad \quad - 5 - 3c + \quad a \\
 \hline
 2x - 7 \quad \quad + 11a \\
 4x - 13 - 8c + 11a
 \end{array}$$

$$\begin{array}{r}
 22. \quad 2x + 3y + 5z \\
 \quad \quad x - y + 3z \\
 \hline
 3x - 2y + 10z \\
 6x \quad \quad + 18z
 \end{array}$$

$$\begin{array}{r}
 23. \quad 4x - 20z \\
 \quad \quad 8x \quad \quad + 9y \\
 \quad \quad \quad \quad 5z - 3y \\
 \hline
 12x - 15z + 6y
 \end{array}$$

$$\begin{array}{r}
 24. \quad a - 3(x - y) + z \\
 - 10a + 4(x - y) \quad + 5 \\
 \quad \quad - 2(x - y) \quad + 6 \\
 \hline
 - 9a - (x - y) + z + 11
 \end{array}$$

$$\begin{array}{r}
 25. \quad a + d + 2(b - c) \\
 - 12a + 6d + 7(b - c) \\
 \hline
 11a \quad \quad - 5(b - c) \\
 \quad \quad + 7d + 4(b - c)
 \end{array}$$

$$\begin{array}{r}
 26. \quad 7a^2 - 13b^2 + 12c^2 \\
 - a^2 + 15b^2 - 7c^2 \\
 \hline
 \quad \quad + 3b^2 + 5c^2 \\
 6a^2 + 5b^2 + 10c^2
 \end{array}$$

$$\begin{array}{r}
 27. \quad x^2 + 2xy + y^2 \\
 - 4x^2 - 5xy - y^2 \\
 \hline
 + 16x^2 - 8xy + y^2 \\
 13x^2 - 11xy + y^2
 \end{array}$$

$$\begin{array}{r}
 28. \quad 5ab - a^2 + b^2 \\
 + 4ab + 5a^2 - 9b^2 \\
 - 2ab \quad \quad - 2b^2 \\
 \hline
 7ab + 4a^2 - 10b^2
 \end{array}$$

$$\begin{array}{r}
 29. \quad 5x^2 - 6x + 11 \\
 - 3x^2 - 4x - 8 \\
 \quad \quad + 7x - 18 \\
 \quad \quad + 13x \\
 \hline
 2x^2 + 10x - 15
 \end{array}$$

$$\begin{array}{r}
 30. \quad 12c^2 - 10bc + 8b^2 \\
 - c^2 + bc - 6b^2 + c^3 \\
 \hline
 11c^2 - 9bc + 2b^2 - 10c^3
 \end{array}$$

$$\begin{array}{r}
 31. \quad 4x^2y - xy + y^2 \\
 - 2x^2y - 3xy \quad + 4xy^2 \\
 + x^2y \quad \quad + 4y^2 + 3xy^2 \\
 \hline
 3x^2y - 4xy + 5y^2 + 7xy^2
 \end{array}$$

$$\begin{array}{r}
 32. \quad \frac{a}{2} + \frac{b}{3} - \frac{c}{4} \\
 - \frac{2a}{3} + \frac{b}{2} + c \\
 \hline
 \frac{3a}{4} - b + \frac{c}{2} + 7 \\
 \hline
 \frac{7a}{12} - \frac{b}{6} + \frac{5c}{4} + 7
 \end{array}$$

$$33. ax + 3x = (a + 3)x.$$

$$34. 2ax + x = (2a + 1)x.$$

$$35. 3cx - x = (3c - 1)x.$$

$$36. ax + bx + cx = (a + b + c)x.$$

$$37. 2ax - 3x + bx = (2a - 3 + b)x.$$

$$38. 3ax - 4cx + x = (3a - 4c + 1)x.$$

$$39. 3ax - bx - x + a^2x = (3a - b - 1 + a^2)x.$$

$$40. bx - 5cx - x - 4bx = (-3b - 5c - 1)x.$$

$$41. a(b + c) + 3(b + c) = (a + 3)(b + c).$$

$$42. 4(a - x) - 5b(a - x) = (4 - 5b)(a - x).$$

$$43. 8a(a + 3b) - 1(a + 3b) = (8a - 1)(a + 3b).$$

$$44. 7b(x^2 + y^2) - a(x^2 + y^2) + (x^2 + y^2) = (7b - a + 1)(x^2 + y^2).$$

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- | | |
|---|--|
| <p>1. $x + 5 = 7$.
Subtracting 5 in both members,
$x = 2$.</p> <p>2. $y - 9 = 6$.
Adding 9 in both members,
$y = 15$.</p> <p>3. $4x - 6 = 14$.
Adding 6 in both members,
$4x = 20$.
Whence $x = 5$.</p> <p>4. $4x + 2 = 3x - 4$.
Subtracting $3x$ and 2 in both
members, $x = -6$.</p> <p>5. $5x - 2 = 4x + 5$.
Subtracting $4x$ and adding 2 in
both members,
$x = 7$.</p> <p>6. $5p = 3p + 8$.
Subtracting $3p$ in both members,
$2p = 8$.
Whence $p = 4$.</p> <p>7. $4x + 2 = 2x$.
Subtracting $2x$ and 2 in both
members, $2x = -2$.
Whence $x = -1$.</p> <p>8. $3n = n + 3$.
$2n = 3$.
$n = \frac{3}{2}$.</p> <p>9. $4x + 2 = x + 5$.
$3x = 3$.
$x = 1$.</p> <p>10. $5x + 6 = 3x + 10$.
$2x = 4$.
$x = 2$.</p> | <p>11. $4x - 7 = 2x + 3$.
$2x = 10$.
$x = 5$.</p> <p>12. $3k - 5 = 7 - k$.
$4k = 12$.
$k = 3$.</p> <p>13. $6 - a = 9 - 4a$.
$3a = 3$.
$a = 1$.</p> <p>14. $2x = 1 - x$.
$3x = 1$.
$x = \frac{1}{3}$.</p> <p>15. $5 + 3y = y + 4$.
$2y = -1$.
$y = -\frac{1}{2}$.</p> <p>16. $2x + 8 = 5x - 1$.
$9 = 3x$.
$x = 3$.</p> <p>17. $5 + 2q = 5q - 1$.
$6 = 3q$.
$q = 2$.</p> <p>18. $2 - 3n = n + 1$.
$1 = 4n$.
$n = \frac{1}{4}$.</p> <p>19. $4x - 2 = 7 + x$.
$3x = 9$.
$x = 3$.</p> <p>20. $4x + 3 - x + 5 = x - 10$.
Combining, $3x + 8 = x - 10$.
$2x = -18$.
$x = -9$.</p> <p>21. $3 + 2z - 1 = 12 - 3z$.
$2z + 2 = 12 - 3z$.
$5z = 10$.
$z = 2$.</p> |
|---|--|

$$\begin{array}{ll}
 22. \quad 2x + 3x + x = x + 9 + 2x. & 25. \quad 3x + 12x + 17 - x + 4x = 107. \\
 \quad \quad 6x = 3x + 9. & \quad \quad 18x + 17 = 107. \\
 \quad \quad 3x = 9. & \quad \quad 18x = 90. \\
 \quad \quad x = 3. & \quad \quad x = 5. \\
 23. \quad 4y - 3 + 3y - 4 = 6y - 8. & 26. \quad 4k - 3 = 5k - 16 - 43k - 71. \\
 \quad \quad 7y - 7 = 6y - 8. & \quad \quad 4k - 3 = -38k - 87. \\
 \quad \quad y = -1. & \quad \quad 42k = -84. \\
 24. \quad 36 - 8x + 4 - 9x = 2x + 10. & \quad \quad k = -2. \\
 \quad \quad 40 - 17x = 2x + 10. & \\
 \quad \quad 30 = 19x. & \\
 \quad \quad x = \frac{30}{19}. &
 \end{array}$$

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1. Let x = the number of papers sold by second boy.
 Then $x + 5$ = the number of papers sold by first boy.
 But by the conditions of the problem,
 $x + x + 5 = 67$.
 Whence $x = 31$; $x + 5 = 36$.
2. Let x = the number of students in smaller group.
 Then $x + 3$ = the number of students in larger group.
 But by the conditions of the problem,
 $x + x + 3 = 51$.
 Whence $x = 24$; $x + 3 = 27$.
3. Let x = the less number.
 Then $5x$ = the greater number.
 But $x + 5x = 30$.
 Whence $x = 5$; $5x = 25$.
4. Let x = the less number.
 Then $3x$ = the greater number.
 But $x + 3x = 72$.
 Whence $x = 18$; $3x = 54$.
5. Let x = the less number.
 Then $5x$ = the greater number.
 But $5x - x = 52$.
 Whence $x = 13$; $5x = 65$.
6. Let x = the number of cars in shorter train.
 Then $2x$ = the number of cars in longer train.
 But $x + 2x = 72$.
 Whence $x = 24$; $2x = 48$.

7. Let $x =$ the side in feet.
 Then $x + x + x + x = 24$.
 Whence $x = 6$.
8. Let $x =$ the width in feet.
 Then $3x =$ the length in feet.
 But $x + 3x + x + 3x = 64$.
 Whence $x = 8$; $3x = 24$.
9. Let $x =$ the breadth in feet.
 Then $5x =$ the length in feet.
 But $x + 5x + x + 5x = 78$.
 Whence $x = 6\frac{1}{2}$; $5x = 32\frac{1}{2}$.
10. Let $x =$ the width in feet.
 Then $x + 12 =$ the length in feet.
 But $x + x + 12 + x + x + 12 = 96$.
 Whence $x = 18$; $x + 12 = 30$.
11. Let $x =$ the length in feet of the shorter piece.
 Then $x + 8 =$ the length in feet of the longer piece.
 But $x + x + 8 = 36$.
 Whence $x = 14$; $x + 8 = 22$.
12. Let $x =$ the length in inches of the shorter piece.
 Then $3x + 14 =$ the length in inches of the longer piece.
 But $x + 3x + 14 = 120$.
 Whence $x = 26\frac{1}{2}$; $3x + 14 = 93\frac{1}{2}$.
13. Let $x =$ the first number.
 Then the other numbers will be $2x$ and $8x$.
 But $x + 2x + 8x = 22$.
 Whence $x = 2$; $2x = 4$; $8x = 16$.
14. Let $x =$ the third number.
 Then $2x =$ the first number,
 and $4x =$ the second number.
 But $x + 2x + 4x = 63$.
 Whence $x = 9$; $2x = 18$; $4x = 36$.
15. Let $x =$ the number of men in the first squad.
 Then $x + 5 =$ the number in the second squad,
 and $x + 5 + 10 = x + 15$, the number in the third squad.
 But $x + x + 5 + x + 15 = 95$.
 Whence $x = 25$; $x + 5 = 30$; $x + 15 = 40$.

16. Let x = the number of cars in the shorter section.
 Then $2x + 2$ = the number of cars in the longer section.
 But $x + 2x + 2 = 62$.
 Whence $x = 20$; $2x + 2 = 42$.
17. Let x = the less number.
 Then $x + 1$ = the greater number.
 But $x + x + 1 = 45$.
 Whence $x = 22$; $x + 1 = 23$.
18. Let x = the less number.
 Then $x + 2$ = the greater number.
 But $x + x + 2 = 36$.
 Whence $x = 17$; $x + 2 = 19$.
19. Let $x, x + 1, x + 2$, be the numbers.
 Then $x + x + 1 + x + 2 = 72$.
 Whence $x = 23$; $x + 1 = 24$; $x + 2 = 25$.
20. Let $x, x + 2, x + 4$, be the numbers.
 Then $x + x + 2 + x + 4 = 48$.
 Whence $x = 14$; $x + 2 = 16$; $x + 4 = 18$.
21. Let $x, x + 2, x + 4, x + 6, x + 8$, be the numbers.
 Then $x + x + 2 + x + 4 + x + 6 + x + 8 = 85$.
 Whence $x = 13$; $x + 2$, etc. = 15, 17, 19, 21.
22. Let x = the length in inches of the shorter piece.
 Then $x + 1$ and $x + 2$ = the lengths in inches of the other pieces.
 But $x + x + 1 + x + 2 = 36$.
 Whence $x = 11$; $x + 1 = 12$; $x + 2 = 13$.

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|--|---|--|--|
| 1. $a + 3$
$\frac{a + 2}{1}$ | 4. $-4a + 9$
$\frac{3a + 7}{-7a + 2}$ | 7. $4x - 9$
$\frac{9x + 4}{-5x - 13}$ | 10. $5xy$
$\frac{3xy - a}{2xy + a}$ |
| 2. $a - 6$
$\frac{a - 4}{-2}$ | 5. $-2x + 5$
$\frac{2x - 5}{-4x + 10}$ | 8. $4a - 2$
$\frac{4a}{-2}$ | 11. $x + 4$
$\frac{0}{x + 4}$ |
| 3. $3a + 7$
$\frac{2a - 3}{a + 10}$ | 6. $3x + 6$
$\frac{6x - 3}{-3x + 9}$ | 9. $5c$
$\frac{2ab - 5c}{-2ab + 10c}$ | 12. 0
$\frac{2x - 5}{-2x + 5}$ |

$$\begin{array}{r} 13. \quad 2x - 2y + z \\ \quad x - 2y - 3z \\ \hline \quad x \quad \quad + 4z \end{array}$$

$$\begin{array}{r} 16. \quad 3a - 5b \\ \quad 5a - 4b + 3c \\ \hline -2a - \quad b - 3c \end{array}$$

$$\begin{array}{r} 19. \quad -x - y + b \\ \quad a - x + y \\ \hline -a \quad \quad -2y + b \end{array}$$

$$\begin{array}{r} 14. \quad 4x - 5y - 3z \\ \quad 4x - 8y + 2z \\ \hline \quad 3y - 5z \end{array}$$

$$\begin{array}{r} 17. \quad 5x^2 + 8x - 2 \\ \quad 3x^2 - x - 5 \\ \hline 2x^2 + 9x + 3 \end{array}$$

$$\begin{array}{r} 20. \quad 4a - b + 5c \\ \quad 3a - 2b + c + 6 \\ \hline a + b + 4c - 6 \end{array}$$

$$\begin{array}{r} 15. \quad 4a + b + 2c \\ \quad 3a - 2b \\ \hline a + 3b + 2c \end{array}$$

$$\begin{array}{r} 18. \quad -5x^2 + 8x - 13 \\ \quad 7x^2 - 4x + 11 \\ \hline -12x^2 + 12x - 24 \end{array}$$

$$\begin{array}{r} 21. \quad a - 3b \\ \quad 2a + 2b - 2c - 4 \\ \hline -a - 5b + 2c + 4 \end{array}$$

$$\begin{array}{r} 22. \quad -3a + 4x - 3c - 11 \\ \quad 2a - 3x + 4c \\ \hline -5a + 7x - 7c - 11 \end{array}$$

$$\begin{array}{r} 25. \quad 4x^2y - 5xy^2 - y^3 \\ \quad -3x^2y - 3xy^2 + 3x^3 \\ \hline 7x^2y - 2xy^2 - y^3 - 3x^3 \end{array}$$

$$\begin{array}{r} 23. \quad c - a - b \\ \quad -c + a + b \\ \hline 2c - 2a - 2b \end{array}$$

$$\begin{array}{r} 26. \quad 3a^3 + 4c^3 + 5a^2c + 2ac^2 \\ \quad a^3 - c^3 - 3a^2c + 3ac^2 \\ \hline 2a^3 + 5c^3 + 8a^2c - ac^2 \end{array}$$

$$\begin{array}{r} 24. \quad -7a^2 + 8ab + 3b^2 \\ \quad 2a^2 - 3ab + b^2 \\ \hline -9a^2 + 11ab + 2b^2 \end{array}$$

$$\begin{array}{r} 27. \quad 2x + 5y - 3z \\ \quad x - 2y + z \\ \hline x + 7y - 4z \end{array}$$

$$\begin{array}{r} 28. \quad 5x - 2y - 4z \\ \quad 7x - 9y - 3z \\ \hline -2x + 7y - z \end{array}$$

$$\begin{array}{r} 33. \quad 0 \\ \quad 2x - 4y - z \\ \hline -2x + 4y + z \end{array}$$

$$\begin{array}{r} 38. \quad x - y - z \\ \quad 5x + 3y - 8z \\ \hline -4x - 4y + 7z \end{array}$$

$$\begin{array}{r} 29. \quad 6a - 3b \\ \quad 5a - 4b + 6c \\ \hline a + b - 6c \end{array}$$

$$\begin{array}{r} 34. \quad 2c^2 - 3ac + 4a^2 \\ \quad 3c^2 - 2ac + a^2 \\ \hline -c^2 - ac + 3a^2 \end{array}$$

$$\begin{array}{r} 39. \quad 4 - 8x^2 \\ \quad -6 + x^2 + 5x \\ \hline 10 - 9x^2 - 5x \end{array}$$

$$\begin{array}{r} 30. \quad 5 \\ \quad a - b + 2c \\ \hline 5 - a + b - 2c \end{array}$$

$$\begin{array}{r} 35. \quad xy^2 + x^2y - z \\ \quad -3xy^2 + 2x^2y \\ \hline +4xy^2 - x^2y - z \end{array}$$

$$\begin{array}{r} 40. \quad a^4 + a^2c^2 + c^4 \\ \quad -4a^4 - 3a^2c^2 + 2c^4 \\ \hline 5a^4 + 4a^2c^2 - c^4 \end{array}$$

$$\begin{array}{r} 31. \quad 2bx - c \\ \quad ax^2 + bx + c \\ \hline -ax^2 + bx - 2c \end{array}$$

$$\begin{array}{r} 36. \quad x - 2y - z \\ \quad 3x - 2y - z \\ \hline -2x \end{array}$$

$$\begin{array}{r} 41. \quad 3a + 5b - c \\ \quad 0 \\ \hline 3a + 5b - c \end{array}$$

$$\begin{array}{r} 32. \quad 3xy - z \\ \quad 3ab + c \\ \hline 3xy - z - 3ab - c \end{array}$$

$$\begin{array}{r} 37. \quad x^2 - 7x - 10 \\ \quad 3x^2 + 14x - 8 \\ \hline -2x^2 - 21x - 2 \end{array}$$

$$\begin{array}{r} 42. \quad 4a + 6b - 8y \\ \quad a \quad \quad \quad -12 \\ \hline 3a + 6b - 8y + 12 \end{array}$$

$$\begin{array}{r}
 43. \quad a^2 + 2ab - 1 \\
 \quad a^2 + 12ab - 20 \\
 \hline
 2a^2 + 14ab - 21
 \end{array}$$

$$\begin{array}{r}
 a^2 + 13ab - 30 \\
 2a^2 + 14ab - 21 \\
 \hline
 -a^2 - ab - 9
 \end{array}$$

$$\begin{array}{r}
 44. \quad a - 3b + c \\
 \quad 4a + 5b - 6c + 4 \\
 \hline
 5a + 2b - 5c + 4
 \end{array}$$

$$\begin{array}{r}
 a - b + c - x \\
 5a + 2b - 5c + 4 \\
 \hline
 -4a - 3b + 6c - x - 4
 \end{array}$$

$$\begin{array}{r}
 45. \quad 5x + 3x^2y - 15xy^2 \\
 - 6x + 7x^2y - 12xy^2 \\
 \hline
 - x + 10x^2y - 27xy^2 \\
 \quad 11x - 5x^2y + 7xy^2 \\
 \hline
 -12x + 15x^2y - 34xy^2
 \end{array}$$

$$\begin{array}{r}
 46. \quad 4abc^2 - 3ab^2c + 2a^2bc \\
 6abc^2 - 5ab^2c - 4a^2b^2c \\
 \hline
 10abc^2 - 8ab^2c + 2a^2bc - 4a^2b^2c \\
 \quad 2abc^2 + 7ab^2c - 3a^2bc \\
 \hline
 8abc^2 - 15ab^2c + 5a^2bc - 4a^2b^2c
 \end{array}$$

$$\begin{array}{r}
 47. \quad 3x - 4xy - 2z \\
 - 3x + 7xy - 4z \\
 \hline
 3xy - 6z \\
 \quad 9x - 2xy + 4z - 7a^2bc \\
 \hline
 -9x + 5xy - 10z + 7a^2bc
 \end{array}$$

$$\begin{array}{r}
 5z - 2xy - a^2bc \\
 - z - 6a^2bc + 9x \\
 \hline
 4z - 2xy - 7a^2bc + 9x
 \end{array}$$

$$\begin{array}{r}
 48. \quad 4x - 3y + 6 \\
 \quad 3x + 5y - 10 \\
 \hline
 7x + 2y - 4
 \end{array}$$

$$\begin{array}{r}
 51. \quad x + 3y - 2z \\
 \quad 4x - 5y + 3z \\
 \hline
 5x - 2y + z \\
 \quad 3x - 2y - 6z \\
 \hline
 2x + 7z
 \end{array}$$

$$\begin{array}{r}
 53. \quad 4x - 3y + 7 \\
 \quad 4x - 8 \\
 \hline
 8x - 3y - 1 \\
 \quad 2x - 5y - 4 \\
 \hline
 6x + 2y + 3
 \end{array}$$

$$\begin{array}{r}
 49. \quad 7c + 5d - e \\
 \quad 4c + 5d - 9e \\
 \hline
 3c + 8e
 \end{array}$$

$$\begin{array}{r}
 52. \quad 5x + 3y - z \\
 \quad 4y + 7z \\
 \hline
 5x + 7y + 6z \\
 \quad x - y + 3z \\
 \hline
 4x + 8y + 3z
 \end{array}$$

$$\begin{array}{r}
 54. \quad 3c - 5d - e \\
 \quad 5c + 6d + 11e \\
 \hline
 -2c - 11d - 12e \\
 \quad 5c + 4e \\
 \hline
 -7c - 11d - 16e
 \end{array}$$

$$\begin{array}{r}
 50. \quad x^2 + 2x + 5 \\
 \quad 2x^2 + x - 10 \\
 \hline
 3x^2 + 3x - 5 \\
 \quad x^2 - 5x + 3 \\
 \hline
 2x^2 + 8x - 8
 \end{array}$$

$$\begin{array}{r}
 55. \quad 3a + 3b - 4c \\
 \quad x + 4a - 3b - 11c - 4 \\
 \hline
 -x - a + 6b + 7c + 4
 \end{array}$$

$$\begin{array}{r}
 -3b - 3c - 4 \\
 -8c + 4a + x \\
 \hline
 -3b - 11c - 4 + 4a + x
 \end{array}$$

$$\begin{array}{r}
 56. \quad x^3 - c^3 + 3cx^2 - 3c^2x \\
 \quad x^3 + c^3 - 4cx^2 - 2c^2x \\
 \hline
 -2c^3 + 7cx^2 - c^2x
 \end{array}$$

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1. $4x - 1 = 3x + 3.$
Transposing, $4x - 3x = 3 + 1.$
Combining, $x = 4.$
2. $4x + 5 = x - 10.$
Transposing, $4x - x = -10 - 5.$
Combining, $3x = -15.$
Whence $x = -5.$
3. $-6y - 2 = 1 - 9y.$
Transposing, $9y - 6y = 2 + 1.$
Combining, $3y = 3.$
Whence $y = 1.$
4. $6x + 3 - 2x = 27.$
Combining, $4x + 3 = 27.$
Transposing and combining, $4x = 24.$
Whence $x = 6.$
5. $4y + 3 = 2 + 5y.$
Transposing, $3 - 2 = 5y - 4y.$
Combining, $y = 1.$
6. $4k + 1 - 2k = 4.$
Combining, $2k + 1 = 4.$
Transposing and combining, $2k = 3.$
Whence $k = \frac{3}{2}.$
7. $16 + 4x - 4 = x + 12.$
Combining, $12 + 4x = x + 12.$
Transposing and combining, $3x = 0.$
Whence $x = 0.$
8. $4x - 15 = 15 - x - 5.$ 10. $2x - 1 = 29 + 7x.$
 $4x - 15 = 10 - x.$ $-5x = 30.$
 $5x = 25.$ $x = -6.$
 $x = 5.$
9. $5n + 11 - 2n = 6 - 4.$ 11. $3k + 9 + 5k - 33 = 0.$
 $3n + 11 = 2.$ $8k - 24 = 0.$
 $3n = -9.$ $8k = 24.$
 $n = -3.$ $k = 3.$

12. $3h - 20 + 8h - 24 = 0.$

$11h - 44 = 0.$

$11h = 44.$

$h = 4.$

17. $9 - 8h + 2 = 3 - 4h.$

$11 - 8h = 3 - 4h.$

$8 = 4h.$

$h = 2.$

13. $6x - 13 + 2x + 3 = 0.$

$8x - 10 = 0.$

$8x = 10.$

$x = \frac{10}{8} = \frac{5}{4}.$

18. $3 - 5x + 2 = 5 + 7x.$

$5 - 5x = 5 + 7x.$

$0 = 12x.$

$x = 0.$

14. $x + 2 - 5x = -9x + 12.$

$-4x + 2 = -9x + 12.$

$5x = 10.$

$x = 2.$

19. $8x - 3x = 15x + 4 - 13x.$

$5x = 2x + 4.$

$3x = 4.$

$x = \frac{4}{3}.$

15. $4x + 3 = 15 + 2x + 6.$

$4x + 3 = 2x + 21.$

$2x = 18.$

$x = 9.$

16. $3y + 5 + y + 3 = 0.$

$4y + 8 = 0.$

$4y = -8.$

$y = -2.$

20. $8 + 7y - 13 = y - 27 - 5y.$

$7y - 5 = -27 - 4y.$

$11y = -22.$

$y = -2.$

21. $3x - 15 - 10x - 9 + 16x - 21 = 0.$

$9x - 45 = 0.$

$9x = 45.$

$x = 5.$

22. $18 + 5x - 6 - 2x + 1 + 3x - 25 = 0.$

$6x - 12 = 0.$

$6x = 12.$

$x = 2.$

23. $0 = 18 - 4x + 27 + 9x - 3 + 16x.$

$0 = 42 + 21x.$

$-21x = 42.$

$x = -2.$

24. $5n - 8 + 4n + 5 = 7n - 3 - 2n + 5.$

$9n - 3 = 5n + 2.$

$4n = 5.$

$n = \frac{5}{4}.$

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- | | |
|-------------------------------------|-----------------------------|
| 1. Let $n =$ the number. | 8. Let $n =$ the number. |
| Then $n + 16 = 37$. | Then $3n - 23 = 2n - 8$. |
| Whence $n = 21$. | Whence $n = 15$. |
| 2. Let $n =$ the number. | 9. Let $n =$ the number. |
| Then $n - 23 = 9$. | Then $5n + 12 = 8n - 6$. |
| Whence $n = 32$. | Whence $n = 6$. |
| 3. Let $n =$ the number. | 10. Let $n =$ the number. |
| Then $n + 28 = 15$. | Then $4n - 7 = 2n + 21$. |
| Whence $n = -13$. | Whence $n = 14$. |
| 4. Let $n =$ the number. | 11. Let $n =$ the number. |
| Then $n - 15 = 24$. | Then $90 - n = n - 16$. |
| Whence $n = 39$. | Whence $n = 53$. |
| 5. Let $n =$ the number. | 12. Let $x =$ the number. |
| Then $2n - 13 = 49$. | Then $x - 16 = 2(52 - x)$ |
| Whence $n = 31$. | $\qquad\qquad = 104 - 2x$. |
| 6. Let $n =$ the number. | Whence $x = 40$. |
| Then $2n + 16 = 12$. | 13. Let $n =$ the number. |
| Whence $n = -2$. | Then $14 + n = 58 - n$. |
| 7. Let $n =$ the number. | Whence $n = 22$. |
| Then $3n - 16 = n$. | 14. Let $n =$ the number. |
| Whence $n = 8$. | Then $2n + 9 = 5n - 12$. |
| | Whence $n = 7$. |
| 15. Let $n =$ the number. | |
| Then $5n + 16 = 3n + 10$. | |
| Whence $n = -3$. | |
| 17. Let $n =$ the less number. | |
| Then $n + 12 =$ the greater number, | |
| and $n + n + 12 = 58$. | |
| Whence $n = 23; n + 12 = 35$. | |
| 18. Let $n =$ the less number. | |
| Then $n + 42 =$ the greater number, | |
| and $n + n + 42 = 8$. | |
| Whence $n = -17; n + 42 = 25$. | |
| 19. Let $n =$ the first number. | |
| Then $n + 5 =$ the second number, | |
| and $n + n + 5 = 21$. | |
| Whence $n = 8; n + 5 = 13$. | |

20. Let $n =$ the less number.
 Then $n + 41 =$ the greater number,
 and $n + n + 41 = 85$.
 Whence $n = 22$; $n + 41 = 63$.
21. Let $n =$ the first number.
 Then $n + 5 =$ the second number,
 and $n + 17 =$ the third number.
 But $n + n + 5 + n + 17 = 55$.
 Whence $n = 11$; $n + 5 = 16$; $n + 17 = 28$.
22. Let $n =$ the first number.
 Then $n - 10 =$ the second number,
 and $2n - 2 =$ the third number.
 But $n + n - 10 + 2n - 2 = 16$.
 Whence $n = 7$; $n - 10 = -3$; $2n - 2 = 12$.
23. Let $n, n + 1, n + 2$ be the three numbers in question.
 Then $n + n + 1 + n + 2 = 90$.
 Whence $n = 29$; $n + 1 = 30$; $n + 2 = 31$.
24. Let $n, n + 2, n + 4$ be the three numbers in question.
 Then $n + n + 2 + n + 4 = 33$.
 Whence $n = 9$; $n + 2 = 11$; $n + 4 = 13$.
25. Let $x =$ the width in feet.
 Then $3x =$ the length in feet,
 and $x + 3x + x + 3x = 32$.
 Whence $x = 4$; $3x = 12$.
26. Let $x =$ the width in feet.
 Then $2x =$ the length in feet,
 and $x + 2x + x + 2x = 102$.
 Whence $x = 17$; $2x = 34$.
27. Let $x =$ the width in feet.
 Then $2x + 7 =$ the length in feet,
 and $x + 2x + 7 + x + 2x + 7 = 110$.
 Whence $x = 16$; $2x + 7 = 39$.
28. Let $x =$ the bookkeeper's salary in dollars.
 Then $\frac{2}{3}x =$ the stenographer's salary in dollars,
 and $x + \frac{2}{3}x = 40 - 5$.
 $\frac{5}{3}x = 35$,
 and, dividing by $\frac{5}{3}$, $x = 21$;
 $\frac{2}{3}x = 14$.

29. Let $x =$ James's allowance in cents.
 Then $2x =$ Elizabeth's allowance in cents,
 and $2x + 50 =$ John's allowance in cents.
 But $x + 2x + 2x + 50 = 150$.
 Whence $x = 20$; $2x = 40$; $2x + 50 = 90$.
30. Let $x =$ the number of courses laid the first day.
 Then $x - 2 =$ the number laid the second day,
 and $x - 4, x - 6, x - 8, x - 10$, etc.
 $=$ the number laid on each succeeding day.
 But $x + x - 2 + x - 4 + x - 6 + x - 8 + x - 10 = 120$.
 Whence $x = 25$.

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1. $14 - (6 - 3) - 5 = 14 - 6 + 3 - 5 = 6$.
2. $10 + (7 - 4) - (9 - 7) = 10 + 7 - 4 - 9 + 7 = 11$.
3. $(7 - 5 + 2) - (6 - 4) + 12 = 7 - 5 + 2 - 6 + 4 + 12 = 14$.
4. $11a - (4a - 9a) + (6a - a) = 11a - 4a + 9a + 6a - a = 21a$.
5. $(2b - 5a) - (4a - b - 7a) = 2b - 5a - 4a + b + 7a = 3b - 2a$.
6. $a - (b - a) + (2b - 3c) = a - b + a + 2b - 3c = 2a + b - 3c$.
7. $a - b - (c - d) + (a - b) - (b - c)$
 $= a - b - c + d + a - b - b + c = 2a - 3b + d$.
8. $(x - y) - (2y - 3x) + (x - 4y) = x - y - 2y + 3x + x - 4y = 5x - 7y$.
9. $x - (x - y - 2z) - (3z + y + 4) + (x - 6)$
 $= x - x + y + 2z - 3z - y - 4 + x - 6 = x - z - 10$.
10. $7 - [8 - (3 - 10)] - (13 - 25) = 7 - [8 - 3 + 10] - 13 + 25$
 $= 7 - 8 + 3 - 10 - 13 + 25 = 4$.
11. $8 - [5 - (4 - 6)] - (6 - 15) = 8 - [5 - 4 + 6] - 6 + 15$
 $= 8 - 5 + 4 - 6 - 6 + 15 = 10$.
12. $12 - (8 - 17) - [(4 - 1) - 8] = 12 - 8 + 17 - 4 + 1 + 8 = 26$.
13. $16 - [3 - (2 - 5)] + (3 - 5) - (8 - 4)$
 $= 16 - [3 - 2 + 5] + 3 - 5 - 8 + 4$
 $= 16 - 3 + 2 - 5 + 3 - 5 - 8 + 4 = 4$.
14. $a + [2a - (3a - 2b)] + (3b - 2a)$
 $= a + [2a - 3a + 2b] + 3b - 2a$
 $= a + 2a - 3a + 2b + 3b - 2a = 5b - 2a$.
15. $a + [5a - (3x - 2a)] - (4a - 3x)$
 $= a + 5a - 3x + 2a - 4a + 3x = 4a$.
16. $2a - [6a - (5x - 4a)] + (8a - 7x) = 2a - [6a - 5x + 4a] + 8a - 7x$
 $= 2a - 6a + 5x - 4a + 8a - 7x = -2x$.

17. Two; two or three; three or four; one; two; two; two; two.

$$\begin{aligned} 18. (5x - 6y) - [-4x - (4z - y) - 2z] \\ = 5x - 6y - [-4x - 4z + y - 2z] \\ = 5x - 6y + 4x + 4z - y + 2z = 9x - 7y + 6z. \end{aligned}$$

$$\begin{aligned} 19. [3x - (2y + z)] - [- (3y - 2x) + 5x] \\ = 3x - 2y - z - [-3y + 2x + 5x] \\ = 3x - 2y - z + 3y - 2x - 5x = -4x + y - z. \end{aligned}$$

$$\begin{aligned} 20. [(a + 3) - (x - 5)] - [a + 3 + (x - 5)] \\ = a + 3 - x + 5 - a - 3 - x + 5 \\ = -2x + 10. \end{aligned}$$

$$\begin{aligned} 21. 7 - [-6 - \{-4 + (9 - 10)\} + 11] = 7 - [-6 - \{-4 + 9 - 10\} + 11] \\ = 7 - [-6 + 4 - 9 + 10 + 11] \\ = 7 + 6 - 4 + 9 - 10 - 11 = -3. \end{aligned}$$

$$\begin{aligned} 22. 5a - [2a + (-3a + 4b) - (a - 8b) + 4a] \\ = 5a - [2a - 3a + 4b - a + 8b + 4a] \\ = 5a - 2a + 3a - 4b + a - 8b - 4a \\ = 3a - 12b. \end{aligned}$$

$$\begin{aligned} 23. 2x - 3y - [\{3z - 7x - (y - 4z) - 9x\} + z] \\ = 2x - 3y - [3z - 7x - y + 4z - 9x + z] \\ = 2x - 3y - 3z + 7x + y - 4z + 9x - z \\ = 18x - 2y - 8z. \end{aligned}$$

$$\begin{aligned} 24. \{4a - [2a - (8a + 2b) + 4] - (4b - 6)\} \\ = \{4a - [2a - 8a - 2b + 4] - 4b + 6\} \\ = \{4a - 2a + 8a + 2b - 4 - 4b + 6\} \\ = 10a - 2b + 2. \end{aligned}$$

$$\begin{aligned} 25. -5x + [15x - \{11x - (2x - 7x - 4) - 3x\} - 22] \\ = -5x + 15x - \{11x - 2x + 7x + 4 - 3x\} - 22 \\ = -5x + 15x - 11x + 2x - 7x - 4 + 3x - 22 \\ = -3x - 26. \end{aligned}$$

$$\begin{aligned} 26. (4y - 7x) - \{3x - [4x - (7y - 4x) - (-5y + 3x)]\} \\ = 4y - 7x - \{3x - [4x - 7y + 4x + 5y - 3x]\} \\ = 4y - 7x - \{3x - 4x + 7y - 4x - 5y + 3x\} \\ = 4y - 7x - 3x + 4x - 7y + 4x + 5y - 3x \\ = 2y - 5x. \end{aligned}$$

$$27. [(a + b) + c] = [a + b + c]; [(a + b) - c] = [a + b - c].$$

$$28. [4x + (3z - 5y)] = [4x + 3z - 5y]; [4x - (3z - 5y)] = [4x - 3z + 5y].$$

$$\begin{aligned} 29. [(a - b) + (b - 2a)] &= [a - b + b - 2a] = -a; [(a - b) - (b - 2a)] \\ &= [a - b - b + 2a] = [3a - 2b]. \end{aligned}$$

$$\begin{aligned} 30. [(3a - 2b) + (2b - 3a)] &= [3a - 2b + 2b - 3a] = 0; [(3a - 2b) \\ &\quad - (2b - 3a)] = [3a - 2b - 2b + 3a] = [6a - 4b]. \end{aligned}$$

31. $[(a - 2b) + (3c - d)] = [a - 2b + 3c - d]; [(a - 2b) - (3c - d)]$
 $= [a - 2b - 3c + d].$
32. $[(4x - 3) + (5y - 7)] = [4x - 3 + 5y - 7] = [4x + 5y - 10];$
 $[(4x - 3) - (5y - 7)] = [4x - 3 - 5y + 7] = [4x - 5y + 4].$
33. $[(x^2 - a^2) + (y^2 - 2a^2)] = [x^2 - a^2 + y^2 - 2a^2] = [x^2 + y^2 - 3a^2];$
 $[(x^2 - a^2) - (y^2 - 2a^2)] = [x^2 - a^2 - y^2 + 2a^2] = [x^2 - y^2 + a^2].$

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1. $x^2 - a^2 - 2a = + (x^2) - (a^2 + 2a).$
2. $12a + x^2 - 9 - 4a^2 = + (x^2) - (4a^2 - 12a + 9).$
3. $y^2 - 9b^2 + 6ab - a^2 = + (y^2) - (9b^2 - 6ab + a^2).$
4. $10ab + x^2 - a^2 - 25b^2 = + (x^2) - (a^2 - 10ab + 25b^2).$
5. $x^2 - b^2 - 4b^2 + y^2 - a^2 - 2xy = + (x^2 - 2xy + y^2) - (a^2 + 5b^2).$
6. $-4ab + x^2 - 4b^2 + y^2 - a^2 - 2xy$
 $= + (x^2 - 2xy + y^2) - (a^2 + 4ab + 4b^2).$
7. $16x^2 - a^2 - 16xy - 4 + 4a + 4y^2$
 $= + (16x^2 - 16xy + 4y^2) - (a^2 - 4a + 4).$
8. $x^2 - b^2 - 10xy + 12ab - 36a^2 + 25y^2$
 $= + (x^2 - 10xy + 25y^2) - (b^2 - 12ab + 36a^2).$
9. $2ab - a^2 + x^2 + 2xy - b^2 + y^2 = + (x^2 + 2xy + y^2) - (a^2 - 2ab + b^2).$
10. $x^2 - 16xy - a^2 + 16a + 64y^2 - 64$
 $= + (x^2 - 16xy + 64y^2) - (a^2 - 16a + 64).$
11. (1) $x^2 - a^2 - 2a = x^2 - (a^2 + 2a).$
11. (2) $12a + x^2 - 9 - 4a^2 = 12a + x^2 - (9 + 4a^2).$
11. (3) $y^2 - 9b^2 + 6ab - a^2 = y^2 - 9b^2 - (-6ab + a^2).$
11. (4) $10ab + x^2 - a^2 - 25b^2 = 10ab + x^2 - (a^2 + 25b^2).$
12. (5) $x^2 - b^2 - 4b^2 + y^2 - a^2 - 2xy = x^2 - 5b^2 - (-y^2 + a^2 + 2xy).$
12. (6) $-4ab + x^2 - 4b^2 + y^2 - a^2 - 2xy$
 $= -4ab + x^2 - 4b^2 - (-y^2 + a^2 + 2xy).$
12. (7) $16x^2 - a^2 - 16xy - 4 + 4a + 4y^2$
 $= 16x^2 - a^2 - 16xy - (4 - 4a - 4y^2).$
12. (8) $x^2 - b^2 - 10xy + 12ab - 36a^2 + 25y^2$
 $= x^2 - b^2 - 10xy - (-12ab + 36a^2 - 25y^2).$
12. (9) $2ab - a^2 + x^2 + 2xy - b^2 + y^2$
 $= 2ab - a^2 + x^2 - (-2xy + b^2 - y^2).$
12. (10) $x^2 - 16xy - a^2 + 16a + 64y^2 - 64$
 $= x^2 - 16xy - a^2 - (-16a - 64y^2 + 64).$

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1. $x + 2$

$$\begin{array}{r} 2 \\ \hline 2x + 4 \end{array}$$

2. $x - 4$

$$\begin{array}{r} x \\ \hline x^2 - 4x \end{array}$$

7. $y^3 - 5y^2 + 3y - 1$

$$\begin{array}{r} y^2 \\ \hline y^5 - 5y^4 + 3y^3 - y^2 \end{array}$$

8. $5x^2 - 2x - 4$

$$\begin{array}{r} x^2 \\ \hline 5x^4 - 2x^3 - 4x^2 \end{array}$$

9. $-4x^2 + 6x - 5$

$$\begin{array}{r} 6x^2 \\ \hline -24x^4 + 36x^3 - 30x^2 \end{array}$$

10. $2x^2 - 3x - 2$

$$\begin{array}{r} -4x^2 \\ \hline -8x^4 + 12x^3 + 8x^2 \end{array}$$

11. $x^3 - 3x^2 - 4$

$$\begin{array}{r} -5x^4 \\ \hline -5x^7 + 15x^6 + 20x^4 \end{array}$$

12. $2x - 3x^2 - 2x^3$

$$\begin{array}{r} -2x^3 \\ \hline -4x^4 + 6x^5 + 4x^6 \end{array}$$

13. $7xy - x + y$

$$\begin{array}{r} 3xy \\ \hline 21x^2y^2 - 3x^2y + 3xy^2 \end{array}$$

3. $x^2 + 5$

$$\begin{array}{r} 2x \\ \hline 2x^3 + 10x \end{array}$$

4. $3x^2 + 4$

$$\begin{array}{r} 2x \\ \hline 6x^3 + 8x \end{array}$$

5. $x^2 - 2x$

$$\begin{array}{r} x^2 \\ \hline x^4 - 2x^3 \end{array}$$

6. $y^2 - 3y + 2$

$$\begin{array}{r} 3y \\ \hline 3y^3 - 9y^2 + 6y \end{array}$$

14. $x^2 - 2xy + y^2$

$$\begin{array}{r} -3xy \\ \hline -3x^3y + 6x^2y^2 - 3xy^3 \end{array}$$

15. $a^4 - a^2b^2 + b^4$

$$\begin{array}{r} -a^2b^2 \\ \hline -a^6b^2 + a^4b^4 - a^2b^6 \end{array}$$

16. $-a^2x^2 - 2ax + 7b^2$

$$\begin{array}{r} -4abx \\ \hline 4a^3bx^3 + 8a^2bx^2 - 28ab^3x \end{array}$$

17. $7x^3 - 8x^2 + 12x - 6$

$$\begin{array}{r} -4x^3 \\ \hline -28x^6 + 32x^5 - 48x^4 + 24x^3 \end{array}$$

18. $-9a^2 - 12ax + 42x^2$

$$\begin{array}{r} 3ax^3 \\ \hline -27a^3x^3 - 36a^2x^4 + 126ax^5 \end{array}$$

19. $7(2x - 3) = 14x - 21.$

20. $5x(x - y) = 5x^2 - 5xy.$

21. $-8(3x - 7) = -24x + 56.$

22. $-9(-4a + b) = 36a - 9b.$

23. $-3x(2x - 7) = -6x^2 + 21x.$

24. $6x^2(9x^3 - 4x) = 54x^5 - 24x^3.$

25. $-3(x^2 - 2x - 6) = -3x^2 + 6x + 18.$

26. $5xy(x^2 - 6x + 9) = 5x^3y - 30x^2y + 45xy.$

27. $-3x(ax - bx - 3cx^2) = -3ax^2 + 3bx^2 + 9cx^3.$

28. $(x^3 - ax + a^2)(-2a^2x) = -2a^2x^4 + 2a^3x^2 - 2a^4x.$

29. $-7ab(ax^2 - bx + c) = -7a^2bx^2 + 7ab^2x - 7abc.$

30. $4x^2(-3x + 7x^2 - x^4) = -12x^3 + 28x^4 - 4x^6.$

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$$\begin{array}{r}
 1. \quad x + 4 \\
 \quad x + 3 \\
 \hline
 x^2 + 4x \\
 \quad + 3x + 12 \\
 \hline
 x^2 + 7x + 12
 \end{array}$$

$$\begin{array}{r}
 4. \quad 3x - 5 \\
 \quad 3x + 8 \\
 \hline
 9x^2 - 15x \\
 \quad + 24x - 40 \\
 \hline
 9x^2 + 9x - 40
 \end{array}$$

$$\begin{array}{r}
 7. \quad 2x + y \\
 \quad x + 3y \\
 \hline
 2x^2 + xy \\
 \quad + 6xy + 3y^2 \\
 \hline
 2x^2 + 7xy + 3y^2
 \end{array}$$

$$\begin{array}{r}
 2. \quad 2x + 3 \\
 \quad x + 3 \\
 \hline
 2x^2 + 3x \\
 \quad + 6x + 9 \\
 \hline
 2x^2 + 9x + 9
 \end{array}$$

$$\begin{array}{r}
 5. \quad 3x - 2 \\
 \quad 2x + 3 \\
 \hline
 6x^2 - 4x \\
 \quad + 9x - 6 \\
 \hline
 6x^2 + 5x - 6
 \end{array}$$

$$\begin{array}{r}
 8. \quad 2x - 3y \\
 \quad 3x - 2y \\
 \hline
 6x^2 - 9xy \\
 \quad - 4xy + 6y^2 \\
 \hline
 6x^2 - 13xy + 6y^2
 \end{array}$$

$$\begin{array}{r}
 3. \quad 4x + 7 \\
 \quad 3x + 2 \\
 \hline
 12x^2 + 21x \\
 \quad + 8x + 14 \\
 \hline
 12x^2 + 29x + 14
 \end{array}$$

$$\begin{array}{r}
 6. \quad 6 - 4a \\
 \quad - 7 + 5a \\
 \hline
 -42 + 28a \\
 \quad + 30a - 20a^2 \\
 \hline
 -42 + 58a - 20a^2
 \end{array}$$

$$\begin{array}{r}
 9. \quad 3x - 2 \\
 \quad 2x - 3 \\
 \hline
 6x^2 - 4x \\
 \quad - 9x + 6 \\
 \hline
 6x^2 - 13x + 6
 \end{array}$$

$$\begin{array}{r}
 10. \quad -3x + 11a \\
 \quad 5x - a \\
 \hline
 -15x^2 + 55ax \\
 \quad + 3ax - 11a^2 \\
 \hline
 -15x^2 + 58ax - 11a^2
 \end{array}$$

$$\begin{array}{r}
 15. \quad 3x^2 - 3x - 7 \\
 \quad 2x + 4 \\
 \hline
 6x^3 - 6x^2 - 14x \\
 \quad + 12x^2 - 12x - 28 \\
 \hline
 6x^3 + 6x^2 - 26x - 28
 \end{array}$$

$$\begin{array}{r}
 11. \quad ax - bx \\
 \quad cx + dx \\
 \hline
 acx^2 - bcx^2 + adx^2 - bdx^2
 \end{array}$$

$$\begin{array}{r}
 16. \quad x^2 - xy + y^2 \\
 \quad x + y \\
 \hline
 x^3 - x^2y + xy^2 \\
 \quad + x^2y - xy^2 + y^3 \\
 \hline
 x^3 + y^3
 \end{array}$$

$$\begin{array}{r}
 12. \quad -cx + d \\
 \quad -cx^2 + bx \\
 \hline
 c^2x^3 - cdx^2 - bcx^2 + bdx
 \end{array}$$

$$\begin{array}{r}
 17. \quad a^2x^2 - 2a^2x + 4a^2 \\
 \quad ax + 2a \\
 \hline
 a^3x^3 - 2a^3x^2 + 4a^3x \\
 \quad + 2a^3x^2 - 4a^3x + 8a^3 \\
 \hline
 a^3x^3 + 8a^3
 \end{array}$$

$$\begin{array}{r}
 13. \quad 4x - 3y^2 \\
 \quad 6x + 5y \\
 \hline
 24x^2 - 18xy^2 + 20xy - 15y^3
 \end{array}$$

$$\begin{array}{r}
 14. \quad x^2 - 5x + 6 \\
 \quad x - 3 \\
 \hline
 x^3 - 5x^2 + 6x \\
 \quad - 3x^2 + 15x - 18 \\
 \hline
 x^3 - 8x^2 + 21x - 18
 \end{array}$$

$$\begin{array}{r}
 18. \quad 3x^3 - x^2 - 5x \\
 \quad 2x^3 - 5x^2 \\
 \hline
 6x^6 - 2x^5 - 10x^4 \\
 \quad - 15x^5 + 5x^4 + 25x^3 \\
 \hline
 6x^6 - 17x^5 - 5x^4 + 25x^3
 \end{array}$$

$$\begin{array}{r}
 19. \quad 2x^2 - 7x + 12 \\
 \quad \quad x^2 - 3x - 5 \\
 \hline
 2x^4 - 7x^3 + 12x^2 \\
 \quad - 6x^3 + 21x^2 - 36x \\
 \quad \quad - 10x^2 + 35x - 60 \\
 \hline
 2x^4 - 13x^3 + 23x^2 - x - 60
 \end{array}$$

$$\begin{array}{r}
 20. \quad x^2 - 2x - 3 \\
 \quad \quad x^2 - 2x - 3 \\
 \hline
 x^4 - 2x^3 - 3x^2 \\
 \quad - 2x^3 + 4x^2 + 6x \\
 \quad \quad - 3x^2 + 6x + 9 \\
 \hline
 x^4 - 4x^3 - 2x^2 + 12x + 9
 \end{array}$$

$$\begin{array}{r}
 21. \quad 3x^2 - 5x - 1 \\
 \quad \quad 2x - 3 \\
 \hline
 6x^3 - 10x^2 - 2x \\
 \quad - 9x^2 + 15x + 3 \\
 \hline
 6x^3 - 19x^2 + 13x + 3
 \end{array}$$

$$\begin{array}{r}
 22. \quad x^2 - 3x - 2 \\
 \quad \quad x^2 - 2x + 3 \\
 \hline
 x^4 - 3x^3 - 2x^2 \\
 \quad - 2x^3 + 6x^2 + 4x \\
 \quad \quad + 3x^2 - 9x - 6 \\
 \hline
 x^4 - 5x^3 + 7x^2 - 5x - 6
 \end{array}$$

$$\begin{array}{r}
 23. \quad t^2 - 3t + 2 \\
 \quad \quad t^2 - 3t + 2 \\
 \hline
 t^4 - 3t^3 + 2t^2 \\
 \quad - 3t^3 + 9t^2 - 6t \\
 \quad \quad + 2t^2 - 6t + 4 \\
 \hline
 t^4 - 6t^3 + 13t^2 - 12t + 4
 \end{array}$$

$$\begin{array}{r}
 24. \quad -y^2 + 3y - 2 \\
 \quad \quad -y^2 + 3y - 2 \\
 \hline
 y^4 - 3y^3 + 2y^2 \\
 \quad - 3y^3 + 9y^2 - 6y \\
 \quad \quad + 2y^2 - 6y + 4 \\
 \hline
 y^4 - 6y^3 + 13y^2 - 12y + 4
 \end{array}$$

$$\begin{array}{r}
 25. \quad -2x^2 + 4x - 5 \\
 \quad \quad -2x^2 + 4x - 5 \\
 \hline
 4x^4 - 8x^3 + 10x^2 \\
 \quad - 8x^3 + 16x^2 - 20x \\
 \quad \quad + 10x^2 - 20x + 25 \\
 \hline
 4x^4 - 16x^3 + 36x^2 - 40x + 25
 \end{array}$$

$$\begin{array}{r}
 26. \quad 3x^2 - 4x + 7 \\
 \quad \quad 3x^2 - 4x + 7 \\
 \hline
 9x^4 - 12x^3 + 21x^2 \\
 \quad - 12x^3 + 16x^2 - 28x \\
 \quad \quad + 21x^2 - 28x + 49 \\
 \hline
 9x^4 - 24x^3 + 58x^2 - 56x + 49
 \end{array}$$

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$$\begin{array}{r}
 1. \quad 3x^2 - 5x - 2 \\
 \quad \quad 2x - 3 \\
 \hline
 6x^3 - 10x^2 - 4x \\
 \quad - 9x^2 + 15x + 6 \\
 \hline
 6x^3 - 19x^2 + 11x + 6
 \end{array}$$

$$\begin{array}{r}
 2. \quad 3x^2 + 2x - 7 \\
 \quad \quad 2x^2 - 5x - 3 \\
 \hline
 6x^4 + 4x^3 - 14x^2 \\
 \quad - 15x^3 - 10x^2 + 35x \\
 \quad \quad - 9x^2 - 6x + 21 \\
 \hline
 6x^4 - 11x^3 - 33x^2 + 29x + 21
 \end{array}$$

$$\begin{array}{r}
 3. \quad x^2 + 2x - 4 \\
 \quad \quad x^2 + 2x + 4 \\
 \hline
 x^4 + 2x^3 - 4x^2 \\
 \quad + 2x^3 + 4x^2 - 8x \\
 \quad \quad + 4x^2 + 8x - 16 \\
 \hline
 x^4 + 4x^3 + 4x^2 - 16
 \end{array}$$

$$\begin{array}{r}
 4. \quad 3a^2 - 8a - 1 \\
 \quad \quad a^2 + 2a - 3 \\
 \hline
 3a^4 - 8a^3 - a^2 \\
 \quad + 6a^3 - 16a^2 - 2a \\
 \quad \quad - 9a^2 + 24a + 3 \\
 \hline
 3a^4 - 2a^3 - 26a^2 + 22a + 3
 \end{array}$$

5. $2x^2 - 7x - 2$

$2x^2 - 7x - 2$

$4x^4 - 14x^3 - 4x^2$

$-14x^3 + 49x^2 + 14x$

$-4x^2 + 14x + 4$

$4x^4 - 28x^3 + 41x^2 + 28x + 4$

6. $2x^3 - 3x + 2$

$2x^3 - 3x + 2$

$4x^6 - 6x^4 + 4x^3$

$-6x^4 + 9x^2 - 6x$

$+4x^3 - 6x + 4$

$4x^6 - 12x^4 + 8x^3 + 9x^2 - 12x + 4$

7. $a^2 - \frac{a}{2} + \frac{1}{4}$

$a^2 - a + 1$

$a^4 - \frac{a^3}{2} + \frac{a^2}{4}$

$-a^3 + \frac{a^2}{2} - \frac{a}{4}$

$+a^2 - \frac{a}{2} + \frac{1}{4}$

$a^4 - \frac{3a^3}{2} + \frac{7a^2}{4} - \frac{3a}{4} + \frac{1}{4}$

8. $x^2 - xy + y^2$

$x^2 + xy + y^2$

$x^4 - x^3y + x^2y^2$

$+x^3y - x^2y^2 + xy^3$

$+x^2y^2 - xy^3 + y^4$

$x^4 + x^2y^2 + y^4$

9. $3x^3 + 5x^2 - x + 2$

$x^2 - 2x + 3$

$3x^5 + 5x^4 - x^3 + 2x^2$

$-6x^4 - 10x^3 + 2x^2 - 4x$

$+9x^3 + 15x^2 - 3x + 6$

$3x^5 - x^4 - 2x^3 + 19x^2 - 7x + 6$

10. $-a^3 - 2a^2 + a - 3$

$-3a^2 + 2a - 1$

$3a^5 + 6a^4 - 3a^3 + 9a^2$

$-2a^4 - 4a^3 + 2a^2 - 6a$

$+a^3 + 2a^2 - a + 3$

$3a^5 + 4a^4 - 6a^3 + 13a^2 - 7a + 3$

11. $-a^3 + 3a + 6$

$-3a^2 + 4a - 5$

$3a^5 - 9a^3 - 18a^2$

$-4a^4 + 12a^2 + 24a$

$+5a^3 - 15a - 30$

$3a^5 - 4a^4 - 4a^3 - 6a^2 + 9a - 30$

12. $x^3 - x - 5$

$$\begin{array}{r}
 3x^2 - 2x - 4 \\
 \hline
 3x^5 \qquad - 3x^3 - 15x^2 \\
 \qquad - 2x^4 \qquad + 2x^2 + 10x \\
 \qquad \qquad - 4x^3 \qquad + 4x + 20 \\
 \hline
 3x^5 - 2x^4 - 7x^3 - 13x^2 + 14x + 20
 \end{array}$$

13. $-x^3 + x^2 + 3x - 6$

$$\begin{array}{r}
 -x^2 - 3x + 5 \\
 \hline
 x^5 - x^4 - 3x^3 + 6x^2 \\
 \qquad + 3x^4 - 3x^3 - 9x^2 + 18x \\
 \qquad \qquad - 5x^3 + 5x^2 + 15x - 30 \\
 \hline
 x^5 + 2x^4 - 11x^3 + 2x^2 + 33x - 30
 \end{array}$$

14. $(6x - 4a)0 = 0.$

15. $a^3 - 5a^2 + 4a + 7$

$$\begin{array}{r}
 a^3 + a^2 - a + 3 \\
 \hline
 a^6 - 5a^5 + 4a^4 + 7a^3 \\
 \qquad a^5 - 5a^4 + 4a^3 + 7a^2 \\
 \qquad \qquad - a^4 + 5a^3 - 4a^2 - 7a \\
 \qquad \qquad \qquad + 3a^3 - 15a^2 + 12a + 21 \\
 \hline
 a^6 - 4a^5 - 2a^4 + 19a^3 - 12a^2 + 5a + 21
 \end{array}$$

16. $2x^3 - 5x^2 - 9x + 8$

$$\begin{array}{r}
 8x^3 + 5x - 3 \\
 \hline
 16x^6 - 40x^5 - 72x^4 + 64x^3 \\
 \qquad \qquad 10x^4 - 25x^3 - 45x^2 + 40x \\
 \qquad \qquad \qquad - 6x^3 + 15x^2 + 27x - 24 \\
 \hline
 16x^6 - 40x^5 - 62x^4 + 33x^3 - 30x^2 + 67x - 24
 \end{array}$$

17. $x^2 - 2xy + 3y^2$

$$\begin{array}{r}
 x^2 + 2xy + 3y^2 \\
 \hline
 x^4 - 2x^3y + 3x^2y^2 \\
 \qquad + 2x^3y - 4x^2y^2 + 6xy^3 \\
 \qquad \qquad + 3x^2y^2 - 6xy^3 + 9y^4 \\
 \hline
 x^4 \qquad + 2x^2y^2 \qquad + 9y^4
 \end{array}$$

18. $4xy - 5x^2y$

$$\begin{array}{r}
 x^2y - xy^2 \\
 \hline
 4x^3y^2 - 5x^4y^2 - 4x^2y^3 + 5x^3y^3 \\
 3x^2y - 2xy^2 \\
 \hline
 12x^5y^3 - 15x^6y^3 - 12x^4y^4 + 15x^5y^4 \\
 \qquad \qquad - 8x^4y^4 + 10x^5y^4 + 8x^3y^5 - 10x^4y^5 \\
 \hline
 12x^5y^3 - 15x^6y^3 - 20x^4y^4 + 25x^5y^4 + 8x^3y^5 - 10x^4y^5
 \end{array}$$

19. $x^2 - xy - xz - yz + y^2 + z^2$

$$\begin{array}{r}
 x + y + z \\
 \hline
 x^3 - x^2y - x^2z - xyz + xy^2 + xz^2 \\
 + x^2y \quad - xyz - xy^2 \quad - y^2z + y^3 + yz^2 \\
 + x^2z - xyz \quad - xz^2 + y^2z \quad - yz^2 + z^3 \\
 \hline
 x^3 \quad - 3xyz \quad + y^3 \quad + z^3
 \end{array}$$

20. $a + b + c$

$$\begin{array}{r}
 a + b + c \\
 \hline
 a^2 + ab + ac \\
 \quad ab \quad + b^2 + bc \\
 \quad \quad ac \quad + bc + c^2 \\
 \hline
 a^2 + 2ab + 2ac + b^2 + 2bc + c^2
 \end{array}$$

21. $2a - 3b + 4c$

$$\begin{array}{r}
 2a - 3b + 4c \\
 \hline
 4a^2 - 6ab + 8ac \\
 \quad - 6ab \quad + 9b^2 - 12bc \\
 \quad \quad + 8ac \quad - 12bc + 16c^2 \\
 \hline
 4a^2 - 12ab + 16ac + 9b^2 - 24bc + 16c^2
 \end{array}$$

22. $c + d - \frac{1}{2}$

$$\begin{array}{r}
 c + d - \frac{1}{2} \\
 \hline
 c^2 + cd - \frac{c}{2} \\
 \quad + cd \quad + d^2 - \frac{d}{2} \\
 \quad \quad - \frac{c}{2} \quad - \frac{d}{2} + \frac{1}{4} \\
 \hline
 c^2 + 2cd - c + d^2 - d + \frac{1}{4}
 \end{array}$$

23. $2a - 4b + 3$

$$\begin{array}{r}
 2a - 4b + 3 \\
 \hline
 4a^2 - 8ab + 6a \\
 \quad - 8ab \quad + 16b^2 - 12b \\
 \quad \quad + 6a \quad - 12b + 9 \\
 \hline
 4a^2 - 16ab + 12a + 16b^2 - 24b + 9
 \end{array}$$

24. $a - 2b + 3c - 4d$

$$\begin{array}{r}
 a - 2b + 3c - 4d \\
 \hline
 a^2 - 2ab + 3ac - 4ad \\
 \quad - 2ab \quad + 4b^2 - 6bc + 8bd \\
 \quad \quad + 3ac \quad - 6bc \quad + 9c^2 - 12cd \\
 \quad \quad \quad - 4ad \quad + 8bd \quad - 12cd + 16d^2 \\
 \hline
 a^2 - 4ab + 6ac - 8ad + 4b^2 - 12bc + 16bd + 9c^2 - 24cd + 16d^2
 \end{array}$$

25. $x + y + z$

$$\begin{array}{r}
 x + y + z \\
 \hline
 x^2 + xy + xz \\
 + xy + y^2 + yz \\
 + xz + yz + z^2 \\
 \hline
 x^2 + 2xy + 2xz + y^2 + 2yz + z^2 \\
 x + y + z \\
 \hline
 x^3 + 2x^2y + 2x^2z + xy^2 + 2xyz + xz^2 \\
 + x^2y + 2xy^2 + 2xyz + y^3 + 2y^2z + yz^2 \\
 + x^2z + 2xyz + 2xz^2 + y^2z + 2yz^2 + z^3 \\
 \hline
 x^3 + 3x^2y + 3x^2z + 3xy^2 + 6xyz + 3xz^2 + y^3 + 3y^2z + 3yz^2 + z^3
 \end{array}$$

26. $x + 2y$

$$\begin{array}{r}
 x + 2y \\
 \hline
 x^2 + 2xy \\
 + 2xy + 4y^2 \\
 \hline
 x^2 + 4xy + 4y^2 \\
 x^2 - 4xy + 4y^2 \\
 \hline
 8xy \quad (\text{Ans.})
 \end{array}$$

$x - 2y$

$$\begin{array}{r}
 x - 2y \\
 \hline
 x^2 - 2xy \\
 - 2xy + 4y^2 \\
 \hline
 x^2 - 4xy + 4y^2
 \end{array}$$

27. $x - 3$

$$\begin{array}{r}
 x - 3 \\
 \hline
 x^2 - 3x \\
 - 3x + 9 \\
 \hline
 x^2 - 6x + 9 \\
 x - 3 \\
 \hline
 x^3 - 6x^2 + 9x \\
 - 3x^2 + 18x - 27 \\
 \hline
 x^3 - 9x^2 + 27x - 27 \\
 - x^3 + 9x^2 - 27x + 27 \\
 \hline
 0 \quad (\text{Ans.})
 \end{array}$$

$3 - x$

$$\begin{array}{r}
 3 - x \\
 \hline
 3 - x \\
 9 - 3x \\
 - 3x + x^2 \\
 \hline
 9 - 6x + x^2 \\
 3 - x \\
 \hline
 27 - 18x + 3x^2 \\
 - 9x + 6x^2 - x^3 \\
 \hline
 27 - 27x + 9x^2 - x^3
 \end{array}$$

28. $x + y$

$$\begin{array}{r}
 x + y \\
 \hline
 x^2 + xy \\
 + xy + y^2 \\
 \hline
 x^2 + 2xy + y^2 \\
 x + y \\
 \hline
 x^3 + 2x^2y + xy^2 \\
 + x^2y + 2xy^2 + y^3 \\
 \hline
 x^3 + 3x^2y + 3xy^2 + y^3 \\
 x^3 - 3x^2y + 3xy^2 - y^3 \\
 \hline
 2x^3 + 6xy^2 \quad (\text{Ans.})
 \end{array}$$

$x - y$

$$\begin{array}{r}
 x - y \\
 \hline
 x^2 - xy \\
 - xy + y^2 \\
 \hline
 x^2 - 2xy + y^2 \\
 x - y \\
 \hline
 x^3 - 2x^2y + xy^2 \\
 - x^2y + 2xy^2 - y^3 \\
 \hline
 x^3 - 3x^2y + 3xy^2 - y^3
 \end{array}$$

29. $2x - 3a$
 $\frac{2x - 3a}{4x^2 - 6ax}$
 $\quad - 6ax + 9a^2$
 $\frac{4x^2 - 12ax + 9a^2}{2x - 3a}$
 $\frac{8x^3 - 24ax^2 + 18a^2x}{- 12ax^2 + 36a^2x - 27a^3}$
 $\frac{8x^3 - 36ax^2 + 54a^2x - 27a^3}{- 8x^3 + 36ax^2 - 54a^2x + 27a^3}$
 $\frac{16x^3 - 72ax^2 + 108a^2x - 54a^3}{16x^3 - 72ax^2 + 108a^2x - 54a^3} \text{ (Ans.)}$
30. $ax - 5ay$
 $\frac{ax - 5ay}{a^2x^2 - 5a^2xy}$
 $\quad - 5a^2xy + 25a^2y^2$
 $\frac{a^2x^2 - 10a^2xy + 25a^2y^2}{ax - 5ay}$
 $\frac{a^3x^3 - 10a^3x^2y + 25a^3xy^2}{- 5a^3x^2y + 50a^3xy^2 - 125a^3y^3}$
 $\frac{a^3x^3 - 15a^3x^2y + 75a^3xy^2 - 125a^3y^3}{a^3x^3 - 3a^3x^2y + 3a^3xy^2 - a^3y^3}$
 $\frac{- 12a^3x^2y + 72a^3xy^2 - 124a^3y^3}{- 12a^3x^2y + 72a^3xy^2 - 124a^3y^3} \text{ (Ans.)}$
31. $4x - 3y$
 $\frac{4x - 3y}{16x^2 - 12xy}$
 $\quad - 12xy + 9y^2$
 $\frac{16x^2 - 24xy + 9y^2}{4x^2 + 20xy + 25y^2}$
 $\frac{12x^2 - 44xy - 16y^2}{12x^2 - 44xy - 16y^2} \text{ (Ans.)}$
32. From Ex. 27 $(x - 3)^3 = x^3 - 9x^2 + 27x - 27$
 $\frac{4x^2 - 4x + 1}{x^3 - 13x^2 + 31x - 28} \text{ (Ans.)}$
33. $2x^2 - 3a$
 $\frac{2x^2 + 3a}{4x^4 - 6ax^2}$
 $\quad + 6ax^2 - 9a^2$
 $\frac{4x^4}{- 9a^2}$
 $\frac{2x^2}{+ 3a}$
 $\frac{8x^6}{- 18a^2x^2 + 12ax^4 - 27a^3}$
- $3a - 2x$
 $\frac{3a - 2x}{9a^2 - 6ax}$
 $\quad - 6ax + 4x^2$
 $\frac{9a^2 - 12ax + 4x^2}{3a - 2x}$
 $\frac{27a^3 - 36a^2x + 12ax^2}{- 18a^2x + 24ax^2 - 8x^3}$
 $\frac{27a^3 - 54a^2x + 36ax^2 - 8x^3}{27a^3 - 54a^2x + 36ax^2 - 8x^3}$
- $ax - ay$
 $\frac{ax - ay}{a^2x^2 - a^2xy}$
 $\quad - a^2xy + a^2y^2$
 $\frac{a^2x^2 - 2a^2xy + a^2y^2}{ax - ay}$
 $\frac{a^3x^3 - 2a^3x^2y + a^3xy^2}{- a^3x^2y + 2a^3xy^2 - a^3y^3}$
 $\frac{a^3x^3 - 3a^3x^2y + 3a^3xy^2 - a^3y^3}{a^3x^3 - 3a^3x^2y + 3a^3xy^2 - a^3y^3}$
- $2x + 5y$
 $\frac{2x + 5y}{4x^2 + 10xy}$
 $\quad + 10xy + 25y^2$
 $\frac{4x^2 + 20xy + 25y^2}{4x^2 + 20xy + 25y^2}$
- $2x - 1$
 $\frac{2x - 1}{4x^2 - 2x}$
 $\quad - 2x + 1$
 $\frac{4x^2 - 4x + 1}{4x^2 - 4x + 1}$

34. $x^3 - 5$

$$\begin{array}{r}
 x^3 + 4 \\
 \hline
 x^6 - 5x^3 \\
 + 4x^3 - 20 \\
 \hline
 x^6 - x^3 - 20 \\
 + x^5 + x^2 \\
 \hline
 x^6 - x^5 - x^3 - x^2 - 20
 \end{array}$$

36. $a - x^2 + 3$

$$\begin{array}{r}
 a + x^2 - 3 \\
 \hline
 a^2 - ax^2 + 3a \\
 + ax^2 \quad - x^4 + 3x^2 \\
 - 3a \quad + 3x^2 - 9 \\
 \hline
 a^2 \quad - x^4 + 6x^2 - 9
 \end{array}$$

37. $x^2 + x - a - 2$

$$\begin{array}{r}
 -x^2 + x - a + 2 \\
 \hline
 -x^4 - x^3 + ax^2 + 2x^2 \\
 + x^3 \quad + x^2 - ax - 2x \\
 - ax^2 \quad - ax \quad + a^2 + 2a \\
 + 2x^2 \quad + 2x \quad - 2a - 4 \\
 \hline
 -x^4 \quad + 5x^2 - 2ax \quad + a^2 \quad - 4
 \end{array}$$

38. $2x - 3$

$$\begin{array}{r}
 3x^2 - 5x \\
 \hline
 6x^3 - 9x^2 \\
 - 10x^2 + 15x \\
 \hline
 6x^3 - 19x^2 + 15x \\
 8x^3 - 25x^2 - 15x + 36 \\
 \hline
 -2x^3 + 6x^2 + 30x - 36 \text{ (Ans.)}
 \end{array}$$

$4x^2 - 3x$

$$\begin{array}{r}
 2x - 7 \\
 \hline
 8x^3 - 6x^2 \\
 - 28x^2 + 21x \\
 \hline
 8x^3 - 34x^2 + 21x \\
 9x^2 - 36x + 36 \\
 \hline
 8x^3 - 25x^2 - 15x + 36
 \end{array}$$

$3x - 6$

$$\begin{array}{r}
 3x - 6 \\
 \hline
 9x^2 - 18x \\
 - 18x + 36 \\
 \hline
 9x^2 - 36x + 36
 \end{array}$$

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1. $2(x + 3) = 12.$

Removing parenthesis,

$2x + 6 = 12.$

$2x = 6.$

$x = 3.$

2. $5(x - 1) = 30.$

Removing parenthesis,

$5x - 5 = 30.$

$5x = 35.$

$x = 7.$

3. $4(x + 6) = 16.$

$4x + 24 = 16.$

$4x = -8.$

$x = -2.$

4. $2(3 - x) + 1 = 2.$

$6 - 2x + 1 = 2.$

$5 = 2x.$

$x = \frac{5}{2}.$

5. $5(x - 3) + 14 = 4.$
 $5x - 15 + 14 = 4.$
 $5x = 5.$
 $x = 1.$
6. $5(x - 7) + 8x = 4.$
 $5x - 35 + 8x = 4.$
 $13x - 35 = 4.$
 $13x = 39.$
 $x = 3.$
7. $4(4x - 1) + 3 = x.$
 $16x - 4 + 3 = x.$
 $16x - 1 = x.$
 $15x = 1.$
 $x = \frac{1}{15}.$
8. $16 + 2(4y - 7) - 12y = 0.$
 $16 + 8y - 14 - 12y = 0.$
 $2 - 4y = 0.$
 $2 = 4y.$
 $y = \frac{1}{2}.$
9. $5x - 4(4 - x) - 11 = 0.$
 $5x - 16 + 4x - 11 = 0.$
 $9x = 27.$
 $x = 3.$
10. $2(x + 1) - 3 = 3(x - 1).$
 $2x + 2 - 3 = 3x - 3.$
 $2 = x.$
11. $4(2x - 5) + 15 = 3(x + 10).$
 $8x - 20 + 15 = 3x + 30.$
 $5x = 35.$
 $x = 7.$
12. $3(x + 6) + 8 = 5(6 + x).$
 $3x + 18 + 8 = 30 + 5x.$
 $-2x = 4.$
 $x = -2.$
13. $7(x + 5) = 4(x + 8) + 3.$
 $7x + 35 = 4x + 32 + 3.$
 $3x = 0.$
 $x = 0.$
14. $7(y - 2) - 2(3 + y) = 0.$
 $7y - 14 - 6 - 2y = 0.$
 $5y = 20.$
 $y = 4.$
15. $9y - 3(2y - 4) = 2(5 - y) + 7.$
 $9y - 6y + 12 = 10 - 2y + 7.$
 $5y = 5.$
 $y = 1.$
16. $2n - 9(2n + 4) = 2(n - 9).$
 $2n - 18n - 36 = 2n - 18.$
 $-18n = 18.$
 $n = -1.$
17. $7x - 12 - 2(x - 5) = x - 14.$
 $7x - 12 - 2x + 10 = x - 14.$
 $4x = -12.$
 $x = -3.$
18. $5(3h - 1) - 7h = 3(h + 7) - 1.$
 $15h - 5 - 7h = 3h + 21 - 1.$
 $5h = 25.$
 $h = 5.$
20. $(x + 3)^2 - (x + 5)^2 = -40.$
 $x^2 + 6x + 9 - x^2 - 10x - 25 = -40.$
 $24 = 4x.$
 $x = 6.$
21. $(x + 2)^2 - (x - 4)^2 + 48 = 0.$
 $x^2 + 4x + 4 - x^2 + 8x - 16 + 48 = 0.$
 $12x = -36.$
 $x = -3.$

22. $(2x - 4)(3x - 6) = 6x^2 + 72.$
 $6x^2 - 24x + 24 = 6x^2 + 72.$
 $-24x = 48.$
 $x = -2.$
23. $(x + 4)(x + 6) = (x + 18)(x + 13).$
 $x^2 + 10x + 24 = x^2 + 31x + 234.$
 $-21x = 210.$
 $x = -10.$
24. $(h + 2)(h + 3) = (h - 5)(h - 2).$
 $h^2 + 5h + 6 = h^2 - 7h + 10.$
 $12h = 4.$
 $h = \frac{1}{3}.$
25. $(k - 7)(5 + k) - (k - 5)(k + 7) + 5 = 0.$
 $(k^2 - 2k - 35) - (k^2 + 2k - 35) + 5 = 0.$
 $k^2 - 2k - 35 - k^2 - 2k + 35 + 5 = 0.$
 $-4k + 5 = 0.$
 $4k = 5.$
 $k = \frac{5}{4}.$

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1. Let $x =$ the greater number.
Then $49 - x =$ the less number,
and $2x = 7 + 5(49 - x).$
Whence $x = 36; 49 - x = 13.$
2. Let $x =$ the less number.
Then $45 - x =$ the greater number,
and $10x = 300 - 5(45 - x).$
Whence $x = 15; 45 - x = 30.$
3. Let $x =$ the first number.
Then $15 - x =$ the second number,
and $2x - 10(15 - x) = 0.$
Whence $x = 12\frac{1}{2}; 15 - x = 2\frac{1}{2}.$
4. Let $x =$ the less part.
Then $75 - x =$ the greater part,
and $29 + 4x = 3(75 - x).$
Whence $x = 28; 75 - x = 47.$
5. Let $x, x + 1,$ be the integers.
Then $2x + 4(x + 1) = 106.$
Whence $x = 17; x + 1 = 18.$

6. Let $x, x + 1$, be the integers.

Then $4(x + 1) - 3x = 17.$

Whence $x = 13; x + 1 = 14.$

7. Let

$x =$ the first number.

Then

$15 - x =$ the second number,

and

$7x = 54 + 10(15 - x).$

Whence

$x = 12; 15 - x = 3.$

8. Let

$x =$ the first number.

Then

$15 - x =$ the second number,

and

$2x = 240 + 5(15 - x).$

Whence

$x = 45; 15 - x = -30.$

9. Let

$x =$ the greater side.

Since the sum of the two sides = half the perimeter,

then

$45 - x =$ the less side,

and

$5x + 4(45 - x) = 207.$

Whence

$x = 27; 45 - x = 18.$

10. Let $x, x + 1$, be the integers.

Then $(x + 1)^2 - x^2 = 35.$

Whence

$x = 17; x + 1 = 18.$

11. Let $x, x + 1$, be the integers.

Then $(x + 1)^2 - x^2 = 71.$

Whence

$x = 35; x + 1 = 36.$

12. Let $x, x + 2$, be the integers.

Then $(x + 2)^2 - x^2 = 104.$

Whence

$x = 25; x + 2 = 27.$

13. Let $x, x + 1$, be the integers.

Then $x^2 + (x + 1)^2 = 2(x + 1)^2 - 17.$

Whence

$x = 8; x + 1 = 9.$

14. Let $x, x + 2$, be the integers.

Then $(x + 2)^2 - x^2 = 48.$

Whence

$x = 11; x + 2 = 13.$

15. Let $x, x + 2$, be the integers.

Then $x(x + 2) = (x + 2)^2 - 42.$

Whence

$x = 19; x + 2 = 21.$

16. Let $x, x + 2$, be the integers.

Then $x(x + 2) = 44 + x^2.$

Whence

$x = 22; x + 2 = 24.$

17. Let $b = \text{B's age in years.}$
 Then $3b = \text{A's age in years,}$
 and $b + 10 = \text{C's age in years.}$
 But $b + 3b + b + 10 = 45.$
 Whence $b = 7; 3b = 21; b + 10 = 17.$

18. Let $b = \text{B's age in years, now.}$
 Then $2b = \text{A's age in years, now,}$
 and $2b + 7 = \text{C's age in years, now.}$
 Then $b + 6, 2b + 6, 2b + 13,$ will represent their respective ages 6 years hence.

Therefore $b + 6 + 2b + 6 + 2b + 13 = 85.$

Whence $b = 12; 2b = 24; 2b + 7 = 31.$

19. Let $b = \text{B's age in years, now.}$
 Then $b + 10 = \text{A's age in years, now,}$
 and $b - 6 = \text{C's age in years, now.}$
 Then $b - 6, b + 4, b - 12,$ will represent their respective ages 6 years ago.

Therefore $b - 6 + b + 4 + b - 12 = 40.$

Whence $b = 18; b + 10 = 28; b - 6 = 12.$

20. Let $b = \text{B's age in years, now.}$
 Then $2b + 2 = \text{A's age in years, now,}$
 and $2b - 5 = \text{C's age in years, now.}$
 Then $b + 6, 2b + 8, 2b + 1,$ will represent their respective ages 6 years hence.

Therefore $b + 6 + 2b + 8 + 2b + 1 = 75.$

Whence $b = 12; 2b + 2 = 26; 2b - 5 = 19.$

21. Let $x = \text{the number of years.}$
 Then $50 - x = \text{A's age } x \text{ years ago,}$
 and $36 - x = \text{B's age } x \text{ years ago.}$
 Therefore $50 - x = 3(36 - x).$
 Whence $x = 29.$

22. Let $x = \text{the number of years.}$
 Then $19 + x = \text{A's age } x \text{ years hence,}$
 and $54 + x = \text{B's age } x \text{ years hence.}$

If A is half as old as B, then B is twice as old as A.

That is, $54 + x = 2(19 + x).$

Whence $x = 16.$

23. Let $x = \text{B's age in years, now.}$
 Then $x + 30 = \text{A's age in years, now.}$
 Then $x + 20, x + 50$, will represent their respective ages 20 years hence.

Therefore $x + 50 = 2(x + 20).$
 Whence $x = 10; x + 30 = 40.$

24. Let $x = \text{B's age in years, now.}$
 Then $3x = \text{A's age in years, now.}$
 Then $x + 15, 3x + 15$, will represent their respective ages 15 years hence.

Therefore $3x + 15 = 2(x + 15).$
 Whence $x = 15; 3x = 45.$

25. Let $x = \text{B's age in years, now.}$
 Then $2x + 8 = \text{A's age in years, now.}$
 Then $x - 16, 2x - 8$, will represent their respective ages 16 years ago.
 Therefore $2x - 8 = 4(x - 16).$
 Whence $x = 28; 2x + 8 = 64.$

27. Let $s = \text{the side of the square in rods.}$
 Then $s + 15 = \text{the length of the rectangle in inches,}$
 and $s - 10 = \text{the width of the rectangle in inches.}$
 Therefore $s^2 = (s + 15)(s - 10).$
 Whence $s = 30 \text{ rods, the side of the square;}$
 $s + 15 = 45 \text{ rods, } s - 10 = 20 \text{ rods, the sides of the rectangle.}$

28. Let $x = \text{the breadth in feet.}$
 Then $3x - 3 = \text{the length in feet,}$
 and $2x + 2(3x - 3) = 210.$
 Whence $x = 27; 3x - 3 = 78.$

29. Let $x = \text{the breadth in feet.}$
 Then $2x + 6 = \text{the length in feet,}$
 and $2x + 2(2x + 6) = 228.$
 Whence $x = 36; 2x + 6 = 78.$

30. Let $x = \text{the breadth in feet.}$
 Then $x + 20 = \text{the length in feet.}$
 But $2x + 2(x + 20) = 240 \text{ (feet).}$
 Whence $x = 50; x + 20 = 70.$

31. Let $x = \text{the breadth in yards.}$
 Then $2x - 5 = \text{the length in yards,}$
 and $2x + 2(2x - 5) = 260 \text{ (yards).}$
 Whence $x = 45; 2x - 5 = 85.$

32. Let x = the width of the flower bed in feet.

Then $24 - 2x$ and $30 - 2x$ = the sides of the grassplot,

and $2(24 - 2x) + 2(30 - 2x)$ = the perimeter of the grassplot.

But the perimeter = $\frac{1}{2}(24 + 24 + 30 + 30) = \frac{1}{2}(108) = 54$.

Therefore $2(24 - 2x) + 2(30 - 2x) = 54$.

Whence $x = 6\frac{3}{4}$.

34. Let d = the number of dimes.

Then $35 - d$ = the number of quarters,

and $10d + 25(35 - d) = 650$.

Whence $d = 15$; $35 - d = 20$.

35. Let n = the number of nickels.

Then $30 - n$ = the number of dimes,

and $5n + 10(30 - n) = 260$.

Whence $n = 8$; $30 - n = 22$.

36. Let d = the number of dimes.

Then $d + 5$ = the number of nickels,

and $2(2d + 5)$ = the number of quarters.

But $10d + 5(d + 5) + (25)(2)(2d + 5) = 1080$.

Whence $d = 7$; $d + 5 = 12$; $2(2d + 5) = 38$.

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$$\begin{array}{r|l} 1. \ x^2 + 10x + 24 & x + 4 \\ x^2 + 4x & x + 6 \\ \hline 6x + 24 & \\ 6x + 24 & \\ \hline \end{array}$$

$$\begin{array}{r} \text{Check. } x + 4 \\ x + 6 \\ \hline x^2 + 4x \\ + 6x + 24 \\ \hline x^2 + 10x + 24 \end{array}$$

$$\begin{array}{r|l} 2. \ x^2 - 2x - 15 & x + 3 \\ x^2 + 3x & x - 5 \\ \hline -5x - 15 & \\ -5x - 15 & \\ \hline \end{array}$$

$$\begin{array}{r} \text{Check. } x + 3 \\ x - 5 \\ \hline x^2 + 3x \\ -5x - 15 \\ \hline x^2 - 2x - 15 \end{array}$$

$$\begin{array}{r|l} 3. \ x^2 + x - 6 & x - 2 \\ x^2 - 2x & x + 3 \\ \hline 3x - 6 & \\ 3x - 6 & \\ \hline \end{array}$$

$$\begin{array}{r} \text{Check. } x - 2 \\ x + 3 \\ \hline x^2 - 2x \\ + 3x - 6 \\ \hline x^2 + x - 6 \end{array}$$

4.
$$\begin{array}{r|l} 3x^2 + 5x + 2 & x + 1 \\ 3x^2 + 3x & 3x + 2 \\ \hline 2x + 2 & \\ 2x + 2 & \\ \hline \end{array}$$
5.
$$\begin{array}{r|l} 2a^2 - 3a - 2 & 2a + 1 \\ 2a^2 + a & a - 2 \\ \hline -4a - 2 & \\ -4a - 2 & \\ \hline \end{array}$$
6.
$$\begin{array}{r|l} 5x^2 - 22x + 8 & x - 4 \\ 5x^2 - 20x & 5x - 2 \\ \hline -2x + 8 & \\ -2x + 8 & \\ \hline \end{array}$$
7.
$$\begin{array}{r|l} 6x^2 + 19x - 7 & 3x - 1 \\ 6x^2 - 2x & 2x + 7 \\ \hline 21x - 7 & \\ 21x - 7 & \\ \hline \end{array}$$
8.
$$\begin{array}{r|l} 3x^2 - ax - 2a^2 & x - a \\ 3x^2 - 3ax & 3x + 2a \\ \hline 2ax - 2a^2 & \\ 2ax - 2a^2 & \\ \hline \end{array}$$
9.
$$\begin{array}{r|l} 4x^2 - 8ax + 3a^2 & 2x - 3a \\ 4x^2 - 6ax & 2x - a \\ \hline -2ax + 3a^2 & \\ -2ax + 3a^2 & \\ \hline \end{array}$$
10.
$$\begin{array}{r|l} x^3 - 8x^2 + 6x + 12 & x - 2 \\ x^3 - 2x^2 & x^2 - 6x - 6 \\ \hline -6x^2 + 6x & \\ -6x^2 + 12x & \\ \hline -6x + 12 & \\ -6x + 12 & \\ \hline \end{array}$$
11.
$$\begin{array}{r|l} 8x^3 - 12x^2 + 6x - 1 & 2x - 1 \\ 8x^3 - 4x^2 & 4x^2 - 4x + 1 \\ \hline -8x^2 + 6x & \\ -8x^2 + 4x & \\ \hline 2x - 1 & \\ 2x - 1 & \\ \hline \end{array}$$
12.
$$\begin{array}{r|l} x^3 - 11x - 6 & x + 3 \\ x^3 + 3x^2 & x^2 - 3x - 2 \\ \hline -3x^2 - 11x & \\ -3x^2 - 9x & \\ \hline -2x - 6 & \\ -2x - 6 & \\ \hline \end{array}$$
- Check.
$$\begin{array}{r} x^2 - 3x - 2 \\ x + 3 \\ \hline x^3 - 3x^2 - 2x \\ + 3x^2 - 9x - 6 \\ \hline x^3 - 11x - 6 \end{array}$$
13.
$$\begin{array}{r|l} x^3 - 14x - 8 & x - 4 \\ x^3 - 4x^2 & x^2 + 4x + 2 \\ \hline 4x^2 - 14x & \\ 4x^2 - 16x & \\ \hline 2x - 8 & \\ 2x - 8 & \\ \hline \end{array}$$
14.
$$\begin{array}{r|l} x^3 - 2x^2 - 5x + 6 & x - 3 \\ x^3 - 3x^2 & x^2 + x - 2 \\ \hline x^2 - 5x & \\ x^2 - 3x & \\ \hline -2x + 6 & \\ -2x + 6 & \\ \hline \end{array}$$
15.
$$\begin{array}{r|l} x^3 - 5x + 2 & x^2 + 2x - 1 \\ x^3 + 2x^2 - x & x - 2 \\ \hline -2x^2 - 4x + 2 & \\ -2x^2 - 4x + 2 & \\ \hline \end{array}$$
16.
$$\begin{array}{r|l} x^3 - 11x + 6 & x^2 + 3x - 2 \\ x^3 + 3x^2 - 2x & x - 3 \\ \hline -3x^2 - 9x + 6 & \\ -3x^2 - 9x + 6 & \\ \hline \end{array}$$

$$\begin{array}{r}
 17. \quad x^4 \qquad - 11x^2 + 2x + 12 \big| x^2 + 2x - 4 \\
 \quad x^4 + 2x^3 - 4x^2 \qquad \qquad \quad x^2 - 2x - 3 \\
 \hline
 \quad - 2x^3 - 7x^2 + 2x \\
 \quad - 2x^3 - 4x^2 + 8x \\
 \hline
 \qquad - 3x^2 - 6x + 12 \\
 \quad - 3x^2 - 6x + 12 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{Check. } x^2 + 2x - 4 \\
 \quad x^2 - 2x - 3 \\
 \hline
 \quad x^4 + 2x^3 - 4x^2 \\
 \quad - 2x^3 - 4x^2 + 8x \\
 \quad - 3x^2 - 6x + 12 \\
 \hline
 \quad x^4 \qquad - 11x^2 + 2x + 12
 \end{array}$$

$$\begin{array}{r}
 18. \quad x^3 \qquad + 8 \big| x + 2 \\
 \quad x^3 + 2x^2 \qquad \quad x^2 - 2x + 4 \\
 \hline
 \quad - 2x^2 \\
 \quad - 2x^2 - 4x \\
 \hline
 \qquad 4x + 8 \\
 \quad 4x + 8 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{Check. } x^2 - 2x + 4 \\
 \quad x + 2 \\
 \hline
 \quad x^3 - 2x^2 + 4x \\
 \quad + 2x^2 - 4x + 8 \\
 \hline
 \quad x^3 \qquad \qquad \qquad + 8
 \end{array}$$

$$\begin{array}{r}
 19. \quad 8x^3 \qquad + 1 \big| 2x + 1 \\
 \quad 8x^3 + 4x^2 \qquad \quad 4x^2 - 2x + 1 \\
 \hline
 \quad - 4x^2 \\
 \quad - 4x^2 - 2x \\
 \hline
 \qquad 2x + 1 \\
 \quad 2x + 1 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 20. \quad 27x^3 \qquad + 8a^3 \big| 3x + 2a \\
 \quad 27x^3 + 18x^2a \qquad \quad 9x^2 - 6ax + 4a^2 \\
 \hline
 \quad - 18x^2a \\
 \quad - 18x^2a - 12a^2x \\
 \hline
 \qquad 12a^2x + 8a^3 \\
 \quad 12a^2x + 8a^3 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 21. \quad ax + 3a - bx - 3b \big| x + 3 \\
 \quad ax + 3a \qquad \quad a - b \\
 \hline
 \quad - bx - 3b \\
 \quad - bx - 3b \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 22. \quad 3ax - ay - 6bx + 2by \big| -a + 2b \\
 \quad 3ax \qquad - 6bx \qquad \quad - 3x + y \\
 \hline
 \quad - ay \qquad + 2by \\
 \quad - ay \qquad + 2by \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 23. \quad 28ax + 9ny - 21ay - 12nx & -4x + 3y \\
 28ax & -21ay \\
 \hline
 & +9ny \quad -12nx \\
 & +9ny \quad -12nx \\
 \hline
 \end{array}$$

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$$\begin{array}{r|l}
 1. \quad 6x^2 - 13x + 6 & 2x - 3 \\
 6x^2 - 9x & 3x - 2 \\
 \hline
 & -4x + 6 \\
 & -2x + 6 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{Check.} \quad 2x - 3 \\
 3x - 2 \\
 \hline
 6x^2 - 9x \\
 -4x + 6 \\
 \hline
 6x^2 - 13x + 6
 \end{array}$$

$$\begin{array}{r|l}
 3. \quad 6x^2 + 11x - 35 & 2x + 7 \\
 6x^2 + 21x & 3x - 5 \\
 \hline
 & -10x - 35 \\
 & -10x - 35 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 4. \quad 12a^2 + 19a - 21 & 4a - 3 \\
 12a^2 - 9a & 3a + 7 \\
 \hline
 & 28a - 21 \\
 & 28a - 21 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 2. \quad 25x^4 + 30x^2 - 7 & 5x^2 + 7 \\
 25x^4 + 35x^2 & 5x^2 - 1 \\
 \hline
 & -5x^2 - 7 \\
 & -5x^2 - 7 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 5. \quad x^3 - 2x^2 + 4x - 8 & x - 2 \\
 x^3 - 2x^2 & x^2 + 4 \\
 \hline
 & 4x - 8 \\
 & 4x - 8 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 6. \quad a^3 - 3a^2b + 3ab^2 - b^3 & a - b \\
 a^3 - a^2b & a^2 - 2ab + b^2 \\
 \hline
 & -2a^2b \\
 & -2a^2b + 2ab^2 \\
 \hline
 & ab^2 - b^3 \\
 & ab^2 - b^3 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 7. \quad x^3 - 15x^2 + 65x + 63 & x - 7 \\
 x^3 - 7x^2 & x^2 - 8x + 9 + \frac{126}{x-7} \\
 \hline
 & -8x^2 + 65x \\
 & -8x^2 + 56x \\
 \hline
 & +9x + 63 \\
 & +9x - 63 \\
 \hline
 & +126
 \end{array}$$

$$\begin{array}{r}
 \text{Check.} \quad x^2 - 8x + 9 \\
 x - 7 \\
 \hline
 x^3 - 8x^2 + 9x \\
 -7x^2 + 56x - 63 \\
 \hline
 126 \\
 x^3 - 15x^2 + 65x + 63
 \end{array}$$

$$\begin{array}{r|l}
 8. \quad -25x^3 + 5x^2 + 5x - 1 & 5x^2 - 1 \\
 \underline{-25x^3 \qquad \qquad + 5x} & \underline{-5x + 1} \\
 & 5x^2 \qquad \qquad - 1 \\
 & \underline{5x^2 \qquad \qquad - 1}
 \end{array}$$

$$\begin{array}{r|l}
 9. \quad 2x^3 - 14x^2 + 14x + 12 & 2x - 4 \\
 \underline{2x^3 - 4x^2} & \underline{x^2 - 5x - 3} \\
 & -10x^2 + 14x \\
 & \underline{-10x^2 + 20x} \\
 & \qquad -6x + 12 \\
 & \underline{-6x + 12}
 \end{array}$$

$$\begin{array}{r|l}
 10. \quad 6x^3 \qquad \qquad - 18x + 12 & 3x - 3 \\
 \underline{6x^3 - 6x^2} & \underline{2x^2 + 2x - 4} \\
 & 6x^2 - 18x \\
 & \underline{6x^2 - 6x} \\
 & \qquad -12x + 12 \\
 & \underline{-12x + 12}
 \end{array}$$

$$\begin{array}{r|l}
 11. \quad 4x^3 \qquad \qquad + 16x + 12 & x^2 + x - 3 \\
 \underline{4x^3 + 4x^2 - 12x} & \underline{4x - 4 + \frac{32x}{x^2 + x - 3}} \\
 & -4x^2 + 28x + 12 \\
 & \underline{-4x^2 - 4x + 12} \\
 & \qquad 32x
 \end{array}$$

$$\begin{array}{r}
 \text{Check. } x^2 + x - 3 \\
 \underline{4x - 4} \\
 4x^3 + 4x^2 - 12x \\
 \underline{-4x^2 - 4x + 12} \\
 4x^3 \qquad \qquad - 16x + 12 \\
 \qquad \qquad \qquad + 32x \\
 \underline{4x^3 \qquad \qquad + 16x + 12}
 \end{array}$$

$$\begin{array}{r|l}
 12. \quad 6x^3 - 2x^2 + 10x + 18 & 3x^2 - 4x + 9 \\
 \underline{6x^3 - 8x^2 + 18x} & \underline{2x + 2} \\
 & 6x^2 - 8x + 18 \\
 & \underline{6x^2 - 8x + 18}
 \end{array}$$

$$\begin{array}{r|l}
 13. \quad 3a^3 + 28a^2 + 29a - 140 & 3a - 5 \\
 \underline{3a^3 - 5a^2} & \underline{a^2 + 11a + 28} \\
 & 33a^2 \\
 & \underline{33a^2 - 55a} \\
 & \qquad 84a - 140 \\
 & \underline{84a - 140}
 \end{array}$$

$$\begin{array}{r|l}
 14. \quad -6x^3 + 23x^2 - 37x - 24 & 2x - 3 \\
 \hline
 -6x^3 + 9x^2 & \\
 \hline
 14x^2 & \\
 14x^2 - 21x & \\
 \hline
 -16x - 24 & \\
 -16x + 24 & \\
 \hline
 -48 &
 \end{array}
 \quad
 \begin{array}{r}
 -3x^2 + 7x - 8 + \frac{-48}{2x-3} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{Check.} \quad -3x^2 + 7x - 8 \\
 2x - 3 \\
 \hline
 -6x^3 + 14x^2 - 16x \\
 + 9x^2 - 21x + 24 \\
 \hline
 -48 \\
 -6x^3 + 23x^2 - 37x - 24
 \end{array}$$

$$\begin{array}{r|l}
 15. \quad 2x^2 + 10xy - 4x - 20y & 2x + 10y \\
 \hline
 2x^2 + 10xy & \\
 \hline
 -4x & \\
 -4x - 20y & \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 x - 2 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 16. \quad 3cx + 6ac + 8ax + 4x^2 & 3c + 4x \\
 \hline
 3cx & + 4x^2 \\
 \hline
 6ac + 8ax & \\
 6ac + 8ax & \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 x + 2a \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 17. \quad 12a^3 - 53a^2 + 53a + 8 & 4a^2 - 7a - 1 \\
 \hline
 12a^3 - 21a^2 - 3a & 3a - 8 \\
 \hline
 -32a^2 + 56a + 8 & \\
 -32a^2 + 56a + 8 & \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 18. \quad -15a^3 + 56a^2 - 99a + 70 & 3a^2 - 7a + 10 \\
 \hline
 -15a^3 + 35a^2 - 50a & -5a + 7 \\
 \hline
 21a^2 - 49a + 70 & \\
 21a^2 - 49a + 70 & \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 19. \quad 3x^4 + 11x^3 - 3x^2 + 17x - 4 & -3x^2 + x - 4 \\
 \hline
 3x^4 - x^3 + 4x^2 & \\
 \hline
 12x^3 - 7x^2 + 17x & \\
 12x^3 - 4x^2 + 16x & \\
 \hline
 -3x^2 + x - 4 & \\
 -3x^2 + x - 4 & \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 -x^2 - 4x + 1 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 20. \quad 6an + 15xy - 10ny - 9ax & -2n + 3x \\
 \hline
 6an & -9ax \\
 \hline
 15xy - 10ny & \\
 15xy - 10ny & \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 -3a + 5y \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 21. \quad a^4 + 11a^3 + 23a^2 - 55a - 140 & a^2 - 5 \\
 \hline
 a^4 & - 5a^2 \\
 \hline
 & 11a^3 + 28a^2 \\
 & 11a^3 & - 55a \\
 \hline
 & 28a^2 & - 140 \\
 & 28a^2 & - 140 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 22. \quad 25x^3 - 10x^2 + 36x + 72 & 5x - 6 \\
 \hline
 25x^3 - 30x^2 & \\
 \hline
 & 20x^2 \\
 & 20x^2 - 24x \\
 \hline
 & 60x + 72 \\
 & 60x - 72 \\
 \hline
 & 144 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{Check.} \quad 5x^2 + 4x + 12 \\
 \quad 5x - 6 \\
 \hline
 \quad 25x^3 + 20x^2 + 60x \\
 \quad - 30x^2 - 24x - 72 \\
 \hline
 \quad \quad \quad + 144 \\
 \quad 25x^3 - 10x^2 + 36x + 72
 \end{array}$$

$$\begin{array}{r|l}
 23. \quad a^4 + 4a^3 + 6a^2 + 4a + 1 & a^2 + 2a + 1 \\
 \hline
 a^4 + 2a^3 + a^2 & \\
 \hline
 & 2a^3 + 5a^2 \\
 & 2a^3 + 4a^2 + 2a \\
 \hline
 & a^2 + 2a + 1 \\
 & a^2 + 2a + 1 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 24. \quad x^4 - 4x^3 - 31x^2 + 40x + 21 & x^2 - 7x - 3 \\
 \hline
 x^4 - 7x^3 - 3x^2 & \\
 \hline
 & 3x^3 - 28x^2 \\
 & 3x^3 - 21x^2 - 9x \\
 \hline
 & - 7x^2 + 49x + 21 \\
 & - 7x^2 + 49x + 21 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 25. \quad a^4 - 8a^3 + 24a^2 - 32a + 16 & a^2 - 4a + 4 \\
 \hline
 a^4 - 4a^3 + 4a^2 & \\
 \hline
 & - 4a^3 + 20a^2 \\
 & - 4a^3 + 16a^2 - 16a \\
 \hline
 & 4a^2 - 16a + 16 \\
 & 4a^2 - 16a + 16 \\
 \hline
 \end{array}$$

$$\begin{array}{r|l}
 26. \quad 10x^4 - 27x^3 - 42x^2 + 224x - 419 & 2x^2 - 5x + 9 \\
 \hline
 10x^4 - 25x^3 + 45x^2 & \\
 - 2x^3 - 87x^2 & \\
 - 2x^3 + 5x^2 - 9x & \\
 \hline
 - 92x^2 + 233x & \\
 - 92x^2 + 230x - 414 & \\
 \hline
 3x - 5 &
 \end{array}$$

$$\begin{array}{r|l}
 27. \quad ax - 2bx + 3cx - 6cy + 4by - 2ay & -x + 2y \\
 ax & -2ay \\
 \hline
 - 2bx + 3cx - 6cy + 4by & \\
 - 2bx & + 4by \\
 \hline
 3cx - 6cy & \\
 3cx - 6cy &
 \end{array}$$

$$\begin{array}{r|l}
 28. \quad x^4 + a^2x^2 & + a^4 \\
 x^4 - ax^3 + a^2x^2 & x^2 - ax + a^2 \\
 \hline
 ax^3 & x^2 + ax + a^2 \\
 ax^3 & \\
 \hline
 - a^2x^2 + a^3x & \\
 a^2x^2 - a^3x & \\
 a^2x^2 - a^3x + a^4 &
 \end{array}$$

$$\begin{array}{r|l}
 29. \quad a^4 + 3a^2b^2 & + 4b^4 \\
 a^4 - a^3b + 2a^2b^2 & a^2 - ab + 2b^2 \\
 \hline
 a^3b + a^2b^2 & a^2 + ab + 2b^2 \\
 a^3b - a^2b^2 + 2ab^3 & \\
 \hline
 2a^2b^2 - 2ab^3 & \\
 2a^2b^2 - 2ab^3 + 4b^4 &
 \end{array}$$

$$\begin{array}{r|l}
 30. \quad x^6 - 4x^3y^3 & + 8y^6 \\
 x^6 + 2x^5y + 2x^4y^2 & x^2 + 2xy + 2y^2 \\
 \hline
 - 2x^5y - 2x^4y^2 - 4x^3y^3 & x^4 - 2x^3y + 2x^2y^2 - 4xy^3 + 4y^4 \\
 - 2x^5y - 4x^4y^2 - 4x^3y^3 & \\
 \hline
 2x^4y^2 & \\
 2x^4y^2 + 4x^3y^3 + 4x^2y^4 & \\
 - 4x^3y^3 - 4x^2y^4 & \\
 - 4x^3y^3 - 8x^2y^4 - 8xy^5 & \\
 \hline
 4x^2y^4 + 8xy^5 & \\
 4x^2y^4 + 8xy^5 + 8y^6 &
 \end{array}$$

$$\begin{array}{r}
 36. \ a^6 \\
 \underline{a^6 + 7a^4b} \\
 -7a^4b \\
 \underline{-7a^4b - 49a^2b^2} \\
 49a^2b^2 + 343b^3 \\
 \underline{49a^2b^2 + 343b^3}
 \end{array}
 \qquad
 \begin{array}{r}
 + 343b^3 \overline{) a^2 + 7b} \\
 \underline{a^4 - 7a^2b + 49b^2}
 \end{array}$$

$$\begin{array}{r}
 37. \ 8a^3 \\
 \underline{8a^3 - 16a^2x^3} \\
 16a^2x^3 \\
 \underline{16a^2x^3 - 32ax^6} \\
 32ax^6 - 64x^9 \\
 \underline{32ax^6 - 64x^9}
 \end{array}
 \qquad
 \begin{array}{r}
 - 64x^9 \overline{) 2a - 4x^3} \\
 \underline{4a^2 + 8ax^3 + 16x^6}
 \end{array}$$

$$\begin{array}{r}
 38. \ x^5 \\
 \underline{x^5 + x^4y} \\
 -x^4y \\
 \underline{-x^4y - x^3y^2} \\
 x^3y^2 \\
 \underline{x^3y^2 + x^2y^3} \\
 -x^2y^3 \\
 \underline{-x^2y^3 - xy^4} \\
 xy^4 + y^5 \\
 \underline{xy^4 + y^5}
 \end{array}
 \qquad
 \begin{array}{r}
 + y^5 \overline{) x + y} \\
 \underline{x^4 - x^3y + x^2y^2 - xy^3 + y^4}
 \end{array}$$

$$\begin{array}{r}
 39. \ x^5 \\
 \underline{x^5 - x^4y} \\
 x^4y \\
 \underline{x^4y - x^3y^2} \\
 x^3y^2 \\
 \underline{x^3y^2 - x^2y^3} \\
 x^2y^3 \\
 \underline{x^2y^3 - xy^4} \\
 xy^4 + y^5 \\
 \underline{xy^4 - y^5} \\
 2y^5
 \end{array}
 \qquad
 \begin{array}{r}
 + y^5 \overline{) x - y} \\
 \underline{x^4 + x^3y + x^2y^2 + xy^3 + y^4 + \frac{2y^5}{x-y}}
 \end{array}$$

40. x^5

$$\begin{array}{r} x^5 + x^4y \\ - x^4y \\ \hline \end{array}$$

$$\begin{array}{r} - x^4y - x^3y^2 \\ \hline \end{array}$$

$$x^3y^2$$

$$\begin{array}{r} x^3y^2 + x^2y^3 \\ - x^2y^3 \\ \hline \end{array}$$

$$- x^2y^3$$

$$\begin{array}{r} - x^2y^3 - xy^4 \\ \hline \end{array}$$

$$xy^4 - y^5$$

$$\begin{array}{r} xy^4 + y^5 \\ - 2y^5 \\ \hline \end{array}$$

$$- 2y^5$$

$$- y^5$$

$$x + y$$

$$\begin{array}{r} x^4 - x^3y + x^2y^2 - xy^3 + y^4 + \frac{-2y^5}{x+y} \\ \hline \end{array}$$

41. x^5

$$\begin{array}{r} x^5 - x^4y \\ \hline \end{array}$$

$$x^4y$$

$$\begin{array}{r} x^4y - x^3y^2 \\ \hline \end{array}$$

$$x^3y^2$$

$$\begin{array}{r} x^3y^2 - x^2y^3 \\ \hline \end{array}$$

$$x^2y^3$$

$$\begin{array}{r} x^2y^3 - xy^4 \\ \hline \end{array}$$

$$xy^4 - y^5$$

$$\begin{array}{r} xy^4 - y^5 \\ \hline \end{array}$$

$$- y^5$$

$$x - y$$

$$\begin{array}{r} x^4 + x^3y + x^2y^2 + xy^3 + y^4 \\ \hline \end{array}$$

42. $27x^6$

$$\begin{array}{r} 27x^6 + 18x^4n^4 \\ - 18x^4n^4 \\ \hline \end{array}$$

$$- 18x^4n^4$$

$$\begin{array}{r} - 18x^4n^4 - 12x^2n^8 \\ \hline \end{array}$$

$$12x^2n^8 + 8n^{12}$$

$$\begin{array}{r} 12x^2n^8 + 8n^{12} \\ \hline \end{array}$$

$$+ 8n^{12}$$

$$3x^2 + 2n^4$$

$$\begin{array}{r} 9x^4 - 6x^2n^4 + 4n^8 \\ \hline \end{array}$$

43. y^5

$$\begin{array}{r} y^5 - 5y^4 \\ \hline \end{array}$$

$$5y^4$$

$$\begin{array}{r} 5y^4 - 25y^3 \\ \hline \end{array}$$

$$25y^3$$

$$\begin{array}{r} 25y^3 - 125y^2 \\ \hline \end{array}$$

$$120y^2$$

$$\begin{array}{r} 120y^2 - 600y \\ \hline \end{array}$$

$$600y - 3000$$

$$\begin{array}{r} 600y - 3000 \\ \hline \end{array}$$

$$- 5y^2$$

$$- 3000$$

$$y - 5$$

$$\begin{array}{r} y^4 + 5y^3 + 25y^2 + 120y + 600 \\ \hline \end{array}$$

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2. $bx + b = 4b$.
Transposing and combining,
 $bx = 3b$.
Whence $x = 3$.
3. $x + a = a + b$.
Transposing and combining,
 $x = b$.
4. $y - b = a - b$.
Transposing and combining,
 $y = a$.
5. $3ax + 4a = 7a$.
Transposing and combining,
 $3ax = 3a$.
Whence $x = 1$.
6. $cz + c^2 = 6c^2$.
Transposing and combining,
 $cz = 5c^2$.
Whence $z = 5c$.
7. $ax + ab = ac$.
 $ax = ac - ab$.
 $x = c - b$.
8. $4bx - b^2 = 5b^2$.
 $4bx = 6b^2$.
 $x = \frac{3}{2}b$.
9. $a^2c^2 + 3cx = 7a^2c^2$.
 $3cx = 6a^2c^2$.
 $x = 2a^2c$.
10. $5a^2x + 6a^2 = a^2$.
 $5a^2x = -5a^2$.
 $x = -1$.
11. $bc - cx = 4bc$.
 $-cx = 3bc$.
 $x = -3b$.
12. $5(b + x) = 10b$.
 $5b + 5x = 10b$.
 $5x = 5b$.
 $x = b$.
13. $6(c - x) + 18c = 0$.
 $6c - 6x + 18c = 0$.
 $24c = 6x$.
 $x = 4c$.
14. $bx - (b + c) = 5b - c$.
 $bx - b - c = 5b - c$.
 $bx = 6b$.
 $x = 6$.
15. $x + 2(b - c) = 4c + 2b$.
 $x + 2b - 2c = 4c + 2b$.
 $x = 6c$.
16. $3ax - ab = 2ax - ac$.
 $ax = ab - ac$.
 $x = b - c$.
17. $4cy - 3ac = 5ac + 2cy$.
 $2cy = 8ac$.
 $y = 4a$.
18. $3cy + 2bc = 6bc + 2cy - 3ac$.
 $cy = 4bc - 3ac$.
 $y = 4b - 3a$.
19. $4by - 7a^2b = 6ab^2 + 3by$.
 $by = 7a^2b + 6ab^2$.
 $y = 7a^2 + 6ab$.
21. $ax + bx = ac + bc$.
Writing with binomial coefficient,
 $(a + b)x = ac + bc$.
Dividing, $x = c$.
22. $5ax + 4cx = 5ab + 4cb$.
Writing with binomial coefficient,
 $(5a + 4c)x = 5ab + 4cb$.
Dividing, $x = b$.
23. $a^2x + 1 - a^4 - x = 0$.
Transposing, $a^2x - x = a^4 - 1$.
Writing with binomial coefficient,
 $(a^2 - 1)x = a^4 - 1$.
Dividing, $x = a^2 + 1$.

$$\begin{aligned}
 24. \quad ax + 2ab &= 2a^2 + bx. \\
 ax - bx &= 2a^2 - 2ab. \\
 (a - b)x &= 2a^2 - 2ab. \\
 x &= 2a.
 \end{aligned}$$

$$\begin{aligned}
 25. \quad ax - a^3 - 4 &= 3a - x. \\
 ax + x &= a^3 + 3a + 4. \\
 (a + 1)x &= a^3 + 3a + 4. \\
 x &= a^2 - a + 4.
 \end{aligned}$$

$$\begin{aligned}
 26. \quad ax - ac + bc &= 2ac - 5bc + 2bx. \\
 ax - 2bx &= 3ac - 6bc. \\
 (a - 2b)x &= 3ac - 6bc. \\
 x &= 3c.
 \end{aligned}$$

$$\begin{aligned}
 27. \quad 4b^2c^2 + (a + bx)c &= (a - bx)c. \\
 4b^2c^2 + ac + bcx &= ac - bcx. \\
 2bcx &= -4b^2c^2. \\
 x &= -2bc.
 \end{aligned}$$

$$\begin{aligned}
 28. \quad (x + a)(x + b) &= x^2 + 2a^2 + 3ab. \\
 x^2 + ax + bx + ab &= x^2 + 2a^2 + 3ab. \\
 ax + bx &= 2a^2 + 2ab. \\
 (a + b)x &= 2a^2 + 2ab. \\
 x &= 2a.
 \end{aligned}$$

$$\begin{aligned}
 29. \quad 15(x - a) - 6(x + a) &= 3(5a - 3x). \\
 15x - 15a - 6x - 6a &= 15a - 9x. \\
 18x &= 36a. \\
 x &= 2a.
 \end{aligned}$$

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1. Let	x = number of hours.
Then	$6x$ = distance by A in miles.
and	$9x$ = distance by B in miles.
But	$6x + 9x = 90.$
Whence	$x = 6.$

2. Let	x = B's rate in miles per hour.
Then	$3x$ = A's rate in miles per hour.
Also	$3x \cdot 6 = 18x$ = distance by A in miles,
and	$6x$ = distance by B in miles.
But	$18x + 6x = 120.$
Whence	$x = 5; 3x = 15.$

3. Let	x = B's rate in miles per hour.
Then	$x + 4$ = A's rate in miles per hour.
Also	$8x$ = distance by B in miles,
and	$8(x + 4)$ = distance by A in miles.
But	$8x + 8(x + 4) = 144.$
Whence	$x = 7; x + 4 = 11.$

4. Let	x = B's rate in miles per hour.
Then	$x - 3$ = A's rate in miles per hour.
Also	$9x$ = distance by B in miles,
and	$9(x - 3)$ = distance by A in miles.
But	$9x + 9(x - 3) = 189.$
Whence	$x = 12; x - 3 = 9.$

5. Let $x = A$'s rate in miles per hour.
 Then $x - 4 = B$'s rate in miles per hour.
 But $x - 4 = \frac{2}{3}x$.
 Transposing, $\frac{1}{3}x = 4$.
 Dividing by $\frac{1}{3}$, $x = 12$; $x - 4 = 8$.
 Now let $y =$ number of hours.
 Then $12y =$ distance by A in miles,
 and $8y =$ distance by B in miles.
 But $12y + 8y = 180$.
 Whence $y = 9$.

6. Let $x = A$'s rate in miles per hour.
 Then $2x = B$'s rate in miles per hour.
 Also $2x =$ distance in miles by A in 2 hours,
 and $10x =$ distance in miles by B in 5 hours.
 But $2x + 10x = 144$.
 Whence $x = 12$; $2x = 24$ (their respective rates),
 and $2x = 24$; $10x = 120$ (their respective distances).

7. Let $x = A$'s rate in miles per hour.
 Then $x - 4 = B$'s rate in miles per hour.
 Also $6x =$ distance in miles by A in 6 hours,
 and $9(x - 4) =$ distance in miles by B in 9 hours.
 But $6x = 9(x - 4)$.
 Whence $x = 12$; $x - 4 = 8$,
 and $6x = 72$, A 's distance,
 $9(x - 4) = 72$, B 's distance.

8. Let $x =$ common rate of A and B in miles per hour.
 Then $8x =$ distance in miles by each.
 But $8x + 8x = 192$.
 Whence $x = 12$.

9. Let $x = B$'s rate in miles per hour.
 Then $x - 2 = A$'s rate in miles per hour.
 Also $12x =$ distance by B in miles,
 and $12(x - 2) =$ distance by A in miles.
 But $12x + 12(x - 2) = 192$.
 Whence $x = 9$; $x - 2 = 7$.

10. Let $x = A$'s rate in miles per hour.
 Then $3x = B$'s rate in miles per hour.
 Also $12x = \text{distance by } A \text{ in miles,}$
 and $36x = \text{distance by } B \text{ in miles.}$
 But $12x + 36x = 192.$
 Whence $x = 4; 3x = 12.$

11. Let $x = \text{distance by } B \text{ in miles.}$
 Then $192 - x = \text{distance by } A \text{ in miles.}$
 But $x - 42 = 192 - x.$
 Whence $x = 117; 192 - x = 75.$
 Also B 's rate $= 1\frac{1}{9}$, or 13 miles per hour,
 and A 's rate $= 7\frac{5}{9}$, or $8\frac{1}{3}$ miles per hour.

12. Let $x = A$'s rate in miles per hour.
 Then $x + 4 = B$'s rate in miles per hour.
 Also $6x = \text{distance by } A \text{ in miles,}$
 and $6(x + 4) = \text{distance by } B \text{ in miles.}$
 But $6x + 6(x + 4) = 192.$
 Whence $x = 14; x + 4 = 18.$

13. Let $x = B$'s rate in miles per hour.
 Then $x + 6 = A$'s rate in miles per hour.
 Also $12x = \text{distance by } B \text{ in miles,}$
 and $12(x + 6) = \text{distance by } A \text{ in miles.}$
 But $12x + 12(x + 6) = 192.$
 Whence $x = 5; x + 6 = 11.$

14. Since B goes twice as far as A in the same time, B 's rate is double A 's rate.

Let $x = A$'s rate in miles per hour,
 and $2x = B$'s rate in miles per hour.
 Then $x + 4 = 2x.$
 Whence $x = 4; 2x = 8.$ Thus they approached each other
 at the rate of 12 miles per hour;
 hence $144 \div 12 = 12,$ the number of hours they traveled.

15. Let $x = \text{the number of hours } A \text{ travels.}$
 Then $x - 4 = \text{the number of hours } B \text{ travels.}$
 Also $6x = \text{rate} \times \text{time} = \text{distance by } A \text{ in miles,}$
 and $9(x - 4) = \text{rate} \times \text{time} = \text{distance by } B \text{ in miles.}$
 Therefore $6x + 9(x - 4) = 144.$
 Whence $x = 12 \text{ hours, } A$'s time and the total time elapsed.

16. Let x = total time in hours.
Then $x - 3$ = number of hours A travels,
and $x - 5$ = number of hours B travels.
Then $16(x - 3)$ = distance by A in miles,
and $8(x - 5)$ = distance by B in miles.
But $16(x - 3) + 8(x - 5) = 144$.
Whence $x = 9\frac{2}{3}$.
17. Let x = the number of hours.
Then $45x$ = distance in miles by passenger train,
and $12x$ = distance in miles by freight train.
But $45x + 12x = 285$.
Whence $x = 5$.
18. Let x = A's time in hours.
Then $x - 6$ = B's time in hours.
Also $4x$ = distance by A in miles,
and $6(x - 6)$ = distance by B in miles.
But $4x = 6(x - 6)$.
Whence $x = 18$.
19. Let x = first cyclist's running time.
Then $x - 2$ = second cyclist's running time.
Also $10x$ = distance in miles by first cyclist,
and $12(x - 2)$ = distance in miles by second cyclist.
But $10x + 12(x - 2) = 108$.
Whence $x = 6$; $10x = 60$; $12(x - 2) = 48$.
20. Let x = running time of passenger train in hours.
Then $x + 2$ = running time of freight train in hours.
Also $42x$ = distance by passenger train in miles,
and $24(x + 2)$ = distance by freight train in miles.
But $42x + 24(x + 2) = 246$.
Whence $x = 3$.
21. Let x = second messenger's rate in miles per hour.
Then $8x$ = distance in miles by second messenger.
Since the first messenger will have traveled for 10 hours when overtaken, he will have gone $10 \times 8 = 80$ miles.
But both messengers must go the same distance.
Therefore $8x = 80$.
Whence $x = 10$.

22. Let x = distance in miles on trolley.

Then $x - 1$ = distance in miles on foot.

Also $\frac{d}{r} = \frac{x}{10} = \frac{1}{10}x$, number of hours on trolley,

and $\frac{d}{r} = \frac{x-1}{3} = \frac{x}{3} - \frac{1}{3} = \frac{1}{3}x - \frac{1}{3}$, number of hours walked.

But $\frac{1}{10}x + \frac{1}{3}x - \frac{1}{3} = 4$.

Combining and transposing,

$$\frac{1}{30}x = \frac{13}{3},$$

or $x = 10$.

Therefore $\frac{x}{10} = 1$, number of hours on trolley.

23. Let x = average velocity of bullet in feet per second.

Then since the time necessary for the return of the sound is

$$\frac{d}{r} = \frac{1650}{1100} = \frac{3}{2} \text{ seconds,}$$

$$\frac{5}{2} - \frac{3}{2} = 1 \text{ second, time of bullet's flight.}$$

Therefore $x = \frac{d}{t} = \frac{1650}{1}$, or 1650 feet per second.

24. Let x = average velocity of bullet in feet per second.

Then, as in Ex. 23,

$$\frac{825}{1100} = \frac{3}{4}, \text{ number of seconds for sound to return,}$$

and $\frac{7}{4} - \frac{3}{4} = 1$ second, time of bullet's flight.

Therefore $x = \frac{d}{t} = \frac{825}{1}$, or 825 feet per second.

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1. $(2c - 3d)^2 = 4c^2 - 12cd + 9d^2$.
2. $(5x - 2y)^2 = 25x^2 - 20xy + 4y^2$.
3. $(3a - 2b^2)^2 = 9a^2 - 12ab^2 + 4b^4$.
4. $(4x + 5y^3)^2 = 16x^2 + 40xy^3 + 25y^6$.
5. $(7x^2 + 2y)^2 = 49x^4 + 28x^2y + 4y^2$.
6. $(5x^2 - 3y)^2 = 25x^4 - 30x^2y + 9y^2$.
7. $(9a^2 + 2b^2)^2 = 81a^4 + 36a^2b^2 + 4b^4$.
8. $(11a^2b + 2)^2 = 121a^4b^2 + 44a^2b + 4$.
9. $(12ab^2 - 3c)^2 = 144a^2b^4 - 72ab^2c + 9c^2$.
10. $(5a^2b - 2a^2)^2 = 25a^4b^2 - 20a^4b + 4a^4$.
11. $(-2a^2c^5 + 3cd^2)^2 = 4a^4c^{10} - 12a^2c^6d^2 + 9c^2d^4$.

12. $(-4xy^2 - 3xz^2)^2 = 16x^2y^4 + 24x^2y^2z^2 + 9x^2z^4$.
14. Change $-mn$ to $-2mn$.
16. Change $+12x$ to $-12x$ or change -6 to $+6$.
18. Change $-x^2$ to $+x^2$.
19. Change -64 to $+64$.
20. Change -25 to $+25$.
21. Change $-6x$ to $+6x$ or change $+1$ in the denominator to -1 .
22. Change $+12ab$ to $-12ab$ or change $-3b$ to $+3b$.

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- | | |
|--|---|
| 1. $\frac{x^2 + 5x + 6}{x + 3} = x + 2$. | 8. $\frac{x^2 - 4x - 12}{x + 2} = x - 6$. |
| 2. $\frac{a^2 + 7a + 10}{a + 2} = a + 5$. | 9. $\frac{a^2 - 3a - 28}{a - 7} = a + 4$. |
| 3. $\frac{a^2 + 11a + 10}{a + 10} = a + 1$. | 10. $\frac{a^2 - 3ab + 2b^2}{a - b} = a - 2b$. |
| 4. $\frac{b^2 + 7b + 6}{b + 1} = b + 6$. | 11. $\frac{x^2 - xy - 6y^2}{x - 3y} = x + 2y$. |
| 5. $\frac{c^2 - 6c + 8}{c - 2} = c - 4$. | 12. $\frac{m^2 - 4mn - 21n^2}{m + 3n} = m - 7n$. |
| 6. $\frac{c^2 - 9c + 14}{c - 7} = c - 2$. | 13. $\frac{r^2 - 7rs - 18s^2}{r - 9s} = r + 2s$. |
| 7. $\frac{x^2 - 3x + 2}{x - 2} = x - 1$. | 14. $\frac{r^2 - rs - 90s^2}{r - 10s} = r + 9s$. |

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1. $(2a + 1)(a + 3) = 2a^2 + 7a + 3$.
2. $(3a + 2)(a + 5) = 3a^2 + 17a + 10$.
3. $(x + 4)(2x + 3) = 2x^2 + 11x + 12$.
4. $(2x + 3)(x + 7) = 2x^2 + 17x + 21$.
5. $(2c + 3)(c + 8) = 2c^2 + 19c + 24$.
6. $(4c + 3)(4c + 3) = 16c^2 + 24c + 9$.
7. $(5m + 2)(5m + 2) = 25m^2 + 20m + 4$.
8. $(4m - 1)(2m + 3) = 8m^2 + 10m - 3$.
9. $(5n - 3)(3n - 5) = 15n^2 - 34n + 15$.
10. $(2n + 7)(3n - 2) = 6n^2 + 17n - 14$.
11. $(7h + 3)(4h - 5) = 28h^2 - 23h - 15$.
12. $(5h - 4)(5h - 4) = 25h^2 - 40h + 16$.

13. $(5k - 6)(5k - 6) = 25k^2 - 60k + 36.$
14. $(3 + m)(4 + 2m) = 12 + 10m + 2m^2.$
15. $(7 - n)(7 - 3n) = 49 - 28n + 3n^2.$
16. $(2a - b)(3a + 2b) = 6a^2 + ab - 2b^2.$
17. $(5a + 9c)(2a - 9c) = 10a^2 - 27ac - 81c^2.$
18. $(2x^2 - 7y)(3x^2 - 2y) = 6x^4 - 25x^2y + 14y^2.$
19. $(c^2 - 5d)(5c^2 + 2d) = 5c^4 - 23c^2d - 10d^2.$
20. $(4c + 9d^2)(2c - 5d^2) = 8c^2 - 2cd^2 - 45d^4.$

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1. $3ax^2 + 18a = 3a(x^2 + 6).$
2. $8x^3 + 8x = 8x(x^2 + 1).$
3. $2x^4 - 6x^2 = 2x^2(x^2 - 3).$
4. $a^3b^2 + a^3b^3 = a^3b^2(1 + b).$
5. $3r^2s - 27r^2 = 3r^2(s - 9).$
6. $10x^3 - 4x^7 = 2x^3(5 - 2x^4).$
7. $18x^2 + 27 + 9x = 9(2x^2 + 3 + x).$
8. $x^3 - x^2 + x^4 = x^2(x - 1 + x^2).$
9. $2c^5 - 18c + 2c^3 = 2c(c^4 - 9 + c^2).$
10. $4x^2 - 8ax + 20x = 4x(x - 2a + 5).$
11. $5a^2 + 10a^4 - 25a^3 = 5a^2(1 + 2a^2 - 5a).$
12. $3y^4 + 6y^3c - 3y^3 = 3y^3(y + 2c - 1).$
13. $a^2 + a^3 - a^4 + 2a = a(a + a^2 - a^3 + 2).$
14. $5r^2 + 10r + 15r^3 = 5r(r + 2 + 3r^2).$
15. $6a^5 - 12a^4 + 6a - 18a^6 = 6a(a^4 - 2a^3 + 1 - 3a^5).$
16. $16m^3 - 32m^2n + 24m^2n^2 = 8m^2(2m - 4n + 3n^2).$
17. $-a^2b - 2ab^2 + a^2b^2 = ab(ab - a - 2b).$
18. $m + n.$
19. $b + c.$
20. $r + s + t.$
21. $h - k - l.$
22. $2 - b + a.$

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1. $2a(a - x) + 3(x - a) = 2a(a - x) - 3(a - x) = (a - x)(2a - 3).$
2. $7m(m - 2n) - 2(2n - m) = 7m(m - 2n) + 2(m - 2n)$
 $= (m - 2n)(7m + 2).$
3. $3h(5h - k) - k(k - 5h) - hk(5h - k)$
 $= 3h(5h - k) + k(5h - k) - hk(5h - k)$
 $= (5h - k)(3h + k - hk).$
4. $7x(2x - 5y) - 2y(2x - 5y) + (5y - 2x)$
 $= 7x(2x - 5y) - 2y(2x - 5y) - (2x - 5y)$
 $= (2x - 5y)(7x - 2y - 1).$
5. $(3a - 2x) + 9a(-2x + 3a) + x(2x - 3a)$
 $= (3a - 2x) + 9a(3a - 2x) - x(3a - 2x)$
 $= (3a - 2x)(1 + 9a - x).$
6. $ah + bh + ak + bk = h(a + b) + k(a + b) = (a + b)(h + k).$
7. $mr + nr + ms + ns = r(m + n) + s(m + n) = (m + n)(r + s).$

8. $2ac + bc + 2ad + bd = c(2a + b) + d(2a + b) = (2a + b)(c + d).$
9. $3ac + cx + 9ay + 3xy = c(3a + x) + 3y(3a + x)$
 $= (3a + x)(c + 3y).$
10. $6ax + 15bx + 4ay + 10by = 3x(2a + 5b) + 2y(2a + 5b)$
 $= (2a + 5b)(3x + 2y).$
11. $2a^3 + a^2x + 2ax + x^2 = a^2(2a + x) + x(2a + x) = (2a + x)(a^2 + x).$
12. $6h^3 + 4h^2k + 3hk^2 + 2k^3 = 2h^2(3h + 2k) + k^2(3h + 2k)$
 $= (3h + 2k)(2h^2 + k^2).$
13. $mh - nh + mk - nk = h(m - n) + k(m - n) = (m - n)(h + k).$
14. $2ax - 6ay + bx - 3by = 2a(x - 3y) + b(x - 3y)$
 $= (x - 3y)(2a + b).$
15. $ar - cr + as - cs = r(a - c) + s(a - c) = (a - c)(r + s).$
16. $mr - nr + ms - ns = r(m - n) + s(m - n) = (m - n)(r + s).$
17. $2hx - 2kx + hy - ky = 2x(h - k) + y(h - k) = (h - k)(2x + y).$
18. $3x^3 - 3x^2y + xy - y^2 = 3x^2(x - y) + y(x - y) = (x - y)(3x^2 + y).$
19. $2a^3 - 6a^2b^2 + ab^3 - 3b^5 = 2a^2(a - 3b^2) + b^3(a - 3b^2)$
 $= (a - 3b^2)(2a^2 + b^3).$
20. $5r^3 - 2rs^2 + 10r^2s - 4s^3 = r(5r^2 - 2s^2) + 2s(5r^2 - 2s^2)$
 $= (5r^2 - 2s^2)(r + 2s).$
21. $20x^3 - 5x^2y^2 + 4xy - y^3 = 5x^2(4x - y^2) + y(4x - y^2)$
 $= (4x - y^2)(5x^2 + y).$
22. $6ac - 3c + 2ad - d = 3c(2a - 1) + d(2a - 1) = (2a - 1)(3c + d).$
24. $ac - bc - ad + bd = c(a - b) - d(a - b) = (a - b)(c - d).$
25. $ac - ad - bc + bd = a(c - d) - b(c - d) = (c - d)(a - b).$
26. $2am - 6bm - 3an + 9bn = 2m(a - 3b) - 3n(a - 3b)$
 $= (a - 3b)(2m - 3n).$
27. $3hx - 15kx - hy + 5ky = 3x(h - 5k) - y(h - 5k)$
 $= (h - 5k)(3x - y).$
28. $x^3 - 3xy - 2x^2y^3 + 6y^4 = x(x^2 - 3y) - 2y^3(x^2 - 3y)$
 $= (x^2 - 3y)(x - 2y^3).$
29. $2a^3 - 6a^2b^2 - ab^2 + 3b^4 = 2a^2(a - 3b^2) - b^2(a - 3b^2)$
 $= (a - 3b^2)(2a^2 - b^2).$
30. $10a^4 - 25a^3b^3 + 5b^4 - 2ab = 5a^3(2a - 5b^3) - b(2a - 5b^3)$
 $= (2a - 5b^3)(5a^3 - b).$
31. $14x^4 - 35ax^2 + 10a - 4x^2 = 7x^2(2x^2 - 5a) - 2(2x^2 - 5a)$
 $= (2x^2 - 5a)(7x^2 - 2).$
32. $30x^5 - 10x^2 + 1 - 3x^3 = 10x^2(3x^3 - 1) - (3x^3 - 1)$
 $= (3x^3 - 1)(10x^2 - 1).$
33. $ab^4 - 2b^3 - 5ab + 10 = b^3(ab - 2) - 5(ab - 2)$
 $= (ab - 2)(b^3 - 5).$

$$34. \quad x(a+b) = r(a+b) + s(a+b). \\ x = r + s.$$

$$35. \quad mx + nx = mr + nr + ms + ns. \\ x(m+n) = r(m+n) + s(m+n). \\ x = r + s.$$

$$36. \quad ay + by = ac + bc - ab - b^2. \\ y(a+b) = c(a+b) - b(a+b). \\ y = c - b.$$

$$37. \quad kz - lz = hk - hl - k^2 + lk. \\ z(k-l) = h(k-l) - k(k-l). \\ z = h - k.$$

$$38. \quad cy + ad - ae = dc - ce + ay. \\ cy - ay = dc - ad - ce + ae. \\ y(c-a) = d(c-a) - e(c-a). \\ y = d - e.$$

$$39. \quad 2k + hl - mk = kl + 2h - mh. \\ mh - mk = 2h - 2k - hl + kl. \\ m(h-k) = 2(h-k) - l(h-k). \\ m = 2 - l.$$

Page 123 (First set)

- | | |
|---|---------------------------------------|
| 1. $9r^2 + 6rs + s^2 = (3r + s)^2.$ | 4. $16x^2 - 8xm + m^2 = (4x - m)^2.$ |
| 2. $4a^2 + 4ax + x^2 = (2a + x)^2.$ | 5. $m^2 + 10mn + 25n^2 = (m + 5n)^2.$ |
| 3. $9x^2 - 6xy + y^2 = (3x - y)^2.$ | 6. $25y^2 - 10yx + x^2 = (5y - x)^2.$ |
| 7. $81a^2 + 18ab + b^2 = (9a + b)^2.$ | |
| 8. $m^2 - 26mn + 169n^2 = (m - 13n)^2.$ | |
| 9. $9h^2 - 60hk + 100k^2 = (3h - 10k)^2.$ | |
| 10. $121a^2 - 44ab + 4b^2 = (11a - 2b)^2.$ | |
| 11. $81x^2 + 126xy + 49y^2 = (9x + 7y)^2.$ | |
| 12. $36x^2 + 25y^2 - 60xy = (6x - 5y)^2.$ | |
| 13. $169a^2 + 9b^2 - 78ab = (13a - 3b)^2.$ | |
| 14. $49d^2 + 210cd + 225c^2 = (7d + 15c)^2.$ | |
| 15. $196a^2 - 140ab + 25b^2 = (14a - 5b)^2.$ | |
| 16. $9a^2b^2 - 12ab + 4 = (3ab - 2)^2.$ | |
| 17. $16c^2x^2 + 56cx + 49 = (4cx + 7)^2.$ | |
| 18. $9m^2n^2 - 24mnp + 16p^2 = (3mn - 4p)^2.$ | |

Page 123 (Second set)

1. $x^3 + 6x^2 + 9x = x(x^2 + 6x + 9) = x(x + 3)^2$.
2. $2x^2 + 4x + 2 = 2(x^2 + 2x + 1) = 2(x + 1)^2$.
3. $x^3 - 10x^2 + 25x = x(x^2 - 10x + 25) = x(x - 5)^2$.
4. $a^3 + 2a^2b + ab^2 = a(a^2 + 2ab + b^2) = a(a + b)^2$.
5. $a^3 + 2a^2x + ax^2 = a(a^2 + 2ax + x^2) = a(a + x)^2$.
6. $2c^3 - 20c^2 + 50c = 2c(c^2 - 10c + 25) = 2c(c - 5)^2$.
7. $100x - 80x^2 + 16x^3 = 4x(25 - 20x + 4x^2) = 4x(5 - 2x)^2$.
8. $98c + 28c^2 + 2c^3 = 2c(49 + 14c + c^2) = 2c(7 + c)^2$.
9. $80r - 40r^2 + 5r^3 = 5r(16 - 8r + r^2) = 5r(4 - r)^2$.
10. $245x^5 - 140x^4 + 20x^3 = 5x^3(49x^2 - 28x + 4) = 5x^3(7x - 2)^2$.
11. $45m^2 - 60mn + 20n^2 = 5(9m^2 - 12mn + 4n^2) = 5(3m - 2n)^2$.
12. $2ax + 4ax^2 + 2ax^3 = 2ax(1 + 2x + x^2) = 2ax(1 + x)^2$.
13. $126c^2d^2 + 147c^3d + 27cd^3 = 3cd(42cd + 49c^2 + 9d^2) = 3cd(7c + 3d)^2$.
14. $2ac + 4cx + 2ay + 4xy = 2[c(a + 2x) + y(a + 2x)]$
 $= 2(a + 2x)(c + y)$.
15. $2ax - 6bx + 2ay - 6by = 2[x(a - 3b) + y(a - 3b)]$
 $= 2(a - 3b)(x + y)$.
16. $3mr - 6nr + 15ms - 30ns = 3(mr - 2nr + 5ms - 10ns)$
 $= 3[r(m - 2n) + 5s(m - 2n)]$
 $= 3(m - 2n)(r + 5s)$.
17. $12ab - 6bc - 6ax + 3cx = 3(4ab - 2bc - 2ax + cx)$
 $= 3[2b(2a - c) - x(2a - c)]$
 $= 3(2a - c)(2b - x)$.
18. $ax + cx = m(a + c) - n(a + c)$
 $x = m - n$.
19. $(a + b)x = a^2 + 2ab + b^2$
 $x = a + b$.
20. $mh - mk = h^2 - 2hk + k^2$
 $m(h - k) = (h - k)^2$
 $m = h - k$.
21. $nr - 2ns = r^2 - 4rs + 4s^2$
 $n(r - 2s) = (r - 2s)^2$
 $n = r - 2s$.
22. $ky - 4kl - l^2 = 4k^2 - ky - ly$
 $2ky + ly = 4k^2 + 4kl + l^2$
 $y(2k + l) = (2k + l)^2$
 $y = 2k + l$.
23. $acy - ad^2 - ac^2 = ady - 2acd$
 $acy - ady = ac^2 - 2acd + ad^2$
 $cy - dy = c^2 - 2cd + d^2$
 $y(c - d) = (c - d)^2$
 $y = c - d$.
24. $2ade - ae^2 = ad^2 - adz + aez$
 $adz - aez = ad^2 - 2ade + ae^2$
 $dz - ez = d^2 - 2de + e^2$
 $z(d - e) = (d - e)^2$
 $z = d - e$.

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1. $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2) = (a^2 + b^2)(a + b)(a - b).$
2. $x^4 - 1 = (x^2 + 1)(x^2 - 1) = (x^2 + 1)(x + 1)(x - 1).$
3. $a^4 - 16 = (a^2 + 4)(a^2 - 4) = (a^2 + 4)(a + 2)(a - 2).$
4. $r^4 - 81 = (r^2 + 9)(r^2 - 9) = (r^2 + 9)(r + 3)(r - 3).$
5. $16a^4 - 1 = (4a^2 + 1)(4a^2 - 1) = (4a^2 + 1)(2a + 1)(2a - 1).$
6. $81 - c^4 = (9 + c^2)(9 - c^2) = (9 + c^2)(3 + c)(3 - c).$
7. $625x^4 - 1 = (25x^2 + 1)(25x^2 - 1) = (25x^2 + 1)(5x + 1)(5x - 1).$
8. $625 - a^4 = (25 + a^2)(25 - a^2) = (25 + a^2)(5 + a)(5 - a).$
9. $625x^4 - 16y^4 = (25x^2 + 4y^2)(25x^2 - 4y^2)$
 $= (25x^2 + 4y^2)(5x + 2y)(5x - 2y).$
10. $a^{12} - c^{16} = (a^6 + c^8)(a^6 - c^8) = (a^6 + c^8)(a^3 + c^4)(a^3 - c^4).$
11. $c^4d^4 - 1 = (c^2d^2 + 1)(c^2d^2 - 1) = (c^2d^2 + 1)(cd + 1)(cd - 1).$
12. $c^8 - 81 = (c^4 + 9)(c^4 - 9) = (c^4 + 9)(c^2 + 3)(c^2 - 3).$
13. $c^8 - d^8 = (c^4 + d^4)(c^4 - d^4) = (c^4 + d^4)(c^2 + d^2)(c + d)(c - d).$
14. $a^4b^8 - 16 = (a^2b^4 + 4)(a^2b^4 - 4) = (a^2b^4 + 4)(ab^2 + 2)(ab^2 - 2).$
15. $x^4y^8 - z^4 = (x^2y^4 + z^2)(x^2y^4 - z^2) = (x^2y^4 + z^2)(xy^2 + z)(xy^2 - z).$
16. $(x + y)^2 - z^2 = (x + y + z)(x + y - z).$
17. $(x - y)^2 - 25 = (x - y + 5)(x - y - 5).$
18. $(2x + 5)^2 - 4y^2 = (2x + 5 + 2y)(2x + 5 - 2y).$
19. $(3a - 7)^2 - x^2 = (3a - 7 + x)(3a - 7 - x).$
20. $4(x - 9)^2 - 9a^2 = (2x - 18 + 3a)(2x - 18 - 3a).$
21. $9(a - b)^2 - 25c^2 = (3a - 3b + 5c)(3a - 3b - 5c).$
22. $25x^2(c + d)^2 - 9y^2 = (5xc + 5xd + 3y)(5xc + 5xd - 3y).$
23. $4a^2 - (2c - d)^2 = (2a + 2c - d)(2a - 2c + d).$
24. $25x^2 - (a - 3b)^2 = (5x + a - 3b)(5x - a + 3b).$
25. $49a^2 - (3x + 2)^2 = (7a + 3x + 2)(7a - 3x - 2).$
26. $81a^2 - 4(c^4 + 2)^2 = (9a + 2c^4 + 4)(9a - 2c^4 - 4).$
27. $100 - a^2(b + c)^2 = (10 + ab + ac)(10 - ab - ac).$
28. $121a^2 - 4b^2(c - 3d^2)^2$
 $= (11a + 2bc - 6bd^2)(11a - 2bc + 6bd^2).$
29. $y^2z^2 - (y + z)^2 = (yz + y + z)(yz - y - z).$
30. $c^2e^2 - (c - e)^2 = (ce + c - e)(ce - c + e).$
31. $c^2d^2 - (c^2 - d)^2 = (cd + c^2 - d)(cd - c^2 + d).$
32. $9a^2b^2 - 4(a - b)^2 = (3ab + 2a - 2b)(3ab - 2a + 2b).$
33. $25a^2b^2 - 9(a + b)^2 = (5ab + 3a + 3b)(5ab - 3a - 3b).$
34. $4c^2d^2 - (c - 2d)^2 = (2cd + c - 2d)(2cd - c + 2d).$
35. $16c^2d^2 - (a^2 + x)^2 = (4cd + a^2 + x)(4cd - a^2 - x).$
36. $25c^2d^2 - 3^2(c + d)^2 = (5cd + 3c + 3d)(5cd - 3c - 3d).$
37. $49r^2s^2 - 9(r^2 - s^2)^2 = (7rs + 3r^2 - 3s^2)(7rs - 3r^2 + 3s^2).$

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2. $x^2 + 2x + 1 - y^2 = (x + 1)^2 - y^2 = (x + 1 + y)(x + 1 - y).$
3. $a^2 - 6a + 9 - b^2 = (a - 3)^2 - b^2 = (a - 3 + b)(a - 3 - b).$
4. $c^2 - 10c + 25 - 4d^2 = (c - 5)^2 - 4d^2 = (c - 5 + 2d)(c - 5 - 2d).$
5. $4a^2 + 4a + 1 - 9b^2 = (2a + 1)^2 - 9b^2 = (2a + 1 + 3b)(2a + 1 - 3b).$
6. $16 + 24a + 9a^2 - 25b^2c^2 = (4 + 3a)^2 - 25b^2c^2$
 $= (4 + 3a + 5bc)(4 + 3a - 5bc).$
7. $4a^2 - 12ab + 9b^2 - 9b^4 = (2a - 3b)^2 - 9b^4$
 $= (2a - 3b + 3b^2)(2a - 3b - 3b^2).$
8. $25x^2 + 4y^2 - 20xy - 16z^2 = (5x - 2y)^2 - 16z^2$
 $= (5x - 2y + 4z)(5x - 2y - 4z).$
9. $9x^2 + 25y^4 - 81z^4 - 30xy^2 = (3x - 5y^2)^2 - 81z^4$
 $= (3x - 5y^2 + 9z^2)(3x - 5y^2 - 9z^2).$
10. $60ab - 25c^4 + 9b^2 + 100a^2 = (3b + 10a)^2 - 25c^4$
 $= (3b + 10a + 5c^2)(3b + 10a - 5c^2).$
11. $1 - 14ab^2 + 49a^2b^4 - b^4 = (1 - 7ab^2)^2 - b^4$
 $= (1 - 7ab^2 + b^2)(1 - 7ab^2 - b^2).$
12. $4 - 20ab^2c^3 + 25a^2b^4c^6 - 4c^8 = (2 - 5ab^2c^3)^2 - 4c^8$
 $= (2 - 5ab^2c^3 + 2c^4)(2 - 5ab^2c^3 - 2c^4).$
13. $9x^2 - 30xy + 25y^2 - 16z^4 = (3x - 5y)^2 - 16z^4$
 $= (3x - 5y + 4z^2)(3x - 5y - 4z^2).$
14. $16x^6 - 8x^3y^2 + y^4 - z^4 = (4x^3 - y^2)^2 - z^4$
 $= (4x^3 - y^2 + z^2)(4x^3 - y^2 - z^2).$
16. $x^2 - 4y^2 + 4yz - z^2 = x^2 - (4y^2 - 4yz + z^2)$
 $= x^2 - (2y - z)^2 = (x + 2y - z)(x - 2y + z).$
17. $x^2 - y^2 + 10yz - 25z^2 = x^2 - (y^2 - 10yz + 25z^2)$
 $= x^2 - (y - 5z)^2 = (x + y - 5z)(x - y + 5z).$
18. $c^2 - 4a^2 - 12ab - 9b^2 = c^2 - (4a^2 + 12ab + 9b^2)$
 $= c^2 - (2a + 3b)^2 = (c + 2a + 3b)(c - 2a - 3b).$
19. $4c^2 - a^2 + 10ab - 25b^2 = 4c^2 - (a^2 - 10ab + 25b^2)$
 $= 4c^2 - (a - 5b)^2$
 $= (2c + a - 5b)(2c - a + 5b).$
20. $9d^2 - a^2 + 6ab - 9b^2 = 9d^2 - (a^2 - 6ab + 9b^2)$
 $= 9d^2 - (a - 3b)^2$
 $= (3d + a - 3b)(3d - a + 3b).$
21. $16e^2 - 25m^2 + 10mn - n^2 = 16e^2 - (25m^2 - 10mn + n^2)$
 $= 16e^2 - (5m - n)^2$
 $= (4e + 5m - n)(4e - 5m + n).$

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1. $ax^2 - ay^2 = a(x^2 - y^2) = a(x + y)(x - y).$
2. $a^2c - 4b^2c = c(a^2 - 4b^2) = c(a + 2b)(a - 2b).$
3. $a^3 + 6a^2b + 9ab^2 = a(a^2 + 6ab + 9b^2) = a(a + 3b)^2.$
4. $c^6 + 2c^4 + c^2 = c^2(c^4 + 2c^2 + 1) = c^2(c^2 + 1)^2.$
5. $dx^2 - 25d^3 = d(x^2 - 25d^2) = d(x + 5d)(x - 5d).$
6. $25e^4 - 30e^3 + 9e^2 = e^2(25e^2 - 30e + 9) = e^2(5e - 3)^2.$
7. $2x^3 - 18xy^2 = 2x(x^2 - 9y^2) = 2x(x + 3y)(x - 3y).$
8. $r^7 + 2r^5 + r^3 = r^3(r^4 + 2r^2 + 1) = r^3(r^2 + 1)^2.$
9. $a^3b - 6a^2b^3 + 9ab^5 = ab(a^2 - 6ab^2 + 9b^4) = ab(a - 3b^2)^2.$
10. $5a^4c^2 - 45c^4 = 5c^2(a^4 - 9c^2) = 5c^2(a^2 + 3c)(a^2 - 3c).$
11. $a^3x^3 - 2a^2x^2 + ax = ax(a^2x^2 - 2ax + 1) = ax(ax - 1)^2.$
12. $ax^5 - a^5x = ax(x^4 - a^4) = ax(x^2 + a^2)(x + a)(x - a).$
13. $2x^5 - 162x = 2x(x^4 - 81) = 2x(x^2 + 9)(x + 3)(x - 3).$
14. $x^5y - 16xy^5 = xy(x^4 - 16y^4) = xy(x^2 + 4y^2)(x + 2y)(x - 2y).$
15. $5a^4 - 5 = 5(a^4 - 1) = 5(a^2 + 1)(a + 1)(a - 1).$
16. $x^5 - xy^4 = x(x^4 - y^4) = x(x^2 + y^2)(x + y)(x - y).$
17. $32x^4 - 1250 = 2(16x^4 - 625) = 2(4x^2 + 25)(2x + 5)(2x - 5).$
18. $3z(x + y)^2 - 12z^3 = 3z[(x + y)^2 - 4z^2]$
 $= 3z(x + y + 2z)(x + y - 2z).$
19. $20x^2 + 20x^3 + 5x^4 + 5x^5 = 5x^2(4 + 4x + x^2 + x^3)$
 $= 5x^2[4(1 + x) + x^2(1 + x)]$
 $= 5x^2(1 + x)(4 + x^2).$
20. $8r^4 - 8r^2(3s + t)^2 = 8r^2[r^2 - (3s + t)^2]$
 $= 8r^2(r + 3s + t)(r - 3s - t).$
21. $8a^4 - 12a^2bc - 18a^2b^2 - 2a^2c^2 = 2a^2(4a^2 - 6bc - 9b^2 - c^2)$
 $= 2a^2[4a^2 - (9b^2 + 6bc + c^2)]$
 $= 2a^2(2a + 3b + c)(2a - 3b - c).$
22. $x(a + 2) = a^2 - 4.$
 $x = a - 2.$
23. $b^2y + 5y = b^4 - 25.$
 $y(b^2 + 5) = b^4 - 25.$
 $y = b^2 - 5.$
24. $z(c^2 + 25)(c - 5) = c^4 - 625.$
 $z(c^2 + 25)(c - 5) = (c^2 + 25)(c + 5)(c - 5).$
 $z = c + 5.$
25. $mh^2 - mhk = h^3 - hk^2.$
 $mh - mk = h^2 - k^2.$
 $m(h - k) = h^2 - k^2.$
 $m = h + k.$
26. $acn + 2c - a^2c = 2cn - 2c.$
 $an + 2 - a^2 = 2n - 2.$
 $an - 2n = a^2 - 4.$
 $n(a - 2) = a^2 - 4.$
 $n = a + 2.$

Page 129 (First set)

1. $a^2 + 5a + 6 = (a + 3)(a + 2)$.
2. $c^2 + 7c + 12 = (c + 4)(c + 3)$.
3. $x^2 + 4x + 4 = (x + 2)(x + 2)$.
4. $x^2 - 8x + 12 = (x - 2)(x - 6)$.
5. $m^2 - 13m + 30 = (m - 3)(m - 10)$.
6. $h^2 - 16h + 15 = (h - 1)(h - 15)$.
7. $n^2 - 8n + 16 = (n - 4)(n - 4)$.
8. $k^2 - 17k + 30 = (k - 2)(k - 15)$.
9. $m^2 - 10m + 25 = (m - 5)(m - 5)$.
10. $r^2 - 5r - 14 = (r + 2)(r - 7)$.
11. $s^2 + 2s - 35 = (s + 7)(s - 5)$.
12. $s^2 - 6s + 9 = (s - 3)(s - 3)$.
13. $t^2 + t - 42 = (t + 7)(t - 6)$.
14. $x^2 + 5x - 14 = (x + 7)(x - 2)$.
15. $10a - 39 + a^2 = (a + 13)(a - 3)$.
16. $a^4 - 4a^2 - 5 = (a^2 - 5)(a^2 + 1)$.
17. $a^4 - 14a^2 + 49 = (a^2 - 7)(a^2 - 7)$.
19. $-x^2 - x + 12 = -1(x^2 + x - 12) = -1(x + 4)(x - 3) = (x + 4)(3 - x)$.
20. $-m^2 - 8m + 9 = -1(m^2 + 8m - 9) = -1(m + 9)(m - 1) = (m + 9)(1 - m)$.
21. $-2a + 63 - a^2 = -1(a^2 + 2a - 63) = -1(a + 9)(a - 7) = (a + 9)(7 - a)$.
22. $20 - m - m^2 = -1(m^2 + m - 20) = -1(m + 5)(m - 4) = (m + 5)(4 - m)$.
23. $24 + 2n - n^2 = -1(n^2 - 2n - 24) = -1(n - 6)(n + 4) = (6 - n)(n + 4)$.
24. $80 - 2r - r^2 = -1(r^2 + 2r - 80) = -1(r + 10)(r - 8) = (r + 10)(8 - r)$.
25. $x^2 - x - 72 = (x - 9)(x + 8)$.
26. $a^2 - a - 110 = (a - 11)(a + 10)$.
27. $a^2 + 12a + 36 = (a + 6)(a + 6)$.
28. $c^2 - 50 + 5c = c^2 + 5c - 50 = (c + 10)(c - 5)$.
29. $h^2 - 55 + 6h = h^2 + 6h - 55 = (h + 11)(h - 5)$.
30. $-4s - 21 + s^2 = s^2 - 4s - 21 = (s - 7)(s + 3)$.
31. $27 + 6x - x^2 = -1(x^2 - 6x - 27) = -1(x - 9)(x + 3) = (x + 3)(9 - x)$.
32. $x^2 + 2rx - 3r^2 = (x + 3r)(x - r)$.
33. $m^2 + 2mn - 99n^2 = (m + 11n)(m - 9n)$.
34. $h^2 - 3hk - 130k^2 = (h - 13k)(h + 10k)$.
35. $k^2 - 4kl + 4l^2 = (k - 2l)(k - 2l)$.
36. $r^2 + 15rs - 100s^2 = (r + 20s)(r - 5s)$.
37. $5hk + k^2 - 36h^2 = k^2 + 5hk - 36h^2 = (k + 9h)(k - 4h)$.
38. $r^2s^2 - 3rst - 40t^2 = (rs - 8t)(rs + 5t)$.
39. $a^2b^2 + 8abc + 7c^2 = (ab + 7c)(ab + c)$.
40. $33p^2 - m^2n^2 - 8mnp = -1(m^2n^2 + 8mnp - 33p^2)$
 $= -1(mn + 11p)(mn - 3p)$
 $= (mn + 11p)(3p - mn)$.

Page 129 (Second set)

1. $x^4 + 7x^3 + 12x^2 = x^2(x^2 + 7x + 12) = x^2(x + 3)(x + 4).$
2. $a^4 - 12a^3 + 36a^2 = a^2(a^2 - 12a + 36) = a^2(a - 6)(a - 6).$
3. $2x^2 + 10x - 48 = 2(x^2 + 5x - 24) = 2(x + 8)(x - 3).$
4. $5c^3 + 15c^2 - 140c = 5c(c^2 + 3c - 28) = 5c(c + 7)(c - 4).$
5. $3m^4 + 66m^3 + 363m^2 = 3m^2(m^2 + 22m + 121) = 3m^2(m + 11)^2.$
6. $16a^3 - 4ab^2 = 4a(4a^2 - b^2) = 4a(2a + b)(2a - b).$
7. $20c^5 - 5cd^4 = 5c(4c^4 - d^4) = 5c(2c^2 + d^2)(2c^2 - d^2).$
8. $5x^3 - 20xy^2 = 5x(x^2 - 4y^2) = 5x(x + 2y)(x - 2y).$
9. $r^3s + 10r^2s^2 + 21rs^3 = rs(r^2 + 10rs + 21s^2) = rs(r + 7s)(r + 3s).$
10. $2h^3k^2 + 2h^2k^3 - 12hk^4 = 2hk^2(h^2 + hk - 6k^2) = 2hk^2(h + 3k)(h - 2k).$
11. $2r^5 - 162r = 2r(r^4 - 81) = 2r(r^2 + 9)(r + 3)(r - 3).$
12. $45m^3n^2 - 20mn^4 = 5mn^2(9m^2 - 4n^2) = 5mn^2(3m + 2n)(3m - 2n).$
13. $2x^3y + 10x^2y^2 + 12xy^3 = 2xy(x^2 + 5xy + 6y^2)$
 $= 2xy(x + 2y)(x + 3y).$
14. $4c^4d + 4c^3d^2 - 24c^2d^3 = 4c^2d(c^2 + cd - 6d^2) = 4c^2d(c + 3d)(c - 2d).$
15. $5r^4s - 40r^3s^2 + 60r^2s^3 = 5r^2s(r^2 - 8rs + 12s^2)$
 $= 5r^2s(r - 2s)(r - 6s).$
16. $39d^4 - 10cd^3 - c^2d^2 = -d^2(c^2 + 10cd - 39d^2)$
 $= -d^2(c + 13d)(c - 3d)$
 $= d^2(c + 13d)(3d - c).$
17. $18x^5 + 7kx^4 - k^2x^3 = -x^3(k^2 - 7kx - 18x^2)$
 $= -x^3(k - 9x)(k + 2x)$
 $= x^3(9x - k)(k + 2x).$
18. $8abc^5 + a^2b^2c^4 - 65c^6 = c^4(a^2b^2 + 8abc - 65c^2)$
 $= c^4(ab - 5c)(ab + 13c).$
19. $6ah^2 + 3h^2c + 18ahk + 9chk = 3h(2ah + hc + 6ak + 3ck)$
 $= 3h[h(2a + c) + 3k(2a + c)]$
 $= 3h(2a + c)(h + 3k).$
20. $30m^2nr - 10mr + 45m^2n - 15m = 5m(6mnr - 2r + 9mn - 3)$
 $= 5m[2r(3mn - 1) + 3(3mn - 1)]$
 $= 5m(3mn - 1)(2r + 3).$
21. $2a^3 + 4a^2b + 2ab^2 - 2ac^2 = 2a(a^2 + 2ab + b^2 - c^2)$
 $= 2a[(a + b)^2 - c^2]$
 $= 2a(a + b + c)(a + b - c).$
22. $3c^3 + 6c^2d + 3cd^2 - 12c = 3c(c^2 + 2cd + d^2 - 4)$
 $= 3c(c + d + 2)(c + d - 2).$
23. $4x^3 - 4xy^2 - 8xyz - 4xz^2 = 4x(x^2 - y^2 - 2yz - z^2)$
 $= 4x[x^2 - (y + z)^2]$
 $= 4x(x + y + z)(x - y - z).$

24. $x(a+2) = a^2 + 5a + 6.$
 $x(a+2) = (a+2)(a+3).$
 $x = a+3.$
25. $ym - y = m^2 - 4m + 3.$
 $y(m-1) = (m-1)(m-3).$
 $y = m-3.$
26. $rz + r = r^2 + 5z - 20.$
 $rz - 5z = r^2 - r - 20.$
 $z(r-5) = (r-5)(r+4).$
 $z = r+4.$
27. $ar + 3ac - a^2 = 2cr + 2c^2.$
 $ar - 2cr = a^2 - 3ac + 2c^2.$
 $r(a-2c) = (a-2c)(a-c).$
 $r = a-c.$
28. $2as + a^3 + a^4 = 6a^2 + a^2s.$
 $2s + a^2 + a^3 = 6a + as.$
 $2s - as = -a^3 - a^2 + 6a.$
 $s(2-a) = -a(a^2 + a - 6).$
 $s(a-2) = a(a+3)(a-2).$
 $s = a(a+3).$

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1. $3x^2 + 7x + 2 = (3x+1)(x+2).$
2. $2x^2 + 7x + 6 = (2x+3)(x+2).$
3. $3x^2 + 8x + 5 = (3x+5)(x+1).$
4. $2a^2 - 9a + 10 = (2a-5)(a-2).$
5. $5a^2 - 2a - 3 = (5a+3)(a-1).$
6. $6a^2 + 7a - 5 = (3a+5)(2a-1).$
7. $10x^2 + 13x - 3 = (2x+3)(5x-1).$
8. $6c^2 + 7c - 20 = (3c-4)(2c+5).$
9. $10x^2 + 9x + 2 = (5x+2)(2x+1).$
10. $6x^2 + 5x - 6 = (3x-2)(2x+3).$
11. $2x^2 + 13x + 18 = (2x+9)(x+2).$
12. $3d^2 - 10d - 25 = (3d+5)(d-5).$
13. $5x^2 - 38x - 16 = (5x+2)(x-8).$
14. $10x^2 + 7x - 6 = (5x+6)(2x-1).$
15. $4k^2 + 20k + 21 = (2k+7)(2k+3).$
16. $9x^2 + 3x - 2 = (3x+2)(3x-1).$
17. $16c^2 - 8c - 3 = (4c-3)(4c+1).$
18. $10l^2 - 7l - 12 = (5l+4)(2l-3).$
19. $12x^2 - 8x - 15 = (6x+5)(2x-3).$
20. $14r^2 - 39r + 10 = (7r-2)(2r-5).$
21. $21x^2 - 61x - 30 = (7x+3)(3x-10).$
22. $25s^2 - 15s + 2 = (5s-2)(5s-1).$
23. $36x^2 - 36x + 5 = (6x-5)(6x-1).$
24. $36a^2 + 23a - 3 = (9a-1)(4a+3).$
25. $49x^2 - 21x + 2 = (7x-1)(7x-2).$
26. $2 + x - 15x^2 = (2-5x)(1+3x).$
27. $12x^4 + x^2 - 20 = (4x^2-5)(3x^2+4).$
28. $50x^4 + 5x^2 - 3 = (10x^2+3)(5x^2-1).$
29. $6 + 7x^2 - 5x^4 = (3+5x^2)(2-x^2).$

30. $12 + 17x^3 + 6x^6 = (3 + 2x^3)(4 + 3x^3)$.
 31. $a^2 - 3ab + 2b^2 = (a - 2b)(a - b)$.
 32. $a^2 + 2ab - 8b^2 = (a + 4b)(a - 2b)$.
 33. $c^2 - cd - 12d^2 = (c - 4d)(c + 3d)$.
 34. $2x^2 + 5xy + 2y^2 = (2x + y)(x + 2y)$.
 35. $2a^2 - 5ab + 2b^2 = (2a - b)(a - 2b)$.
 36. $3x^2 - 10xy + 3y^2 = (3x - y)(x - 3y)$.
 37. $10x^2 - 27xy + 5y^2 = (2x - 5y)(5x - y)$.
 38. $12x^2 + 23xy - 2y^2 = (12x - y)(x + 2y)$.
 39. $30x^2 - 13xy - y^2 = (15x + y)(2x - y)$.
 40. $30x^2 + 109xy + 30y^2 = (10x + 3y)(3x + 10y)$.

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1. $4x^2 + 10x + 4 = 2(2x^2 + 5x + 2) = 2(2x + 1)(x + 2)$.
 2. $3x^3 + 18x^2 + 27x = 3x(x^2 + 6x + 9) = 3x(x + 3)^2$.
 3. $20x^4 - 60x^3 + 45x^2 = 5x^2(4x^2 - 12x + 9) = 5x^2(2x - 3)^2$.
 4. $50ax^2 - 140ax + 98a = 2a(25x^2 - 70x + 49) = 2a(5x - 7)^2$.
 5. $27x^3 - 36x^4 + 12x^5 = 3x^3(9 - 12x + 4x^2) = 3x^3(3 - 2x)^2$.
 6. $8x^4 + 2x^2 - 1 = (2x^2 + 1)(2x + 1)(2x - 1)$.
 7. $60x^3 - 35x^2 - 60x = 5x(12x^2 - 7x - 12) = 5x(4x + 3)(3x - 4)$.
 8. $3x^3y - 4x^2y^2 + xy^3 = xy(3x^2 - 4xy + y^2) = xy(3x - y)(x - y)$.
 9. $12x^5y + 21x^3y^3 - 6xy^5 = 3xy(4x^4 + 7x^2y^2 - 2y^4)$
 $\quad = 3xy(x^2 + 2y^2)(2x + y)(2x - y)$.
 10. $5x^3y + 10x^2y^2 - 75xy^3 = 5xy(x^2 + 2xy - 15y^2)$
 $\quad = 5xy(x + 5y)(x - 3y)$.
 11. $6x^2r^2 + 12ar^2x + 4rx^2 + 8axr = 2xr(3xr + 6ar + 2x + 4a)$
 $\quad = 2xr[3r(x + 2a) + 2(x + 2a)]$
 $\quad = 2xr(x + 2a)(3r + 2)$.
 12. $-18ax^2y - 45a^2xy - 6ax^2 - 15a^2x = -3ax(6xy + 15ay + 2x + 5a)$
 $\quad = -3ax[3y(2x + 5a) + (2x + 5a)]$
 $\quad = -3ax(2x + 5a)(3y + 1)$.
 13. $45x^4y^2 + 21x^3y - 6x^2 = 3x^2(15x^2y^2 + 7xy - 2)$
 $\quad = 3x^2(5xy - 1)(3xy + 2)$.
 14. $r(a + 1) = 2a^2 + 3a + 1$.
 $r(a + 1) = (a + 1)(2a + 1)$.
 $r = 2a + 1$.
 15. $sb + sc = b^2 - bc - 2c^2$.
 $s(b + c) = (b - 2c)(b + c)$.
 $s = b - 2c$.
 16. $2hx = 6h^2 + h - 2 + x$.
 $2hx - x = 6h^2 + h - 2$.
 $x(2h - 1) = (2h - 1)(3h + 2)$.
 $x = 3h + 2$.
 17. $cy - cd - 2c^2 = dy - 3d^2$.
 $cy - dy = 2c^2 + cd - 3d^2$.
 $y(c - d) = (c - d)(2c + 3d)$.
 $y = 2c + 3d$.

Page 134 (First set)

1. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$.
2. $m^3 - n^3 = (m - n)(m^2 + mn + n^2)$.
3. $a^3 + 8 = a^3 + 2^3 = (a + 2)(a^2 - 2a + 4)$.
4. $b^3 - 64 = b^3 - 4^3 = (b - 4)(b^2 + 4b + 16)$.
5. $c^3 + 8d^3 = c^3 + (2d)^3 = (c + 2d)(c^2 - 2cd + 4d^2)$.
6. $m^3 - 125n^3 = m^3 - (5n)^3 = (m - 5n)(m^2 + 5mn + 25n^2)$.
7. $8 - x^3 = 2^3 - x^3 = (2 - x)(4 + 2x + x^2)$.
8. $64 + y^3 = 4^3 + y^3 = (4 + y)(16 - 4y + y^2)$.
9. $27 + 8a^3 = 3^3 + (2a)^3 = (3 + 2a)(9 - 6a + 4a^2)$.
10. $m^9 - n^3 = (m^3)^3 - n^3 = (m^3 - n)(m^6 + m^3n + n^2)$.
11. $125x^3 - y^3 = (5x)^3 - y^3 = (25x^2 + 5xy + y^2)(5x - y)$.
12. $1 - 27m^3n^9 = 1^3 - (3mn^3)^3 = (1 - 3mn^3)(1 + 3mn^3 + 9m^2n^6)$.
13. $8x^3y^6 + 1 = (2xy^2)^3 + 1 = (2xy^2 + 1)(4x^2y^4 - 2xy^2 + 1)$.
14. $216 - x^3 = 6^3 - x^3 = (6 - x)(36 + 6x + x^2)$.
15. $m^6 + n^6 = (m^2)^3 + (n^2)^3 = (m^2 + n^2)(m^4 - m^2n^2 + n^4)$.
16. $x^6 + y^6 = (x^2)^3 + (y^2)^3 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$.
17. $x^{12} + y^{12} = (x^4)^3 + (y^4)^3 = (x^4 + y^4)(x^8 - x^4y^4 + y^8)$.
18. $64x^6 + y^6 = (4x^2)^3 + (y^2)^3 = (4x^2 + y^2)(16x^4 - 4x^2y^2 + y^4)$.
19. $a^6 + 64 = (a^2)^3 + 4^3 = (a^2 + 4)(a^4 - 4a^2 + 16)$.
20. $m^6 - n^6 = (m^3 + n^3)(m^3 - n^3)$
 $= (m + n)(m - n)(m^2 - mn + n^2)(m^2 + mn + n^2)$.
21. $x^6 - y^6 = (x^3 + y^3)(x^3 - y^3) = (x + y)(x - y)(x^2 - xy + y^2)(x^2 + xy + y^2)$.
22. $a^6 - 64 = (a^3 + 8)(a^3 - 8) = (a + 2)(a - 2)(a^2 - 2a + 4)(a^2 + 2a + 4)$.
23. $x^6 - 64y^6 = (x^3 + 8y^3)(x^3 - 8y^3)$
 $= (x + 2y)(x - 2y)(x^2 - 2xy + 4y^2)(x^2 + 2xy + 4y^2)$.
24. $1 - a^6 = (1 + a^3)(1 - a^3) = (1 + a)(1 - a)(1 - a + a^2)(1 + a + a^2)$.
25. $64x^6 - 1 = (8x^3 + 1)(8x^3 - 1)$
 $= (2x + 1)(2x - 1)(4x^2 - 2x + 1)(4x^2 + 2x + 1)$.
26. $a^{12} - b^6 = (a^6 + b^3)(a^6 - b^3)$
 $= (a^2 + b)(a^4 - a^2b + b^2)(a^2 - b)(a^4 + a^2b + b^2)$.
27. $x^{12} - 1 = (x^6 + 1)(x^6 - 1)$
 $= (x^2 + 1)(x^4 - x^2 + 1)(x + 1)(x - 1)(x^2 - x + 1)(x^2 + x + 1)$.
28. $y^{12} - x^6 = (y^6 + x^3)(y^6 - x^3)$
 $= (y^2 + x)(y^4 - y^2x + x^2)(y^2 - x)(y^4 + y^2x + x^2)$.

Page 134 (Second set)

1. $x^4 + xy^3 = x(x^3 + y^3) = x(x + y)(x^2 - xy + y^2)$.
2. $2x^4y - 2xy^4 = 2xy(x^3 - y^3) = 2xy(x - y)(x^2 + xy + y^2)$.
3. $2a^4 - 54a = 2a(a^3 - 27) = 2a(a - 3)(a^2 + 3a + 9)$.
4. $5x^6 - 40x^3 = 5x^3(x^3 - 8) = 5x^3(x - 2)(x^2 + 2x + 4)$.

5. $x^4 - 2x^3 + 27x - 54 = x^3(x - 2) + 27(x - 2)$
 $= (x - 2)(x^3 + 27)$
 $= (x - 2)(x + 3)(x^2 - 3x + 9).$
6. $x^6 - 7x^3 - 8 = (x^3 - 8)(x^3 + 1)$
 $= (x - 2)(x^2 + 2x + 4)(x + 1)(x^2 - x + 1).$
7. $x^{10} - 64x^4 = x^4(x^6 - 64) = x^4(x^3 + 8)(x^3 - 8)$
 $= x^4(x + 2)(x^2 - 2x + 4)(x - 2)(x^2 + 2x + 4).$
8. $3x^4 + 2x^3 - 24x - 16 = x^3(3x + 2) - 8(3x + 2)$
 $= (3x + 2)(x^3 - 8)$
 $= (3x + 2)(x - 2)(x^2 + 2x + 4).$
9. $3x - ax = 27 - a^3.$
 $x(3 - a) = (3 - a)(9 + 3a + a^2).$
 $x = 9 + 3a + a^2.$
10. $bx - 8 = b^3 - 2x.$
 $bx + 2x = b^3 + 8.$
 $x(b + 2) = (b + 2)(b^2 - 2b + 4).$
 $x = b^2 - 2b + 4.$
11. $c^2x - 27 = c^3 + 3cx - 9x.$
 $c^2x - 3cx + 9x = c^3 + 27.$
 $x(c^2 - 3c + 9) = (c + 3)(c^2 - 3c + 9).$
 $x = c + 3.$
12. $a^3 + a^2r = 8 - 2ar - 4r.$
 $a^2r + 2ar + 4r = 8 - a^3.$
 $r(a^2 + 2a + 4) = (2 - a)(4 + 2a + a^2).$
 $r = 2 - a.$
13. $abs + as + a = ab^3 - ab^2s.$
 $abs + as + ab^2s = ab^3 - a.$
 $bs + s + b^2s = b^3 - 1.$
 $s(b + 1 + b^2) = (b - 1)(b^2 + b + 1).$
 $s = b - 1.$

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1. $x^3 - 4x = x(x^2 - 4) = x(x + 2)(x - 2).$
2. $2a^3 + 4a^2b + 2ab^2 = 2a(a^2 + 2ab + b^2) = 2a(a + b)^2.$
3. $ax^3 + ax^2y + a^2x^2 + a^2xy = ax(x^2 + xy + ax + ay)$
 $= ax[x(x + y) + a(x + y)]$
 $= ax(x + y)(x + a).$
4. $4x^4 + 400x^8 - 116x^6 = 4x^4(1 + 100x^4 - 29x^2)$
 $= 4x^4(1 - 25x^2)(1 - 4x^2)$
 $= 4x^4(1 + 5x)(1 - 5x)(1 + 2x)(1 - 2x).$
5. $2cx^3 + 2cx^2 - 12cx = 2cx(x^2 + x - 6) = 2cx(x + 3)(x - 2).$
6. $5a^3 - 10a^2b - 75ab^2 = 5a(a^2 - 2ab - 15b^2) = 5a(a - 5b)(a + 3b).$
7. $10x^4 - 5x^3 - 30x^2 = 5x^2(2x^2 - x - 6) = 5x^2(2x + 3)(x - 2).$

8. $5x^{10} + 40x^7 = 5x^7(x^3 + 8) = 5x^7(x + 2)(x^2 - 2x + 4).$
9. $x^3y + 3x^3 + 8y + 24 = x^3(y + 3) + 8(y + 3)$
 $= (y + 3)(x^3 + 8)$
 $= (y + 3)(x + 2)(x^2 - 2x + 4).$
10. $x^5 - 4x^3 - 8x^2 + 32 = x^3(x^2 - 4) - 8(x^2 - 4)$
 $= (x^2 - 4)(x^3 - 8)$
 $= (x - 2)^2(x + 2)(x^2 + 2x + 4).$
11. $16x^4 - 8x^3 - 2x + 1 = 8x^3(2x - 1) - 1(2x - 1)$
 $= (2x - 1)(8x^3 - 1)$
 $= (2x - 1)^2(4x^2 + 2x + 1).$
12. $a^5 - 25a^3 - a^2 + 25 = a^3(a^2 - 25) - 1(a^2 - 25)$
 $= (a^2 - 25)(a^3 - 1)$
 $= (a + 5)(a - 5)(a - 1)(a^2 + a + 1).$
13. $-45a^3x^3 - 3a^4x^2 + 18a^5x = -3a^3x(15x^2 + ax - 6a^2)$
 $= 3a^3x(2a + 3x)(3a - 5x).$
14. $9x^2 - 30axy^2 + 25a^2y^4 = (3x - 5ay^2)^2.$
15. $x^{12} - 63x^6 - 64 = (x^6 - 64)(x^6 + 1)$
 $= (x^3 + 8)(x^3 - 8)(x^6 + 1)$
 $= (x + 2)(x^2 - 2x + 4)(x - 2)(x^2 + 2x + 4)(x^2 + 1)(x^4 - x^2 + 1).$
16. $x^6 - 64y^{12} = (x^3 + 8y^6)(x^3 - 8y^6)$
 $= (x + 2y^2)(x^2 - 2xy^2 + 4y^4)(x - 2y^2)(x^2 + 2xy^2 + 4y^4).$
17. $6x^4 - 7x^2y - 5y^2 = (2x^2 + y)(3x^2 - 5y).$
18. $3 + 15x - 2ax - 10ax^2 = 3(1 + 5x) - 2ax(1 + 5x)$
 $= (1 + 5x)(3 - 2ax).$
19. $x^7 - 27x^4 - x^3 + 27 = x^4(x^3 - 27) - (x^3 - 27)$
 $= (x^3 - 27)(x^4 - 1)$
 $= (x - 3)(x^2 + 3x + 9)(x^2 + 1)(x + 1)(x - 1).$
20. $a^3 + 3a^2b - ab^2 - 3b^3 = a^2(a + 3b) - b^2(a + 3b)$
 $= (a + 3b)(a^2 - b^2)$
 $= (a + 3b)(a + b)(a - b).$
21. $20x^4y + 38x^3y^2 - 30x^2y^3 = 2x^2y(10x^2 + 19xy - 15y^2)$
 $= 2x^2y(5x - 3y)(2x + 5y).$
22. $8ax^2 - 2a - 20x^2 + 5 = 2a(4x^2 - 1) - 5(4x^2 - 1)$
 $= (4x^2 - 1)(2a - 5) = (2x + 1)(2x - 1)(2a - 5).$
23. $9a^2 + 4b^2 - 12ab - 4x^2y^2 = (3a - 2b)^2 - 4x^2y^2$
 $= (3a - 2b + 2xy)(3a - 2b - 2xy).$
24. $4x^2 - x^4y^4 - 4xy + y^2 = (2x - y)^2 - x^4y^4$
 $= (2x - y + x^2y^2)(2x - y - x^2y^2).$
25. $x^2 + y^2 - 4a^2 - 2xy = (x - y)^2 - 4a^2$
 $= (x - y + 2a)(x - y - 2a).$

26. $x^5 - 4x^3y^2 + x^2y^3 - 4y^5 = x^3(x^2 - 4y^2) + y^3(x^2 - 4y^2)$
 $= (x^2 - 4y^2)(x^3 + y^3)$
 $= (x + 2y)(x - 2y)(x + y)(x^2 - xy + y^2).$
27. $x^2 + x^3 - 6x^4 = x^2(1 + x - 6x^2) = x^2(1 + 3x)(1 - 2x).$
28. $1 - x^2 + x^3 - x^5 = (1 - x^2) + x^3(1 - x^2)$
 $= (1 - x^2)(1 + x^3)$
 $= (1 + x)^2(1 - x)(1 - x + x^2).$
29. $1 - 4x^2 + 8x^3 - 32x^5 = (1 - 4x^2) + 8x^3(1 - 4x^2)$
 $= (1 - 4x^2)(1 + 8x^3)$
 $= (1 + 2x)^2(1 - 2x)(1 - 2x + 4x^2).$
30. $x^6 - 8x^4 + 16x^2 = x^2(x^4 - 8x^2 + 16) = x^2(x^2 - 4)^2 = x^2(x + 2)^2(x - 2)^2.$
31. $2x^3y + 4x^2y + 2xy - 2xy^3 = 2xy(x^2 + 2x + 1 - y^2)$
 $= 2xy(x + 1 + y)(x + 1 - y).$
32. $8a^2y - 8ay^2 + 2y^3 - 2y = 2y(4a^2 - 4ay + y^2 - 1)$
 $= 2y(2a - y + 1)(2a - y - 1).$
33. $4x^4 + 4x^3 - 4x^6 + x^2 = x^2(4x^2 + 4x + 1 - 4x^4)$
 $= x^2(2x + 1 + 2x^2)(2x + 1 - 2x^2).$
34. $a^6 - 13a^4 + 36a^2 = a^2(a^4 - 13a^2 + 36)$
 $= a^2(a^2 - 9)(a^2 - 4)$
 $= a^2(a + 3)(a - 3)(a + 2)(a - 2).$
35. $x^8y^2 - 4x^6y^2 + 3x^4y^6 - 12x^2y^6 = x^2y^2(x^6 - 4x^4 + 3x^2y^4 - 12y^4)$
 $= x^2y^2[x^4(x^2 - 4) + 3y^4(x^2 - 4)]$
 $= x^2y^2(x^2 - 4)(x^4 + 3y^4)$
 $= x^2y^2(x + 2)(x - 2)(x^4 + 3y^4).$
36. $ax + ad + bd = ac + bc - bx.$
 $ax + bx = ac + bc - ad - bd.$
 $x(a + b) = c(a + b) - d(a + b).$
 $x = c - d.$
37. $ax + bx + 3 = a + b + 3x.$
 $ax + bx - 3x = a + b - 3.$
 $x(a + b - 3) = a + b - 3.$
 $x = 1.$
38. $10r - 25 - 5x = r^2 - rx.$
 $rx - 5x = r^2 - 10r + 25.$
 $x(r - 5) = (r - 5)^2.$
 $x = r - 5.$
39. $4d^2 + 2dx - c^2 = -cx.$
 $cx + 2dx = c^2 - 4d^2.$
 $x(c + 2d) = (c + 2d)(c - 2d).$
 $x = c - 2d.$

$$\begin{aligned}
 40. \quad & 4r^2 + 9 + 3z = 2rz + 12r. \\
 & 3z - 2rz = -(9 - 12r + 4r^2). \\
 & z(3 - 2r) = -(3 - 2r)^2. \\
 & z = -(3 - 2r) = 2r - 3.
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & m + 42 - 7x = m^2 - mx. \\
 & mx - 7x = m^2 - m - 42. \\
 & x(m - 7) = (m - 7)(m + 6). \\
 & x = m + 6.
 \end{aligned}$$

$$\begin{aligned}
 42. \quad & a + 15 + 5x = 2a^2 - 2ax. \\
 & 5x + 2ax = 2a^2 - a - 15. \\
 & x(2a + 5) = (2a + 5)(a - 3). \\
 & x = a - 3.
 \end{aligned}$$

$$\begin{aligned}
 43. \quad & -3c - x = 2c^2 - 2cx - 2. \\
 & 2cx - x = 2c^2 + 3c - 2. \\
 & x(2c - 1) = (2c - 1)(c + 2). \\
 & x = c + 2.
 \end{aligned}$$

$$\begin{aligned}
 44. \quad & 6r^2 - 12r + 4x = r^3 - r^2x + 4rx - 8. \\
 & r^2x - 4rx + 4x = r^3 - 6r^2 + 12r - 8. \\
 & x(r^2 - 4r + 4) = (r^3 - 8) - 6r(r - 2). \\
 & x(r - 2) = r^2 + 2r + 4 - 6r. \\
 & x(r - 2) = r^2 - 4r + 4. \\
 & x = r - 2.
 \end{aligned}$$

$$\begin{aligned}
 45. \quad & a^3 - c^3 = ay - cy. \\
 & y(a - c) = (a - c)(a^2 + ac + c^2). \\
 & y = a^2 + ac + c^2.
 \end{aligned}$$

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$$\begin{aligned}
 1. \quad & x^2 - 9 = 0. \\
 & \text{Factoring,} \\
 & (x - 3)(x + 3) = 0. \\
 & x - 3 = 0, \\
 \text{or} \quad & x + 3 = 0. \\
 & x = 3, -3.
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & x^2 - 25 = 0. \\
 & \text{Factoring,} \\
 & (x - 5)(x + 5) = 0. \\
 & \text{Whence} \quad x = 5, -5.
 \end{aligned}$$

CR

$$\begin{aligned}
 3. \quad & x^2 = 16. \\
 & \text{Transposing,} \\
 & x^2 - 16 = 0. \\
 & (x - 4)(x + 4) = 0. \\
 & \text{Whence} \quad x = 4, -4.
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & x^2 = 49. \\
 & x^2 - 49 = 0. \\
 & (x - 7)(x + 7) = 0. \\
 & x = 7, -7.
 \end{aligned}$$

5. $x^2 - 2x = 0.$
 $x(x - 2) = 0.$
 $x = 0, 2.$
6. $2x^2 + 6x = 0.$
 $2x(x + 3) = 0.$
 $x = 0, -3.$
7. $x^2 = 5x.$
 $x^2 - 5x = 0.$
 $x(x - 5) = 0.$
 $x = 0, 5.$
8. $3x^2 = 9x.$
 $3x^2 - 9x = 0.$
 $3x(x - 3) = 0.$
 $x = 0, 3.$
9. $x^2 - 4x + 3 = 0.$
 $(x - 3)(x - 1) = 0.$
 $x = 3, 1.$
10. $x^2 - 6x + 9 = 0.$
 $(x - 3)^2 = 0.$
 $x = 3$ (for both factors).
11. $x^2 - 7x = -12.$
 $x^2 - 7x + 12 = 0.$
 $(x - 3)(x - 4) = 0.$
 $x = 3, 4.$
12. $x^2 + x = 20.$
 $x^2 + x - 20 = 0.$
 $(x - 4)(x + 5) = 0.$
 $x = 4, -5.$
13. $x^2 + 8 = -6x.$
 $x^2 + 6x + 8 = 0.$
 $(x + 2)(x + 4) = 0.$
 $x = -2, -4.$
14. $x^2 = 3x + 10.$
 $x^2 - 3x - 10 = 0.$
 $(x - 5)(x + 2) = 0.$
 $x = 5, -2.$
15. $4x^2 - 16x = 0.$
 $4x(x - 4) = 0.$
 $x = 0, 4.$
16. $5x^2 + 35x = 0.$
 $5x(x + 7) = 0.$
 $x = 0, -7.$
17. $8 - 9x = -x^2.$
 $x^2 - 9x + 8 = 0.$
 $(x - 1)(x - 8) = 0.$
 $x = 1, 8.$
18. $12x - 28 = -x^2.$
 $x^2 + 12x - 28 = 0.$
 $(x - 2)(x + 14) = 0.$
 $x = 2, -14.$
19. $x^2 - 16x + 64 = 0.$
 $(x - 8)^2 = 0.$
 $x = 8$ (for both factors).
20. $x^2 - 54 = 15x.$
 $x^2 - 15x - 54 = 0.$
 $(x - 18)(x + 3) = 0.$
 $x = 18, -3.$
21. $12 - 25x + 12x^2 = 0.$
 $(4 - 3x)(3 - 4x) = 0.$
 $x = \frac{4}{3}, \frac{3}{4}.$
22. $3x^2 + x = 4.$
 $3x^2 + x - 4 = 0.$
 $(x - 1)(3x + 4) = 0.$
 $x = 1, -\frac{4}{3}.$
23. $18x^2 = 9x + 20.$
 $18x^2 - 9x - 20 = 0.$
 $(3x - 4)(6x + 5) = 0.$
 $x = \frac{4}{3}, -\frac{5}{6}.$

24. $(x + 8)(x + 1) = -12.$
 $x^2 + 9x + 8 = -12.$
 $x^2 + 9x + 20 = 0.$
 $(x + 4)(x + 5) = 0.$
 $x = -4, -5.$
25. $x^2 + 9x - 12 = 3x + 15.$
 $x^2 + 6x - 27 = 0.$
 $(x - 3)(x + 9) = 0.$
 $x = 3, -9.$
26. $x + 12 = x^2.$
 $0 = x^2 - x - 12.$
 $(x - 4)(x + 3) = 0.$
 $x = 4, -3.$
27. $50x + 24 = 25x^2.$
 $0 = 25x^2 - 50x - 24.$
 $(5x - 12)(5x + 2) = 0.$
 $x = \frac{12}{5}, -\frac{2}{5}.$
28. $(x - 11)(x + 3) = 2x + 42.$
 $x^2 - 8x - 33 = 2x + 42.$
 $x^2 - 10x - 75 = 0.$
 $(x - 15)(x + 5) = 0.$
 $x = 15, -5.$
29. $x^2 - a^2 = 0.$
 $(x - a)(x + a) = 0.$
 $x = a, -a.$
30. $x^2 = 4b^2.$
 $x^2 - 4b^2 = 0.$
 $(x - 2b)(x + 2b) = 0.$
 $x = 2b, -2b.$
31. $x^2 = 16k^2.$
 $x^2 - 16k^2 = 0.$
 $(x - 4k)(x + 4k) = 0.$
 $x = 4k, -4k.$
32. $x^2 - 2bx + b^2 = 0.$
 $(x - b)^2 = 0.$
 $x = b$ (for both factors).
33. $x^2 + 4a^2 = 4ax.$
 $x^2 - 4ax + 4a^2 = 0.$
 $(x - 2a)^2 = 0.$
 $x = 2a$ (for both factors).
34. $x^2 - ax = 0.$
 $x(x - a) = 0.$
 $x = 0, a.$
35. $x^2 = 7ax.$
 $x^2 - 7ax = 0.$
 $x(x - 7a) = 0.$
 $x = 0, 7a.$
36. $2x^2 - 7x = 21 - 6x.$
 $2x^2 - x - 21 = 0.$
 $(2x - 7)(x + 3) = 0.$
 $x = \frac{7}{2}, -3.$
37. $x^2 - bx - ax = 0.$
 $x(x - b - a) = 0.$
 $x = 0, b + a.$
38. $x^2 - 5x = 2ax.$
 $x^2 - 2ax - 5x = 0.$
 $x(x - 2a - 5) = 0.$
 $x = 0, 2a + 5.$
39. $x^2 - ax - bx + ab = 0.$
 $x^2 - (a + b)x + ab = 0.$
 $(x - a)(x - b) = 0.$
 $x = a, b.$
40. $x^2 + bx = 4b + 4x.$
 $x^2 - 4x + bx - 4b = 0.$
 $x^2 + (b - 4)x - 4b = 0.$
 $(x - 4)(x + b) = 0.$
 $x = 4, -b.$

Page 142 (First set)

1. $x^3 - 9x = 0.$
 $x(x - 3)(x + 3) = 0.$
 $x = 0, 3, -3.$
2. $3x^3 = 12x.$
 $3x^3 - 12x = 0.$
 $3x(x - 2)(x + 2) = 0.$
 $x = 0, 2, -2.$
3. $x^3 + 4x^2 - 12x = 0.$
 $x(x^2 + 4x - 12) = 0.$
 $x(x - 2)(x + 6) = 0.$
 $x = 0, 2, -6.$
4. $x^3 - 14x = 5x^2.$
 $x^3 - 5x^2 - 14x = 0.$
 $x(x^2 - 5x - 14) = 0.$
 $x(x - 7)(x + 2) = 0.$
 $x = 0, 7, -2.$
5. $5x^2 = 2x - 3x^3.$
 $3x^3 + 5x^2 - 2x = 0.$
 $x(3x^2 + 5x - 2) = 0.$
 $x(3x - 1)(x + 2) = 0.$
 $x = 0, \frac{1}{3}, -2.$
6. $x^3 - x^2 - 4x + 4 = 0.$
 $(x - 1)(x^2 - 4) = 0.$
 $(x - 1)(x - 2)(x + 2) = 0.$
 $x = 1, 2, -2.$
7. $2x^3 - x^2 = 8x - 4.$
 $2x^3 - x^2 - 8x + 4 = 0.$
 $(2x - 1)(x^2 - 4) = 0.$
 $(2x - 1)(x - 2)(x + 2) = 0.$
 $x = \frac{1}{2}, 2, -2.$
8. $3x^3 - 2x^2 - 12x + 8 = 0.$
 $(3x - 2)(x^2 - 4) = 0.$
 $(3x - 2)(x - 2)(x + 2) = 0.$
 $x = \frac{2}{3}, 2, -2.$
9. $x^3 - 25 = 25x - x^2.$
 $x^3 + x^2 - 25x - 25 = 0.$
 $(x^2 - 25)(x + 1) = 0.$
 $(x - 5)(x + 5)(x + 1) = 0.$
 $x = 5, -5, -1.$
10. $2(x^3 - x) = 3(1 - x^2).$
 $2x^3 + 3x^2 - 2x - 3 = 0.$
 $(x^2 - 1)(2x + 3) = 0.$
 $(x - 1)(x + 1)(2x + 3) = 0.$
 $x = 1, -1, -\frac{3}{2}.$
11. $x^4 - 5x^2 + 4 = 0.$
 $(x^2 - 4)(x^2 - 1) = 0.$
 $(x - 2)(x + 2)(x - 1)(x + 1) = 0.$
 $x = 2, -2, 1, -1.$
12. $x^4 - 13x^2 + 36 = 0.$
 $(x^2 - 4)(x^2 - 9) = 0.$
 $(x - 2)(x + 2)(x - 3)(x + 3) = 0.$
 $x = 2, -2, 3, -3.$

13. $x^4 = 4x^2$.
 $x^4 - 4x^2 = 0$.
 $x^2(x - 2)(x + 2) = 0$.
 $x = 0, 0, 2, -2$.
14. $x^4 = 26x^2 - 25$.
 $x^4 - 26x^2 + 25 = 0$.
 $(x^2 - 25)(x^2 - 1) = 0$.
 $(x - 5)(x + 5)(x - 1)(x + 1) = 0$.
 $x = 5, -5, 1, -1$.

Page 142 (Second set)

- | | |
|--|---------------------------------|
| 1. Let $n =$ the number. | 6. Let $n =$ the number. |
| Then $n^2 + n = 42$. | Then |
| Whence $n = 6$ or -7 . | $(15 - n)(25 - n) = 119$. |
| 2. Let $n =$ the number. | Whence $n = 8$ or 32 . |
| Then $n^2 - 3n = 54$. | 7. Let $n =$ the number. |
| Whence $n = 9$ or -6 . | Then |
| 3. Let $n =$ the number. | $(30 - n)(18 + n) = 560$. |
| Then $n^2 + 2n + 9 = 129$. | Whence $n = 2$ or 10 . |
| Whence $n = 10$ or -12 . | 8. Let $n =$ the number. |
| 4. Let $n =$ the number. | Then |
| Then $3n^2 = 4n$. | $(n + 15)(22 - n) = 50n + 36$. |
| Whence $n = 0$ or $\frac{4}{3}$. | Whence $n = 6$ or -49 . |
| 5. Let $n =$ the number. | 9. Let $n =$ the number. |
| Then | Then $(4n)^2 - 5n = 15n^2$. |
| $(n + 16)(n + 21) = 546$. | Whence $n = 0$ or 5 . |
| Whence $n = 5$ or -42 . | |
| 10. Let $x =$ the frontage in feet. | |
| Then $4x =$ the depth in feet, | |
| and $(x)(4x) = 1600$. | |
| Whence $x = 20$ or -20 . | |
| But the answer -20 is absurd and must be rejected. | |
| Therefore $x = 20$; $4x = 80$. | |
| 11. Let $x =$ the breadth in yards. | |
| Then $x + 3 =$ the length in yards, | |
| and $x(x + 3) = 54$. | |
| Whence $x = 6$, root -9 rejected; | |
| $x + 3 = 9$. | |

12. Let x = the width in rods.
 Then $x + 6$ = the length in rods,
 and $x(x + 6) = 216$.
 Whence $x = 12$, root — 18 rejected ;
 $x + 6 = 18$.
13. Let $x, x + 1$, be the integers.
 Then $x^2 + (x + 1)^2 = 313$.
 Whence $x = 12, x + 1 = 13$;
 or $x = -13, x + 1 = -12$.
14. Let $x, x + 2$, be the integers.
 Then $x^2 + (x + 2)^2 = 514$.
 Whence $x = 15, x + 2 = 17$;
 or $x = -17, x + 2 = -15$.
15. Let $x, x + 2, x + 4$, be the integers.
 Then $x^2 + (x + 2)^2 + (x + 4)^2 = 251$.
 Whence $x = 7, x + 2 = 9, x + 4 = 11$;
 or $x = -11, x + 2 = -9, x + 4 = -7$.
16. Let x = the length and breadth in inches.
 Then x^2 = the area of the bottom in square inches,
 and $6x$ = the area of one side in square inches.
 Therefore $x^2 + 24x = 112$.
 Whence $x = 4$, root — 28 rejected.
17. Let x = the edge in inches.
 Then x^2 = the area of a single face in square inches.
 Therefore $6x^2 = 216$.
 Whence $x = 6$, root — 6 rejected.
18. Let x = the depth in inches.
 Then $2x$ = the width, and $3x$ = the length, in inches.
 There will be two sides $x \cdot 2x$ each in area, two sides $x \cdot 3x$ each, and
 top and bottom $2x \cdot 3x$ each.
 Therefore $4x^2 + 6x^2 + 12x^2 = 550$.
 Whence $x = 5$, root — 5 rejected.
 Then $2x = 10$; $3x = 15$.
19. Let x = the depth in inches.
 Then $x + 1$ = the width, and $x + 3$ the length, in inches.
 As in Ex. 18, the total surface =
 $2x(x + 1) + 2x(x + 3) + 2(x + 1)(x + 3) = 62$.
 Whence $(3x + 14)(x - 2) = 0$,
 $x = 2$, root — $\frac{14}{3}$ rejected ;
 $x + 1 = 3, x + 3 = 5$.

20. Let $b =$ the base in feet.
 Then $\frac{8b}{2} = 40$, or $4b = 40$.
 Whence $b = 10$.
21. Let $b =$ the base in feet.
 Then $2b =$ the altitude in feet,
 and $\frac{b \times 2b}{2} = 36$, or $b^2 = 36$.
 Whence $b = 6$, root — 6 rejected ;
 $2b = 12$.
22. Let $a =$ the altitude in feet.
 Then $5a =$ the base in feet,
 and $\frac{a \times 5a}{2} = 40$, or $\frac{5}{2}a^2 = 40$,
 or $a^2 = 16$.
 Whence $a = 4$, root — 4 rejected ;
 $5a = 20$.
23. Let $a =$ the altitude in inches.
 Then $6a =$ the base in inches,
 and $\frac{a \times 6a}{2} = 48$, or $3a^2 = 48$.
 Whence $a = 4$, root — 4 rejected ;
 $6a = 24$.
25. Let $x =$ the base in feet.
 Then $x + 4 =$ the altitude in feet,
 and $\frac{x(x + 4)}{2} = 6$.
 Multiplying by 2, $x(x + 4) = 12$.
 Whence $x = 2$, root — 6 rejected ;
 $x + 4 = 6$.
26. Let $x =$ the shorter leg in yards.
 Then $x + 4 =$ the longer leg in yards,
 and $\frac{x(x + 4)}{2} = 30$.
 Multiplying by 2, $x(x + 4) = 60$.
 Whence $x = 6$, root — 10 rejected ;
 $x + 4 = 10$.

27. Let x = the shorter leg in feet.
 Then $x + 8$ = the longer leg in feet,
 and $\frac{x(x + 8)}{2} = 9 \times 26 = 234$.
 Multiplying by 2, $x(x + 8) = 468$.
 Whence $x = 18$, root — 26 rejected ;
 $x + 8 = 26$.

28. Let x = the altitude in inches.
 Then $2x + 6$ = the base in inches,
 and $\frac{x(2x + 6)}{2} = \frac{21}{8} \times 144 = 378$.
 Performing the indicated multiplication and division,
 $x^2 + 3x = 378$.
 Whence $x = 18$, root — 21 rejected ;
 $2x + 6 = 42$.

29. Let x = the base in feet.
 Then $2x - 6$ = the altitude in feet,
 and $\frac{x(2x - 6)}{2} = 12 \times 9$.
 Performing the indicated operations,
 $x^2 - 3x = 108$.
 Whence $x = 12$, root — 9 rejected ;
 $2x - 6 = 18$.

30. Let A = the area.
 Then $A = \frac{12(10 + 18)}{2} = \frac{12 \times 28}{2} = 168$.

32. Let x = the altitude in feet.
 Then $3x$ = the second base in feet,
 and $\text{area} = \frac{x(3x + 12)}{2} = 90$.
 Multiplying by 2, $x(3x + 12) = 180$.
 Whence $x = 6$, root — 10 rejected ;
 $3x = 18$.

33. Let x = the longer base in feet.
 Then $\frac{2}{3}x$ = the shorter base in feet,
 and $\frac{1}{2} \cdot \frac{2}{3}x = \frac{1}{3}x$ = the altitude in feet.

Therefore
$$\frac{\frac{1}{3}x(x + \frac{2}{3}x)}{2} = \frac{\frac{1}{3}x \cdot \frac{5}{3}x}{2} = \frac{5}{18}x^2 = 250,$$

or
$$x^2 = 900.$$

Whence
$$x = 30, \text{ root } - 30 \text{ rejected;} \\ \frac{2}{3}x = 20; \frac{1}{3}x = 10.$$

34. Let
$$x = \text{the altitude in feet.}$$

Then
$$x + 12 \text{ and } x + 6 = \text{the bases in feet,}$$

and
$$\frac{x(x + 12 + x + 6)}{2} = 112.$$

Performing the indicated operations,

$$x^2 + 9x = 112.$$

Whence
$$x = 7, \text{ root } - 16 \text{ rejected;} \\ x + 6 = 13; x + 12 = 19.$$

35. Let
$$x = \text{the altitude in feet.}$$

Then
$$x + 1 \text{ and } x + 5 = \text{the bases in feet,}$$

and
$$\frac{x(x + 1 + x + 5)}{2} = 9 \times 30.$$

Performing the indicated operations,

$$x^2 + 3x = 270.$$

Whence
$$x = 15, \text{ root } - 18 \text{ rejected;} \\ x + 1 = 16; x + 5 = 20.$$

36. Let
$$x = \text{the shorter base in feet.}$$

Then
$$x + 6 = \text{the longer base in feet,}$$

and
$$\frac{1}{2}(x + x + 6) = x + 3 = \text{the altitude in feet.}$$

Therefore
$$\frac{(x + 3)(2x + 6)}{2} = 9 \times 9.$$

Performing the indicated operations,

$$x^2 + 6x + 9 = 81.$$

Whence
$$x = 6, \text{ root } - 12 \text{ rejected;} \\ x + 6 = 12; x + 3 = 9.$$

37. Let
$$x = \text{the altitude and the shorter base} \\ \text{in feet.}$$

Then
$$x + 2 = \text{the longer base in feet,}$$

and
$$\frac{x(x + x + 2)}{2} = 8 \times 9.$$

Performing the indicated operations,

$$x^2 + x = 72.$$

Whence
$$x = 8, \text{ root } - 9 \text{ rejected;} \\ x + 2 = 10.$$

38. Let x = the shorter base in feet.
 Then $x + 10$ = the longer base in feet,
 and $5x + 2$ = the altitude in feet.

Therefore $\frac{(5x + 2)(x + x + 10)}{2} = 9 \times 22.$

Performing the indicated operations,

$$5x^2 + 27x + 10 = 198.$$

Whence

$$x = 4, \text{ root } -\frac{47}{5} \text{ rejected;}$$

$$x + 10 = 14;$$

$$5x + 2 = 22.$$

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$$1. \frac{36a}{54a^2} = \frac{\cancel{2}^2 \cdot \cancel{3}^2 \cdot \cancel{a}}{\cancel{2}^2 \cdot \cancel{3}^3 \cdot \cancel{a}^2} = \frac{2}{3a}.$$

$$3. \frac{84m^2}{126mn} = \frac{\cancel{2}^2 \cdot \cancel{3} \cdot \cancel{7}^m \cdot \cancel{m}^2}{\cancel{2}^2 \cdot \cancel{3}^2 \cdot \cancel{7} \cdot \cancel{m}n} = \frac{2m}{3n}.$$

$$2. \frac{52x^2}{130xy} = \frac{\cancel{2}^2 \cdot \cancel{13}^x \cdot \cancel{x}^2}{\cancel{2} \cdot 5 \cdot \cancel{13}xy} = \frac{2x}{5y}.$$

$$4. \frac{x^3y}{xy^3} = \frac{x^2}{y^2}.$$

$$5. \frac{xy^2}{2x^2y} = \frac{y}{2x}.$$

$$7. \frac{10ax^2}{25ax^3} = \frac{2}{5x}.$$

$$9. \frac{108m^2n^3}{252md^2} = \frac{3mn^3}{7d^2}.$$

$$6. \frac{5ab^2c}{3ac^2} = \frac{5b^2}{3c}.$$

$$8. \frac{27cd^2e}{45c^2e^3} = \frac{3d^2}{5ce^2}.$$

$$10. \frac{2a}{a^2 + ab} = \frac{2}{a + b}.$$

$$11. \frac{4c^2 - 8cd}{12c} = \frac{4c(c - 2d)}{12c} = \frac{c - 2d}{3}.$$

$$12. \frac{10x^2y^2 - 5xy^3}{15xy^3} = \frac{5xy^2(2x - y)}{15xy^3} = \frac{2x - y}{3y}.$$

$$13. \frac{9a^2 - 6a}{3a - 2} = \frac{3a(3a - 2)}{3a - 2} = 3a.$$

$$14. \frac{4x^2 - 25}{4x^2 - 20x + 25} = \frac{(2x + 5)(2x - 5)}{(2x - 5)(2x - 5)} = \frac{2x + 5}{2x - 5}.$$

$$15. \frac{9 - 6x + x^2}{9 - x^2} = \frac{(3 - x)(3 - x)}{(3 + x)(3 - x)} = \frac{3 - x}{3 + x}.$$

$$16. \frac{9x^2 - 1}{9x^2 - 9x + 2} = \frac{(3x + 1)(3x - 1)}{(3x - 1)(3x - 2)} = \frac{3x + 1}{3x - 2}.$$

$$17. \frac{10a^2 - 2a}{15a^2 + 7a - 2} = \frac{2a(5a - 1)}{(5a - 1)(3a + 2)} = \frac{2a}{3a + 2}.$$

$$18. \frac{c^3 - 4c}{c^5 - 8c^2} = \frac{c(c + 2)(c - 2)}{c^2(c - 2)(c^2 + 2c + 4)} = \frac{c + 2}{c(c^2 + 2c + 4)}.$$

19. $\frac{c^2 - 6c + 8}{c^2 + c - 6} = \frac{(c - 4)(c - 2)}{(c + 3)(c - 2)} = \frac{c - 4}{c + 3}.$
20. $\frac{2x^3 + x^2 - 3x}{3x^4 - 3x^2} = \frac{x(2x + 3)(x - 1)}{3x^2(x + 1)(x - 1)} = \frac{2x + 3}{3x(x + 1)}.$
21. $\frac{50x - 2x^3}{x^3 + 8x^2 + 15x} = \frac{2x(5 + x)(5 - x)}{x(x + 5)(x + 3)} = \frac{2(5 - x)}{x + 3}.$
22. $\frac{x^3 - 49x}{42x^2 - 27x^3 + 3x^4} = \frac{x(x + 7)(x - 7)}{3x^2(7 - x)(2 - x)} = \frac{x + 7}{3x(x - 2)}.$
23. $\frac{a^3 - a^2 + 2a - 2}{3a^3 - a^2 + 6a - 2} = \frac{(a - 1)(a^2 + 2)}{(3a - 1)(a^2 + 2)} = \frac{a - 1}{3a - 1}.$
24. $\frac{2a^3y + 2a^2xy + 2acy + 2cxy}{2ax + 2a^2 - ay - xy} = \frac{2y(a + x)(a^2 + c)}{(x + a)(2a - y)} = \frac{2y(a^2 + c)}{2a - y}.$
25. $\frac{m^2 - n^2}{m^3 - n^3} = \frac{(m + n)(m - n)}{(m - n)(m^2 + mn + n^2)} = \frac{m + n}{m^2 + mn + n^2}.$
26. $\frac{x^4 - 8x}{x^3 - 2x^2} = \frac{x(x - 2)(x^2 + 2x + 4)}{x^2(x - 2)} = \frac{x^2 + 2x + 4}{x}.$
27. $\frac{a^2 - 4}{(a - 2)^2} = \frac{(a + 2)(a - 2)}{(a - 2)^2} = \frac{a + 2}{a - 2}.$
28. $\frac{a^6 - 1}{a^2 - 1} = \frac{(a + 1)(a^2 - a + 1)(a - 1)(a^2 + a + 1)}{(a + 1)(a - 1)} = (a^2 - a + 1)(a^2 + a + 1).$
29. $\frac{a^4 - x^4}{a^4 + 3a^2x^2 + 2x^4} = \frac{(a^2 + x^2)(a + x)(a - x)}{(a^2 + x^2)(a^2 + 2x^2)} = \frac{a^2 - x^2}{a^2 + 2x^2}.$
30. $\frac{27x^3 + a^3}{9x^2 - a^2} = \frac{(3x + a)(9x^2 - 3ax + a^2)}{(3x + a)(3x - a)} = \frac{9x^2 - 3ax + a^2}{3x - a}.$

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|-----------------------------------|------------------------------------|----------------------------------|
| 1. $96 = 2^5 \cdot 3.$ | 3. $50 = 2 \cdot 5^2.$ | 5. $mn.$ |
| $36 = 2^2 \cdot 3^2.$ | $44 = 2^2 \cdot 11.$ | $m^2.$ |
| $40 = 2^3 \cdot 5.$ | $110 = 2 \cdot 5 \cdot 11.$ | $m^5n^2.$ |
| L.C.M. $= 2^5 \cdot 3^2 \cdot 5.$ | $275 = 11 \cdot 5^2.$ | L.C.M. $= m^5 \cdot n^2.$ |
| | L.C.M. $= 2^2 \cdot 5^2 \cdot 11.$ | |
| 2. $45 = 3^2 \cdot 5.$ | 4. $a^2x.$ | 6. $2x = 2x.$ |
| $105 = 3 \cdot 5 \cdot 7.$ | $ax^3.$ | $6x^2 = 2 \cdot 3x^2.$ |
| $175 = 5^2 \cdot 7.$ | $a^2x^2.$ | $4x^3 = 2^2 \cdot x^3.$ |
| L.C.M. $= 3^2 \cdot 5^2 \cdot 7.$ | L.C.M. $= a^2 \cdot x^3.$ | L.C.M. $= 2^2 \cdot 3 \cdot x^3$ |
| | | $= 12x^3.$ |

$$\begin{aligned}
 7. \quad & 3r = 3r. \\
 & 12 = 2^2 \cdot 3. \\
 & 15r^2 = 3 \cdot 5 \cdot r^2. \\
 & \text{L.C.M.} = 2^2 \cdot 3 \cdot 5r^2 \\
 & = 60r^2.
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & 4a = 2^2 \cdot a. \\
 & 6a^2 = 2 \cdot 3 \cdot a^2. \\
 & 8a^5 = 2^3 \cdot a^5. \\
 & \text{L.C.M.} = 2^3 \cdot 3a^5 \\
 & = 24a^5.
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & 2ax^2. \\
 & 3a^2x. \\
 & a^2cx^3. \\
 & \text{L.C.M.} = 2 \cdot 3a^2cx^3 \\
 & = 6a^2cx^3.
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & 4x^2y = 2^2 \cdot x^2y. \\
 & 6x^3yz^2 = 2 \cdot 3 \cdot x^3yz^2. \\
 & 10xz^2 = 2 \cdot 5xz^2. \\
 & \text{L.C.M.} = 2^2 \cdot 3 \cdot 5x^3yz^2 \\
 & = 60x^3yz^2.
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & 6a^2b = 2 \cdot 3a^2b. \\
 & 9ab^2c = 3^2ab^2c. \\
 & 30b^3c^3 = 2 \cdot 3 \cdot 5b^3c^3. \\
 & \text{L.C.M.} = 2 \cdot 3^2 \cdot 5a^2b^3c^3 \\
 & = 90a^2b^3c^3.
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & 4c^2f = 2^2c^2f. \\
 & 10df^2 = 2 \cdot 5df^2. \\
 & 24cd^2e^3 = 2^3 \cdot 3cd^2e^3. \\
 & \text{L.C.M.} = 2^3 \cdot 3 \cdot 5c^2d^2e^3f^2 \\
 & = 120c^2d^2e^3f^2.
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & 2a^2c = 2a^2c. \\
 & 18ax^2k^3 = 2 \cdot 3^2 \cdot ax^2k^3. \\
 & 45x^3k = 3^2 \cdot 5x^3k. \\
 & \text{L.C.M.} = 2 \cdot 3^2 \cdot 5a^2ck^3x^3 \\
 & = 90a^2ck^3x^3.
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & x^2 - xy = x(x - y). \\
 & 2x^2y^3 = 2x^2y^3. \\
 & \text{L.C.M.} = 2x^2y^3(x - y).
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & a^2 + ab = a(a + b). \\
 & 2ab = 2ab. \\
 & \text{L.C.M.} = 2ab(a + b).
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & 5c - 10 = 5(c - 2). \\
 & 10c = 2 \cdot 5c. \\
 & \text{L.C.M.} = 10c(c - 2).
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & b^2 - bk = b(b - k). \\
 & b^2k = b^2k. \\
 & \text{L.C.M.} = b^2k(b - k).
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & ac^2 = ac^2. \\
 & 3a^2c = 3a^2c. \\
 & 6a^2c - 9ac^2 = 3ac(2a - 3c). \\
 & \text{L.C.M.} = 3a^2c^2(2a - 3c).
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & a^2 + ab = a(a + b). \\
 & ab + b^2 = b(a + b). \\
 & \text{L.C.M.} = ab(a + b).
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & a^2 - ab = a(a - b). \\
 & 2a - 2b = 2(a - b). \\
 & 2a^2 - 2ab = 2a(a - b). \\
 & \text{L.C.M.} = 2a(a - b).
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & ax - ay = a(x - y). \\
 & x^2 + xy = x(x + y). \\
 & x^2 - y^2 = (x + y)(x - y). \\
 & \text{L.C.M.} = ax(x + y)(x - y).
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & a^2 - 4 = (a + 2)(a - 2). \\
 & a^2 + 3a + 2 = (a + 2)(a + 1). \\
 & a^2 - a - 2 = (a - 2)(a + 1). \\
 & \text{L.C.M.} = (a + 2)(a - 2)(a + 1).
 \end{aligned}$$

24. $4x^2 - 9 = (2x + 3)(2x - 3).$
 $2x^2 + 7x + 6 = (2x + 3)(x + 2).$
 $2x^2 + x - 6 = (2x - 3)(x + 2).$
L.C.M. $= (2x + 3)(2x - 3)(x + 2).$
25. $x^2 - 1 = (x + 1)(x - 1).$
 $x^3 - 1 = (x - 1)(x^2 + x + 1).$
 $x^2 + 2x + 1 = (x + 1)^2.$
L.C.M. $= (x + 1)^2(x - 1)(x^2 + x + 1).$
26. $a^2 - 9 = (a + 3)(a - 3).$
 $2a^3 + 6a^2 + 18a = 2a(a^2 + 3a + 9).$
 $2a^4 - 54a = 2a(a - 3)(a^2 + 3a + 9).$
L.C.M. $= 2a(a + 3)(a - 3)(a^2 + 3a + 9).$
27. $x^3 - 4x = x(x + 2)(x - 2).$
 $4x^2 + 2x = 2x(2x + 1).$
 $2x^2 + 4x = 2x(x + 2).$
 $2x^2 - 3x - 2 = (2x + 1)(x - 2).$
L.C.M. $= 2x(x + 2)(x - 2)(2x + 1).$

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1. $\frac{x}{3}, \frac{y}{4}, \frac{z}{5}$. L.C.D. = 60.
 $\frac{x}{3} = \frac{20x}{60}, \frac{y}{4} = \frac{15y}{60}, \frac{z}{5} = \frac{12z}{60}.$
2. $\frac{3x}{8}, \frac{5x^2}{12}, \frac{7x^3}{24}$. L.C.D. = 24.
 $\frac{3x}{8} = \frac{9x}{24}, \frac{5x^2}{12} = \frac{10x^2}{24}, \frac{7x^3}{24} = \frac{7x^3}{24}.$
3. $\frac{5a}{7}, \frac{3ax}{14}, \frac{2ax^2}{21}$. L.C.D. = 42.
 $\frac{5a}{7} = \frac{30a}{42}, \frac{3ax}{14} = \frac{9ax}{42}, \frac{2ax^2}{21} = \frac{4ax^2}{42}.$
4. $\frac{3a^2}{4}, \frac{6ab}{10}, \frac{15a^2b}{24}$. L.C.D. = 40.
 $\frac{3a^2}{4} = \frac{30a^2}{40}, \frac{6ab}{10} = \frac{24ab}{40}, \frac{15a^2b}{24} \text{ or } \frac{5a^2b}{8} = \frac{25a^2b}{40}.$
5. $\frac{3}{5a}, \frac{2c}{3d}, \frac{5a}{10d}$. L.C.D. = 30ad.
 $\frac{3}{5a} = \frac{18d}{30ad}, \frac{2c}{3d} = \frac{20ac}{30ad}, \frac{5a}{10d} = \frac{15a^2}{30ad}.$
6. $\frac{3x}{4y}, \frac{5x}{6y}, \frac{3}{10y}$. L.C.D. = 60y.
 $\frac{3x}{4y} = \frac{45x}{60y}, \frac{5x}{6y} = \frac{50x}{60y}, \frac{3}{10y} = \frac{18}{60y}.$

$$7. \frac{2a+b}{3a}, \frac{a}{2a^2}, \frac{5a^2}{4a}. \quad \text{L.C.D.} = 12a.$$

$$\frac{2a+b}{3a} = \frac{8a+4b}{12a}, \frac{a}{2a^2} \text{ or } \frac{1}{2a} = \frac{6}{12a}, \frac{5a^2}{4a} = \frac{15a^2}{12a}.$$

$$8. \frac{c+d}{12c}, \frac{3c-d}{15c^2}, \frac{4a}{24c^3}. \quad \text{L.C.D.} = 60c^3.$$

$$\frac{c+d}{12c} = \frac{5c^3+5c^2d}{60c^3}, \frac{3c-d}{15c^2} = \frac{12c^2-4cd}{60c^3}, \frac{4a}{24c^3} \text{ or } \frac{a}{6c^3} = \frac{10a}{60c^3}.$$

$$9. \frac{7a}{cde^2}, \frac{5e}{cd^2e}, \frac{4c}{c^2d^2e^2}.$$

Reducing to lowest terms,

$$\frac{7a}{cde^2}, \frac{5}{cd^2}, \frac{4}{cd^2e^2}. \quad \text{L.C.D.} = cd^2e^2.$$

$$\frac{7a}{cde^2} = \frac{7ad}{cd^2e^2}, \frac{5}{cd^2} = \frac{5e^2}{cd^2e^2}, \frac{4}{cd^2e^2} = \frac{4}{cd^2e^2}.$$

$$10. \frac{7x}{3xy^2}, \frac{4y}{5xz}, \frac{2z}{10yz^2}.$$

Reducing to lowest terms,

$$\frac{7}{3y^2}, \frac{4y}{5xz}, \frac{1}{5yz}. \quad \text{L.C.D.} = 15xy^2z.$$

$$\frac{7}{3y^2} = \frac{35xz}{15xy^2z}, \frac{4y}{5xz} = \frac{12y^3}{15xy^2z}, \frac{1}{5yz} = \frac{3xy}{15xy^2z}.$$

$$12. \frac{5}{a-2}, \frac{7}{a+2}. \quad \text{L.C.D.} = (a-2)(a+2).$$

$$\frac{5}{a-2} = \frac{5(a+2)}{a^2-4}, \frac{7}{a+2} = \frac{7(a-2)}{a^2-4}.$$

$$13. \frac{a}{a-b}, \frac{b}{a+b}. \quad \text{L.C.D.} = (a-b)(a+b).$$

$$\frac{a}{a-b} = \frac{a(a+b)}{a^2-b^2}, \frac{b}{a+b} = \frac{b(a-b)}{a^2-b^2}.$$

$$14. \frac{x}{2x-5}, \frac{3x}{2x+5}. \quad \text{L.C.D.} = (2x-5)(2x+5).$$

$$\frac{x}{2x-5} = \frac{x(2x+5)}{4x^2-25}, \frac{3x}{2x+5} = \frac{3x(2x-5)}{4x^2-25}.$$

$$15. \frac{-2c}{c-3}, \frac{3c}{c+3}. \quad \text{L.C.D.} = (c-3)(c+3).$$

$$\frac{-2c}{c-3} = \frac{-2c(c+3)}{c^2-9}, \frac{3c}{c+3} = \frac{3c(c-3)}{c^2-9}.$$

$$17. \frac{x}{x+y}, \frac{2x}{x-y}, \frac{2}{x}. \text{ L.C.D.} = (x+y)(x-y)x.$$

$$\frac{x}{x+y} = \frac{x^2(x-y)}{x(x+y)(x-y)}, \quad \frac{2x}{x-y} = \frac{2x^2(x+y)}{x(x+y)(x-y)}, \quad \frac{2}{x} = \frac{2(x+y)(x-y)}{x(x+y)(x-y)}.$$

$$18. \frac{2a}{a-3}, \frac{3}{a+3}, \frac{-a}{2}. \text{ L.C.D.} = 2(a-3)(a+3).$$

$$\frac{2a}{a-3} = \frac{4a(a+3)}{2(a+3)(a-3)}, \quad \frac{3}{a+3} = \frac{6(a-3)}{2(a+3)(a-3)},$$

$$\frac{-a}{2} = \frac{-a(a+3)(a-3)}{2(a-3)(a+3)}.$$

$$19. \frac{5c}{c-5}, \frac{-2c}{c-5}, \frac{3}{c}. \text{ L.C.D.} = c(c-5).$$

$$\frac{5c}{c-5} = \frac{5c^2}{c(c-5)}, \quad \frac{-2c}{c-5} = \frac{-2c^2}{c(c-5)}, \quad \frac{3}{c} = \frac{3(c-5)}{c(c-5)}.$$

$$20. \frac{m}{m-n}, \frac{-2m}{m-n}, \frac{2}{m}. \text{ L.C.D.} = (m-n)m.$$

$$\frac{m}{m-n} = \frac{m^2}{m(m-n)}, \quad \frac{-2m}{m-n} = \frac{-2m^2}{m(m-n)}, \quad \frac{2}{m} = \frac{2(m-n)}{m(m-n)}.$$

$$21. \frac{5}{a-2}, \frac{2}{3a-6}.$$

$$\frac{5}{a-2}, \frac{2}{3(a-2)}. \text{ L.C.D.} = 3(a-2).$$

$$\frac{5}{a-2} = \frac{15}{3(a-2)}, \quad \frac{2}{3(a-2)} = \frac{2}{3(a-2)}.$$

$$22. \frac{a}{x^2-xy}, \frac{5}{x-y}.$$

$$\frac{a}{x(x-y)}, \frac{5}{x-y}. \text{ L.C.D.} = x(x-y).$$

$$\frac{a}{x(x-y)} = \frac{a}{x(x-y)}, \quad \frac{5}{x-y} = \frac{5x}{x(x-y)}.$$

$$23. \frac{3a}{a^2-b^2}, \frac{2a}{a+b}.$$

$$\frac{3a}{(a+b)(a-b)}, \frac{2a}{a+b}. \text{ L.C.D.} = (a+b)(a-b).$$

$$\frac{3a}{(a+b)(a-b)}, \frac{2a}{a+b} = \frac{2a(a-b)}{(a+b)(a-b)}.$$

$$24. \frac{5a}{a^2 - b^2}, \frac{-4a}{a - b}.$$

$$\frac{5a}{(a+b)(a-b)}, \frac{-4a}{a-b}. \quad \text{L.C.D.} = (a+b)(a-b).$$

$$\frac{5a}{(a+b)(a-b)}, \frac{-4a}{a-b} = \frac{-4a(a+b)}{(a+b)(a-b)}.$$

$$25. \frac{-3x}{x^2 - y^2}, \frac{5x}{x - y}.$$

$$\frac{-3x}{(x+y)(x-y)}, \frac{5x}{x-y}. \quad \text{L.C.D.} = (x+y)(x-y).$$

$$\frac{-3x}{(x+y)(x-y)}, \frac{5x(x+y)}{(x+y)(x-y)}.$$

$$26. \frac{2x+3}{9-x^2}, \frac{3-2x}{9+3x}, \frac{x}{3-x}.$$

$$\frac{2x+3}{(3+x)(3-x)}, \frac{3-2x}{3(3+x)}, \frac{x}{3-x}. \quad \text{L.C.D.} = 3(3+x)(3-x).$$

$$\frac{2x+3}{(3+x)(3-x)} = \frac{3(2x+3)}{3(3+x)(3-x)}, \frac{3-2x}{3(3+x)} = \frac{(3-2x)(3-x)}{3(3+x)(3-x)},$$

$$\frac{x}{3-x} = \frac{3x(3+x)}{3(3-x)(3+x)}.$$

$$27. \frac{-2x}{x-2}, \frac{3}{x^2-4}, \frac{5x}{x^2-5x+6}.$$

$$\frac{-2x}{x-2}, \frac{3}{(x+2)(x-2)}, \frac{5x}{(x-2)(x-3)}. \quad \text{L.C.D.} = (x+2)(x-2)(x-3).$$

$$\frac{-2x}{x-2} = \frac{-2x(x+2)(x-3)}{(x+2)(x-2)(x-3)}, \frac{3}{(x+2)(x-2)} = \frac{3(x-3)}{(x+2)(x-2)(x-3)},$$

$$\frac{5x}{(x-2)(x-3)} = \frac{5x(x+2)}{(x+2)(x-2)(x-3)}.$$

$$28. \frac{2a-7}{a^3-9a}, \frac{3-5a}{a^3-5a^2+6a}, \frac{2}{a-3}.$$

$$\frac{2a-7}{a(a+3)(a-3)}, \frac{3-5a}{a(a-2)(a-3)}, \frac{2}{a-3}. \quad \text{L.C.D.} = a(a+3)(a-3)(a-2).$$

$$\frac{2a-7}{a(a+3)(a-3)} = \frac{(2a-7)(a-2)}{a(a+3)(a-3)(a-2)},$$

$$\frac{3-5a}{a(a-2)(a-3)} = \frac{(3-5a)(a+3)}{a(a-2)(a-3)(a+3)},$$

$$\frac{2}{a-3} = \frac{2a(a-2)(a+3)}{a(a-2)(a+3)(a-3)}.$$

29. $\frac{a}{a^3 - 8}, \frac{2a}{a^3 + 2a^2 + 4a}.$
 $\frac{a}{(a-2)(a^2 + 2a + 4)}, \frac{2}{a^2 + 2a + 4}. \text{ L.C.D.} = (a-2)(a^2 + 2a + 4).$
 $\frac{a}{(a-2)(a^2 + 2a + 4)}, \frac{2}{a^2 + 2a + 4} = \frac{2(a-2)}{(a-2)(a^2 + 2a + 4)}.$
30. $\frac{5x}{2x^2 + 3x}, \frac{3x}{2x^2 - x - 6}.$
 $\frac{5}{(2x+3)}, \frac{3x}{(2x+3)(x-2)}. \text{ L.C.D.} = (2x+3)(x-2).$
 $\frac{5}{2x+3} = \frac{5(x-2)}{(2x+3)(x-2)}, \frac{3x}{(2x+3)(x-2)}.$
31. $\frac{-4x}{3-x}, \frac{5x}{3+8x-3x^2}.$
 $\frac{-4x}{3-x}, \frac{5x}{(3-x)(1+3x)}. \text{ L.C.D.} = (3-x)(1+3x).$
 $\frac{-4x}{3-x} = \frac{-4x(1+3x)}{(3-x)(1+3x)}; \frac{5x}{(3-x)(1+3x)}.$
32. $\frac{-1}{x+y-3}, \frac{5x+y}{x^2+y^2-9+2xy}.$
 $\frac{-1}{x+y-3}, \frac{5x+y}{(x+y+3)(x+y-3)}. \text{ L.C.D.} = (x+y+3)(x+y-3).$
 $\frac{-1}{x+y-3} = \frac{-1(x+y+3)}{(x+y-3)(x+y+3)}; \frac{5x+y}{(x+y+3)(x+y-3)}.$

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1. $\frac{3}{4} + \frac{5}{6}. \text{ L.C.D.} = 12.$
 $\frac{9}{12} + \frac{10}{12} = \frac{19}{12}.$
2. $\frac{3}{10} + \frac{8}{15}. \text{ L.C.D.} = 30.$
 $\frac{9}{30} + \frac{16}{30} = \frac{25}{30}, \text{ or } \frac{5}{6}.$
3. $\frac{9}{25} - \frac{3}{10}. \text{ L.C.D.} = 50.$
 $\frac{18}{50} - \frac{15}{50} = \frac{3}{50}.$
4. $\frac{5}{12} - \frac{7}{18}. \text{ L.C.D.} = 36.$
 $\frac{15}{36} - \frac{14}{36} = \frac{1}{36}.$
5. $\frac{3}{14} + \frac{2}{21}. \text{ L.C.D.} = 42.$
 $\frac{9}{42} + \frac{4}{42} = \frac{13}{42}.$
6. $\frac{5}{6} - \frac{3}{16}. \text{ L.C.D.} = 48.$
 $\frac{40}{48} - \frac{9}{48} = \frac{31}{48}.$
7. $\frac{7}{24} - \frac{1}{3}\frac{5}{2}. \text{ L.C.D.} = 96.$
 $\frac{28}{96} - \frac{45}{96} = \frac{-17}{96}.$
8. $\frac{1}{4}\frac{7}{0} + \frac{1}{4}\frac{9}{8}. \text{ L.C.D.} = 240.$
 $\frac{102}{240} + \frac{95}{240} = \frac{197}{240}.$
9. $\frac{1}{2}\frac{5}{8} - \frac{8}{4}\frac{7}{0}. \text{ L.C.D.} = 280.$
 $\frac{150}{280} - \frac{609}{280} = \frac{-459}{280}.$
10. $\frac{4}{2}\frac{3}{5} + 2\frac{2}{4}\frac{9}{5}. \text{ L.C.D.} = 225.$
 $\frac{387}{225} + \frac{595}{225} = \frac{982}{225}.$
11. $\frac{1}{2}\frac{9}{4} - \frac{8}{5}\frac{9}{4}. \text{ L.C.D.} = 216.$
 $\frac{171}{216} - \frac{356}{216} = \frac{-185}{216}.$

12. $\frac{5}{3}\frac{5}{8} + \frac{4}{5}\frac{3}{7}$. L.C.D. = 114.
 $\frac{1}{11}\frac{6}{4}\frac{5}{4} + \frac{8}{11}\frac{6}{4} = \frac{2}{11}\frac{5}{4}$.
13. $\frac{7}{6} - 1\frac{4}{9} + 3\frac{7}{12}$. L.C.D. = 36.
 $\frac{42}{36} - \frac{52}{36} + \frac{129}{36} = \frac{42 - 52 + 129}{36} = \frac{119}{36}$.
14. $\frac{1}{10} - \frac{1}{12} + 2\frac{5}{16}$. L.C.D. = 240.
 $\frac{264}{240} - \frac{260}{240} + \frac{555}{240} = \frac{264 - 260 + 555}{240} = \frac{559}{240}$.
15. $\frac{1}{3}\frac{3}{2} + 1 - 3\frac{9}{48}$. L.C.D. = 96.
 $\frac{39}{96} + \frac{96}{96} - \frac{306}{96} = \frac{39 + 96 - 306}{96} = -\frac{171}{96}$.
16. $\frac{3}{36} - \frac{4}{32} + \frac{1}{4}\frac{1}{2}$. L.C.D. = 2016.
 $\frac{1736}{2016} - \frac{2583}{2016} + \frac{528}{2016} = \frac{-319}{2016}$.
17. $\frac{1}{2}\frac{5}{6} - \frac{1}{3}\frac{1}{9} - \frac{2}{5}\frac{5}{2}$. L.C.D. = 156.
 $\frac{90}{156} - \frac{44}{156} - \frac{75}{156} = \frac{-29}{156}$.
18. $\frac{2}{3}\frac{3}{5} - \frac{1}{4}\frac{9}{5} + \frac{2}{5}\frac{7}{0}$. L.C.D. = 3150.
 $\frac{2}{3150}\frac{0}{0} - \frac{1}{3150}\frac{3}{0} + \frac{1}{3150}\frac{0}{1} = \frac{2}{3150}\frac{4}{1}$.
19. $\frac{1}{4}\frac{9}{8} - \frac{1}{7}\frac{3}{2} + \frac{4}{3}\frac{3}{6}$. L.C.D. = 144.
 $\frac{5}{144}\frac{7}{4} - \frac{2}{144}\frac{6}{4} + \frac{1}{144}\frac{7}{2} = \frac{2}{144}\frac{0}{3}$.
20. $\frac{1}{6}\frac{1}{4} + 2 - \frac{1}{2}\frac{7}{4}$. L.C.D. = 192.
 $\frac{3}{192}\frac{3}{2} + \frac{3}{192}\frac{8}{2} - \frac{1}{192}\frac{3}{2} = \frac{2}{192}\frac{8}{1}$.
21. $\frac{1}{6}\frac{4}{9} - \frac{1}{4}\frac{5}{6} + \frac{1}{11}\frac{9}{5}$. L.C.D. = 690.
 $\frac{1}{690}\frac{4}{0} - \frac{2}{690}\frac{2}{5} + \frac{1}{690}\frac{1}{0} = \frac{2}{690}\frac{9}{0}$.
22. $\frac{3a}{2} + \frac{4a}{3}$. L.C.D. = 6.
 $\frac{9a}{6} + \frac{8a}{6} = \frac{17a}{6}$.
23. $\frac{2a}{5} + \frac{3a}{2} + \frac{11a}{10}$. L.C.D. = 10.
 $\frac{4a}{10} + \frac{15a}{10} + \frac{11a}{10} = 3a$.
24. $\frac{7a}{10} - \frac{6a}{15} + \frac{3a}{5}$. L.C.D. = 30.
 $\frac{21a}{30} - \frac{12a}{30} + \frac{18a}{30} = \frac{9a}{10}$.
25. $\frac{4x}{7} + \frac{2x}{21} - \frac{3x}{14}$. L.C.D. = 42.
 $\frac{24x}{42} + \frac{4x}{42} - \frac{9x}{42} = \frac{19x}{42}$.
26. $\frac{a+2}{8} - \frac{3a-2}{10}$. L.C.D. = 40.
 $\frac{5a+10}{40} - \frac{12a-8}{40} = \frac{5a+10-12a+8}{40} = \frac{18-7a}{40}$.

$$27. \frac{5x-3}{12} - \frac{2x-7}{14}. \text{ L.C.D.} = 84.$$

$$\frac{35x-21}{84} - \frac{12x-42}{84} = \frac{35x-21-12x+42}{84} = \frac{23x+21}{84}.$$

$$28. \frac{3a-5}{4} - \frac{2-a}{6} + \frac{7a}{8}. \text{ L.C.D.} = 24.$$

$$\frac{18a-30}{24} - \frac{8-4a}{24} + \frac{21a}{24} = \frac{18a-30-8+4a+21a}{24} = \frac{43a-38}{24}.$$

$$29. \frac{x-5a}{10} - \frac{3x+2a}{18} - \frac{a-7x}{20}. \text{ L.C.D.} = 180.$$

$$\begin{aligned} \frac{18x-90a}{180} - \frac{30x+20a}{180} - \frac{9a-63x}{180} \\ = \frac{18x-90a-30x-20a-9a+63x}{180} \\ = \frac{51x-119a}{180}. \end{aligned}$$

$$30. \frac{4m-1}{6} - \frac{a-m}{8} - \frac{m+3a-5}{30}. \text{ L.C.D.} = 120.$$

$$\begin{aligned} \frac{80m-20}{120} - \frac{15a-15m}{120} - \frac{4m+12a-20}{120} \\ = \frac{80m-20-15a+15m-4m-12a+20}{120} \\ = \frac{91m-27a}{120}. \end{aligned}$$

$$31. \frac{3}{x} + \frac{2}{3x^2}. \text{ L.C.D.} = 3x^2.$$

$$\frac{9x}{3x^2} + \frac{2}{3x^2} = \frac{9x+2}{3x^2}.$$

$$32. \frac{2}{3a} + \frac{5}{2a^2} - \frac{10a}{4a^3}. \text{ L.C.D.} = 12a^2.$$

$$\frac{8a}{12a^2} + \frac{30}{12a^2} - \frac{30}{12a^2} = \frac{2}{3a}.$$

$$33. \frac{2a}{5a^3} - \frac{3}{10a} - \frac{5}{6a^2}. \text{ L.C.D.} = 30a^2.$$

$$\frac{12}{30a^2} - \frac{9a}{30a^2} - \frac{25}{30a^2} = \frac{-9a-13}{30a^2}.$$

$$34. \frac{a}{x} + \frac{x}{a} - \frac{2}{ax}. \text{ L.C.D.} = ax.$$

$$\frac{a^2}{ax} + \frac{x^2}{ax} - \frac{2}{ax} = \frac{a^2+x^2-2}{ax}.$$

$$35. \frac{a}{c} - \frac{1}{ac} - \frac{3}{2a}. \text{ L.C.D.} = 2ac.$$

$$\frac{2a^2}{2ac} - \frac{2}{2ac} - \frac{3c}{2ac} = \frac{2a^2-3c-2}{2ac}.$$

$$36. \frac{7}{3a^2} - \frac{2}{ax} + \frac{3}{x^2}. \text{ L.C.D.} = 3a^2x^2.$$

$$\frac{7x^2}{3a^2x^2} - \frac{6ax}{3a^2x^2} + \frac{9a^2}{3a^2x^2} = \frac{7x^2 - 6ax + 9a^2}{3a^2x^2}.$$

$$37. \frac{2a}{3x} + \frac{11}{2ax^2} - \frac{5x}{a}. \text{ L.C.D.} = 6ax^2.$$

$$\frac{4a^2x}{6ax^2} + \frac{33}{6ax^2} - \frac{30x^3}{6ax^2} = \frac{4a^2x + 33 - 30x^3}{6ax^2}.$$

$$38. \frac{3a+1}{5a} - \frac{4a+3}{3a^2} - \frac{5-a}{6a}. \text{ L.C.D.} = 30a^2.$$

$$\begin{aligned} \frac{18a^2 + 6a}{30a^2} - \frac{40a + 30}{30a^2} - \frac{25a - 5a^2}{30a^2} \\ = \frac{18a^2 + 6a - 40a - 30 - 25a + 5a^2}{30a^2} \\ = \frac{23a^2 - 59a - 30}{30a^2}. \end{aligned}$$

$$39. \frac{4x^2-5}{3x^2} - \frac{2-3x}{2x} + \frac{3x-7}{5x^3}. \text{ L.C.D.} = 30x^3.$$

$$\begin{aligned} \frac{40x^3 - 50x}{30x^3} - \frac{30x^2 - 45x^3}{30x^3} + \frac{18x - 42}{30x^3} \\ = \frac{40x^3 - 50x - 30x^2 + 45x^3 + 18x - 42}{30x^3} \\ = \frac{85x^3 - 30x^2 - 32x - 42}{30x^3}. \end{aligned}$$

$$40. \frac{2x}{5x^2y} - \frac{3y-1}{10xy^2} - \frac{4xy+3}{15x^2y^2}. \text{ L.C.D.} = 30x^2y^2.$$

$$\begin{aligned} \frac{12xy}{30x^2y^2} - \frac{9xy-3x}{30x^2y^2} - \frac{8xy+6}{30x^2y^2} = \frac{12xy - 9xy + 3x - 8xy - 6}{30x^2y^2} \\ = \frac{3x - 6 - 5xy}{30x^2y^2}. \end{aligned}$$

$$42. \frac{12}{x^2-9} - \frac{2}{x^2-5x+6}.$$

$$\begin{aligned} \frac{12}{(x+3)(x-3)} - \frac{2}{(x-2)(x-3)}. \text{ L.C.D.} = (x+3)(x-3)(x-2). \\ \frac{12(x-2) - 2(x+3)}{(x+3)(x-3)(x-2)} = \frac{12x - 24 - 2x - 6}{(x+3)(x-3)(x-2)} \\ = \frac{10x - 30}{(x+3)(x-3)(x-2)} \\ = \frac{10}{(x+3)(x-2)}. \end{aligned}$$

$$43. \frac{7}{a^2 - 49} - \frac{2}{a^2 - 6a - 7}.$$

$$\frac{7}{(a+7)(a-7)} - \frac{2}{(a-7)(a+1)}. \text{ L.C.D.} = (a+7)(a-7)(a+1).$$

$$\frac{7(a+1) - 2(a+7)}{(a+7)(a-7)(a+1)} = \frac{7a+7-2a-14}{(a+7)(a-7)(a+1)} = \frac{5a-7}{(a+7)(a-7)(a+1)}.$$

$$44. \frac{5}{4x^2 - 1} - \frac{3x}{8x^3 - 1}.$$

$$\frac{5}{(2x+1)(2x-1)} - \frac{3x}{(2x-1)(4x^2+2x+1)}.$$

$$\text{L.C.D.} = (2x+1)(2x-1)(4x^2+2x+1).$$

$$\frac{5(4x^2+2x+1) - 3x(2x+1)}{(2x+1)(2x-1)(4x^2+2x+1)} = \frac{20x^2+10x+5-6x^2-3x}{(2x+1)(2x-1)(4x^2+2x+1)}$$

$$= \frac{14x^2+7x+5}{(2x+1)(2x-1)(4x^2+2x+1)}.$$

$$45. \frac{4c+3}{1-c^2} + \frac{2c}{c+c^2}.$$

$$\frac{4c+3}{(1+c)(1-c)} + \frac{2c}{c(1+c)}. \text{ L.C.D.} = (1+c)(1-c)c.$$

$$\frac{(4c+3)c + 2c(1-c)}{(1+c)(1-c)c} = \frac{4c^2+3c+2c-2c^2}{c(1+c)(1-c)}$$

$$= \frac{2c^2+5c}{c(1+c)(1-c)}$$

$$= \frac{2c+5}{(1+c)(1-c)}.$$

$$46. \frac{2y+7}{3+y} - \frac{2y^2-35}{y^2-11y-42}.$$

$$\frac{2y+7}{(3+y)} - \frac{2y^2-35}{(y-14)(y+3)}. \text{ L.C.D.} = (y+3)(y-14).$$

$$\frac{(2y+7)(y-14) - (2y^2-35)}{(y+3)(y-14)} = \frac{2y^2-21y-98-2y^2+35}{(y+3)(y-14)}$$

$$= \frac{-21y-63}{(y+3)(y-14)}$$

$$= \frac{-21(y+3)}{(y+3)(y-14)}$$

$$= \frac{-21}{y-14}.$$

$$47. \frac{5a+1}{a+3} - \frac{7}{2a} + \frac{2}{3}. \quad \text{L.C.D.} = 6a(a+3).$$

$$\frac{6a(5a+1) - 21(a+3) + 4a(a+3)}{6a(a+3)} = \frac{30a^2 + 6a - 21a - 63 + 4a^2 + 12a}{6a(a+3)} \\ = \frac{34a^2 - 3a - 63}{6a(a+3)}.$$

$$48. \frac{2}{x^2-7x} - \frac{3}{x} + \frac{3}{x-7}. \quad \text{L.C.D.} = x(x-7).$$

$$\frac{2 - 3(x-7) + 3x}{x(x-7)} = \frac{2 - 3x + 21 + 3x}{(x-7)x} = \frac{23}{x(x-7)}.$$

$$49. \frac{7a+b}{a^2-b^2} + \frac{2}{a-b} - \frac{3}{a+b}. \quad \text{L.C.D.} = a^2-b^2.$$

$$\frac{7a+b + 2(a+b) - 3(a-b)}{a^2-b^2} = \frac{7a+b + 2a+2b - 3a+3b}{a^2-b^2} \\ = \frac{6a+6b}{a^2-b^2} = \frac{6}{a-b}.$$

$$50. \frac{5c^2}{4c^2-1} - \frac{3c+2}{2c+1} + \frac{5c}{2c-1}. \quad \text{L.C.D.} = 4c^2-1.$$

$$\frac{5c^2 - (3c+2)(2c-1) + 5c(2c+1)}{4c^2-1} = \frac{5c^2 - 6c^2 - c + 2 + 10c^2 + 5c}{4c^2-1} \\ = \frac{9c^2 + 4c + 2}{4c^2-1}.$$

$$51. \frac{m^3+m}{m^3-8} - \frac{3m+7}{m^3+2m^2+4m} - \frac{m-3}{m-2}.$$

$$\frac{m(m^2+1)}{(m-2)(m^2+2m+4)} - \frac{3m+7}{m(m^2+2m+4)} - \frac{m-3}{m-2}.$$

$$\text{L.C.D.} = m(m-2)(m^2+2m+4).$$

$$\frac{m^2(m^2+1) - (3m+7)(m-2) - m(m-3)(m^2+2m+4)}{m(m-2)(m^2+2m+4)} \\ = \frac{m^4 + m^2 - 3m^2 - m + 14 - m^4 + m^3 + 2m^2 + 12m}{m(m-2)(m^2+2m+4)} \\ = \frac{m^3 + 11m + 14}{m(m^3-8)}.$$

$$52. \frac{a^2 + 2ab + b^2}{a^2 - b^2} + \frac{ab}{a^2 + ab} - \frac{2b}{ab - b^2}.$$

$$\frac{\cancel{(a+b)}(a+b)}{\cancel{(a+b)}(a-b)} + \frac{\cancel{ab}}{(a+b)\cancel{a}} - \frac{2\cancel{b}}{\cancel{b}(a-b)} \cdot \text{L.C.D.} = (a+b)(a-b).$$

$$\frac{(a+b)^2 + b(a-b) - 2(a+b)}{(a+b)(a-b)} = \frac{a^2 + 2ab + b^2 + ab - b^2 - 2a - 2b}{(a+b)(a-b)}$$

$$= \frac{a^2 + 3ab - 2a - 2b}{(a+b)(a-b)}.$$

$$53. \frac{x^2 + 3}{x^2 - x - 2} - \frac{x}{x - 2} + \frac{x}{x^2 + x}.$$

$$\frac{x^2 + 3}{(x-2)(x+1)} - \frac{x}{x-2} + \frac{1}{x+1} \cdot \text{L.C.D.} = (x-2)(x+1).$$

$$\frac{x^2 + 3 - x(x+1) + x-2}{(x-2)(x+1)} = \frac{x^2 + 3 - x^2 - x + x - 2}{(x-2)(x+1)} = \frac{1}{(x-2)(x+1)}.$$

$$54. \frac{a}{a^2 - ab} + \frac{b+a}{a^2 - 2ab + b^2} + \frac{2b}{-ab + a^2}.$$

$$\frac{1}{a-b} + \frac{a+b}{(a-b)^2} + \frac{2b}{a(a-b)} \cdot \text{L.C.D.} = a(a-b)^2.$$

$$\frac{a(a-b) + a(a+b) + 2b(a-b)}{a(a-b)^2} = \frac{a^2 - ab + a^2 + ab + 2ab - 2b^2}{a(a-b)^2}$$

$$= \frac{2a^2 + 2ab - 2b^2}{a(a-b)^2}.$$

$$55. \frac{cx^2}{c^2 - x^2} - \frac{2cx}{c^2 - cx} - \frac{cx^2(c-2)}{c^3 - cx^2}.$$

$$\frac{cx^2}{(c+x)(c-x)} - \frac{2\cancel{cx}}{\cancel{c}(c-x)} - \frac{\cancel{cx^2}(c-2)}{\cancel{c}(c+x)(c-x)} \cdot \text{L.C.D.} = (c+x)(c-x).$$

$$\frac{cx^2 - 2x(c+x) - x^2(c-2)}{(c+x)(c-x)} = \frac{cx^2 - 2cx - 2x^2 - cx^2 + 2x^2}{(c+x)(c-x)}$$

$$= \frac{-2cx}{c^2 - x^2}.$$

$$56. \frac{a+c}{a-c} - \frac{a^2 - ac + c^2}{a^2 + ac + c^2} - \frac{4a^2c}{a^3 - c^3} \cdot \text{L.C.D.} = a^3 - c^3.$$

$$\frac{(a+c)(a^2 + ac + c^2) - (a-c)(a^2 - ac + c^2) - 4a^2c}{a^3 - c^3}$$

$$= \frac{a^3 + 2a^2c + 2ac^2 + c^3 - a^3 + 2a^2c - 2ac^2 + c^3 - 4a^2c}{a^3 - c^3}$$

$$= \frac{2c^3}{a^3 - c^3}.$$

$$\begin{aligned}
 57. \quad & \frac{x^2 + 3x + 9}{x^2 - 3x + 9} - \frac{x - 3}{x + 3} - \frac{54}{x^3 + 27}. \quad \text{L.C.D.} = x^3 + 27. \\
 & \frac{(x^2 + 3x + 9)(x + 3) - (x - 3)(x^2 - 3x + 9) - 54}{x^3 + 27} \\
 & = \frac{x^3 + 6x^2 + 18x + 27 - x^3 + 6x^2 - 18x + 27 - 54}{x^3 + 27} \\
 & = \frac{12x^2}{x^3 + 27}.
 \end{aligned}$$

$$\begin{aligned}
 58. \quad & \frac{x - 5}{2x + 5} + \frac{-5x + x^2}{2x^2 + 15x + 25} - \frac{2x - 5}{x + 5}. \\
 & \frac{x - 5}{2x + 5} + \frac{x^2 - 5x}{(2x + 5)(x + 5)} - \frac{2x - 5}{x + 5}. \quad \text{L.C.D.} = (2x + 5)(x + 5). \\
 & \frac{(x - 5)(x + 5) + x^2 - 5x - (2x - 5)(2x + 5)}{(2x + 5)(x + 5)} \\
 & = \frac{x^2 - 25 + x^2 - 5x - 4x^2 + 25}{(2x + 5)(x + 5)} = \frac{-2x^2 - 5x}{(2x + 5)(x + 5)}. \\
 & = \frac{-x}{x + 5}.
 \end{aligned}$$

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$$1. \quad \frac{a}{-h} = \frac{-a}{h} = -\frac{-a}{-h} = -\frac{a}{h}.$$

$$2. \quad \frac{-c}{2d} = \frac{c}{-2d} = -\frac{c}{2d} = -\frac{-c}{-2d}.$$

$$3. \quad -\frac{n}{d} = -\frac{-n}{-d} = \frac{-n}{d} = \frac{n}{-d}.$$

$$4. \quad -\frac{2}{a-c} = -\frac{-2}{c-a} = \frac{-2}{a-c} = \frac{2}{c-a}.$$

$$5. \quad \frac{-3}{x-y} = \frac{3}{y-x} = -\frac{3}{x-y} = -\frac{-3}{y-x}.$$

$$6. \quad \frac{x-2}{2x-3} = \frac{2-x}{3-2x} = -\frac{2-x}{2x-3} = -\frac{x-2}{3-2x}.$$

$$7. \quad -\frac{2a-b}{b^2-a^2} = -\frac{b-2a}{a^2-b^2} = \frac{b-2a}{b^2-a^2} = \frac{2a-b}{a^2-b^2}.$$

$$8. \quad \frac{c-3d}{d^2-c^2} = \frac{3d-c}{c^2-d^2} = -\frac{3d-c}{d^2-c^2} = -\frac{c-3d}{c^2-d^2}.$$

$$9. \quad \frac{x-5}{5x-6-x^2} = \frac{5-x}{x^2-5x+6} = -\frac{5-x}{5x-6-x^2} = -\frac{x-5}{x^2-5x+6}.$$

$$10. \quad -\frac{2-x}{1-x^3} = -\frac{x-2}{x^3-1} = \frac{x-2}{1-x^3} = \frac{2-x}{x^3-1}.$$

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$$1. \frac{7}{x-5} + \frac{3}{5-x} = \frac{7}{x-5} - \frac{3}{x-5} = \frac{4}{x-5}.$$

$$2. \frac{a}{a-b} - \frac{b}{b-a} = \frac{a}{a-b} + \frac{b}{a-b} = \frac{a+b}{a-b}.$$

$$3. \frac{x^2-2a}{x-a} + \frac{a+x^2}{a-x} = \frac{x^2-2a}{x-a} - \frac{a+x^2}{x-a} = \frac{-3a}{x-a} = \frac{3a}{a-x}.$$

$$4. \frac{x-2}{3-x} - \frac{2+x}{x-3} + \frac{3x}{x-3} = \frac{2-x}{x-3} - \frac{2+x}{x-3} + \frac{3x}{x-3} = \frac{x}{x-3}.$$

$$5. \frac{a}{5-a} + \frac{a^2+5a}{a^2-25} = \frac{-a(a+5)}{a^2-25} + \frac{a^2+5a}{a^2-25} = 0.$$

$$6. \frac{2}{c^2-9} - \frac{9}{3-c} = \frac{2}{c^2-9} + \frac{9}{c-3} = \frac{2}{c^2-9} + \frac{9(c+3)}{c^2-9} = \frac{9c+29}{c^2-9}.$$

$$7. \frac{3}{7-x} - \frac{4}{x^2-49} = \frac{3}{7-x} + \frac{4}{49-x^2} = \frac{3(7+x)+4}{49-x^2} = \frac{25+3x}{49-x^2}.$$

$$\begin{aligned} 8. \frac{2a+5}{5a+1} + \frac{2a-5}{1-5a} - \frac{46a+1}{25a^2-1} \\ = \frac{(5a-1)(2a+5) - (2a-5)(5a+1) - (46a+1)}{25a^2-1} \\ = \frac{10a^2+23a-5-10a^2+23a+5-46a-1}{25a^2-1} \\ = \frac{-1}{25a^2-1}. \end{aligned}$$

$$\begin{aligned} 9. \frac{a-b}{3a+2b} - \frac{10ab}{4b^2-9a^2} - \frac{a+b}{3a-2b} &= \frac{a-b}{3a+2b} + \frac{10ab}{9a^2-4b^2} - \frac{a+b}{3a-2b} \\ &= \frac{(a-b)(3a-2b) + 10ab - (a+b)(3a+2b)}{9a^2-4b^2} \\ &= \frac{3a^2-5ab+2b^2+10ab-3a^2-5ab-2b^2}{9a^2-4b^2} \\ &= 0. \end{aligned}$$

$$\begin{aligned} 10. \frac{2x+1}{2x-1} - \frac{-6x+1}{1-4x^2} - \frac{2x-1}{2x+1} &= \frac{2x+1}{2x-1} + \frac{1-6x}{4x^2-1} - \frac{2x-1}{2x+1} \\ &= \frac{(2x+1)^2+1-6x-(2x-1)^2}{4x^2-1} \\ &= \frac{4x^2+4x+1+1-6x-4x^2+4x-1}{4x^2-1} \\ &= \frac{1}{2x-1}. \end{aligned}$$

$$\begin{aligned}
 11. \quad & \frac{-11a + 46 - 6a^2}{a^2 + 5a - 14} + \frac{2a - 5}{a + 7} + \frac{7 - 4a}{2 - a} \\
 &= \frac{-11a + 46 - 6a^2}{a^2 + 5a - 14} + \frac{(2a - 5)(a - 2)}{(a + 7)(a - 2)} + \frac{(4a - 7)(a + 7)}{(a - 2)(a + 7)} \\
 &= \frac{-11a + 46 - 6a^2 + 2a^2 - 9a + 10 + 4a^2 + 21a - 49}{a^2 + 5a - 14} \\
 &= \frac{a + 7}{a^2 + 5a - 14} \\
 &= \frac{1}{a - 2}.
 \end{aligned}$$

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1. $a + \frac{2}{x} = \frac{ax + 2}{x}$.
2. $b - \frac{c}{d} = \frac{bd - c}{d}$.
3. $2c + \frac{c}{a} = \frac{2ac + c}{a}$.
4. $5x - \frac{x}{d} = \frac{5dx - x}{d}$.
5. $\frac{m}{n} - 2 = \frac{m - 2n}{n}$.
6. $\frac{c}{5d} + 3 = \frac{c + 15d}{5d}$.
7. $s + \frac{s}{2} = \frac{2s + s}{2} = \frac{3s}{2}$.
8. $x^2 - \frac{3x^2}{5} = \frac{5x^2 - 3x^2}{5} = \frac{2x^2}{5}$.
9. $s^2 - \left(\frac{s}{2}\right)^2 = s^2 - \frac{s^2}{4} = \frac{4s^2 - s^2}{4} = \frac{3s^2}{4}$.
10. $x^2 - \left(\frac{2x}{3}\right)^2 = x^2 - \frac{4x^2}{9} = \frac{9x^2 - 4x^2}{9} = \frac{5x^2}{9}$.
11. $2a^2 - \frac{2a^2}{3} = \frac{6a^2 - 2a^2}{3} = \frac{4a^2}{3}$.
12. $a + 2 + \frac{a - 2}{2} = \frac{2a + 4 + a - 2}{2} = \frac{3a + 2}{2}$.
13. $c - d + \frac{c^2 - d^2}{c + d} = \frac{c^2 - d^2 + c^2 - d^2}{c + d} = \frac{2c^2 - 2d^2}{c + d} = \frac{2(c - d)}{1}$.
14. $x - 2y - \frac{x^2 + y^2}{x + 2y} = \frac{x^2 - 4y^2 - x^2 - y^2}{x + 2y} = \frac{-5y^2}{x + 2y}$.
15. $a^2 + b^2 - \frac{(a + b)^2}{2} = \frac{2a^2 + 2b^2 - a^2 - 2ab - b^2}{2} = \frac{a^2 - 2ab + b^2}{2}$.
16. $5a - 7 - \frac{25a^2}{5a + 7} = \frac{25a^2 - 49 - 25a^2}{5a + 7} = \frac{-49}{5a + 7}$.
17. $m^2 + mn + n^2 - \frac{2m^3 - n^3}{m - n} = \frac{m^3 - n^3 - 2m^3 + n^3}{m - n} = \frac{m^3}{n - m}$.
18. $x^2 - xy - \frac{x^3 - y^3}{x + y} + y^2 = \frac{x^3 + y^3 - x^3 + y^3}{x + y} = \frac{2y^3}{x + y}$.
19. $5 - \frac{125 - 2x^3}{25 + 5x + x^2} - x = \frac{125 - x^3 - 125 + 2x^3}{25 + 5x + x^2} = \frac{x^3}{25 + 5x + x^2}$.

$$\begin{aligned}
 20. \quad \frac{x+2}{x-2} - 3 + \frac{x-3}{2x+3} &= \frac{(x+2)(2x+3) - 3(x-2)(2x+3) + (x-3)(x-2)}{(2x+3)(x-2)} \\
 &= \frac{2x^2 + 7x + 6 - 6x^2 + 3x + 18 + x^2 - 5x + 6}{(2x+3)(x-2)} \\
 &= \frac{-3x^2 + 5x + 30}{(2x+3)(x-2)}.
 \end{aligned}$$

$$21. \quad x^3 + 4x - \frac{x^4 - 18}{x-2} + 8 + 2x^2 = \frac{x^4 - 16 - x^4 + 18}{x-2} = \frac{2}{x-2}.$$

$$\begin{aligned}
 22. \quad b + 2 - \frac{4b}{2b-1} - \left(2b - 1 - \frac{b^2 + 9b}{b+5}\right) \\
 &= b + 2 - \frac{4b}{2b-1} - 2b + 1 + \frac{b^2 + 9b}{b+5} \\
 &= 3 - b - \frac{4b}{2b-1} + \frac{b^2 + 9b}{b+5} \\
 &= \frac{(3-b)(2b-1)(b+5) - 4b(b+5) + (b^2 + 9b)(2b-1)}{(2b-1)(b+5)} \\
 &= \frac{-2b^3 - 3b^2 + 32b - 15 - 4b^2 - 70b + 2b^3 + 17b^2 - 9b}{(2b-1)(b+5)} \\
 &= \frac{-47b - 15}{(2b-1)(b+5)}.
 \end{aligned}$$

$$23. \quad \left(2 + \frac{6a}{b}\right) - \left(3 - \frac{4a}{3b}\right) = 2 + \frac{6a}{b} - 3 + \frac{4a}{3b} = \frac{22a}{3b} - 1 = \frac{22a - 3b}{3b}.$$

$$\begin{aligned}
 24. \quad \left(3 - \frac{7}{2ac}\right) - \left(2 - \frac{5}{3c^2}\right) &= 3 - \frac{7}{2ac} - 2 + \frac{5}{3c^2} = 1 - \frac{7}{2ac} + \frac{5}{3c^2} \\
 &= \frac{6ac^2 - 21c + 10a}{6ac^2}.
 \end{aligned}$$

$$\begin{aligned}
 25. \quad \left(x + \frac{4y^2}{x-2y}\right) - \left(\frac{x^2}{x+2y} + 2y\right) &= x + \frac{4y^2}{x-2y} - \frac{x^2}{x+2y} - 2y \\
 &= \frac{(x-2y)^2(x+2y) + 4y^2(x+2y) - x^2(x-2y)}{(x+2y)(x-2y)} \\
 &= \frac{x^3 - 2x^2y - 4xy^2 + 8y^3 + 4xy^2 + 8y^3 - x^3 + 2x^2y}{(x+2y)(x-2y)} \\
 &= \frac{16y^3}{(x+2y)(x-2y)}.
 \end{aligned}$$

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$$1. \quad \frac{\frac{2}{6} \cdot \frac{10}{15}}{\frac{5}{2}} = \frac{2}{5}.$$

$$2. \quad \frac{\frac{2}{4} \cdot \frac{4}{12}}{\frac{21}{7} \cdot \frac{10}{5}} = \frac{8}{35}.$$

$$3. \quad \frac{\frac{2}{4} \cdot \frac{5}{15}}{\frac{8}{3} \cdot \frac{22}{11}} = \frac{10}{11}.$$

$$4. 1\frac{1}{6} \cdot \frac{8}{21} = \frac{\cancel{7}}{6} \cdot \frac{\cancel{8}}{\cancel{21}} = \frac{4}{9}.$$

$$7. 2\frac{1}{4} \cdot \frac{1}{21} \cdot 1\frac{7}{8} = \frac{9}{4} \cdot \frac{1}{\cancel{21}} \cdot \frac{\cancel{15}}{8} = \frac{45}{224}.$$

$$5. -\frac{\cancel{14}}{\cancel{11}} \cdot \frac{\cancel{11}}{\cancel{42}} = -1.$$

$$8. -1\frac{1}{11} \cdot 1\frac{1}{2} = -\frac{\cancel{12}}{11} \cdot \frac{32}{\cancel{21}} = -\frac{128}{77}.$$

$$6. 2\frac{4}{7} \cdot -1\frac{7}{8} = \frac{\cancel{18}}{7} \cdot -\frac{\cancel{35}}{\cancel{18}} = -5.$$

$$9. \frac{\cancel{14}}{\cancel{24}} \cdot \frac{\cancel{16}}{\cancel{76}} \cdot \frac{32}{5} = \frac{64}{15}.$$

$$10. \frac{22}{7} \cdot 6^2 \cdot \frac{1}{144} = \frac{\cancel{22}}{7} \cdot \cancel{36} \cdot \frac{1}{\cancel{144}} = \frac{11}{14}.$$

$$11. \frac{4}{3} \cdot \frac{22}{7} \cdot (2\frac{1}{2})^3 = \frac{\cancel{4}}{3} \cdot \frac{\cancel{22}}{7} \cdot \frac{125}{\cancel{8}} = \frac{1375}{21}.$$

$$12. \frac{\cancel{5}a^2\cancel{b}}{\cancel{3}c^3} \cdot \frac{\cancel{2}}{\cancel{10}a\cancel{b^3}} = \frac{a}{b^2c^3}.$$

$$14. \frac{\cancel{5}x^2}{\cancel{15}a\cancel{x^3}} \cdot \frac{\cancel{3}}{\cancel{12}a^5} \cdot \frac{2}{\cancel{16}x^2} = 2ax^2.$$

$$13. \frac{\cancel{10}xy^2}{\cancel{18}z^2} \cdot \frac{\cancel{24}z^8}{\cancel{15}x^4} = \frac{8y^2z}{9x^3}.$$

$$15. \frac{\cancel{8}cd^2}{\cancel{3}x} \cdot \frac{\cancel{18}}{\cancel{40}c^3d^5} \cdot \frac{x}{\cancel{5}c^2d^8} = \frac{6x}{c^2d^2}.$$

$$16. \left(\frac{5x}{2ac}\right)^2 \cdot \frac{6a^2}{10x^3} \cdot \frac{5c^5}{3a^4} = \frac{\cancel{25}x^2}{\cancel{4}a^2\cancel{c^2}} \cdot \frac{\cancel{6}a^2}{\cancel{10}x^3} \cdot \frac{\cancel{5}c^5}{\cancel{3}a^4} = \frac{25c^3}{4a^4x}.$$

$$17. 21x^2y \cdot \frac{9ay^2}{14x^3} \left(\frac{2x}{3y^2}\right)^3 = \frac{\cancel{21}x^2y}{\cancel{14}x^3} \cdot \frac{\cancel{9}ay^2}{\cancel{14}x^3} \cdot \frac{\cancel{8}x^3}{\cancel{27}y^6} = \frac{4a}{x^3y^3}.$$

$$18. \left(\frac{a}{5c^2}\right)^2 \cdot \left(\frac{-c}{2d}\right)^3 \cdot \frac{15d}{ac} = \frac{\cancel{a^2}}{\cancel{25}c^4} \cdot -\frac{\cancel{c^3}}{\cancel{8}d^3} \cdot \frac{\cancel{15}d}{\cancel{ac}} = -\frac{3a}{40c^2d^2}.$$

$$19. \frac{10m}{n^7} \cdot \left(\frac{4n^2}{15m}\right)^2 \cdot \left(\frac{5m^3}{6m^2n}\right)^2 = \frac{10m}{\cancel{n^7}} \cdot \frac{\cancel{16}n^4}{\cancel{225}m^2} \cdot \frac{\cancel{25}m^6}{\cancel{36}m^4n^2} = \frac{40m}{81n^5}.$$

$$20. \frac{(2x)^2}{3y} \cdot \frac{(3y)^2}{2x^2} = \frac{\cancel{4}x^2}{\cancel{3}y} \cdot \frac{\cancel{9}y^2}{\cancel{2}x^2} = 6y.$$

$$21. \frac{(6a)^2}{(10b^2)^3} \cdot \frac{125b^4}{2(a^3)^2} = \frac{\cancel{36}a^2}{\cancel{1000}b^6} \cdot \frac{\cancel{125}b^4}{\cancel{2}a^6} = \frac{9}{4a^4b^2}.$$

$$22. \left(\frac{8m^2}{10mn^2}\right)^2 \cdot \left(\frac{15n^3}{4m}\right)^3 = \frac{\cancel{64}m^4}{\cancel{100}m^2n^4} \cdot \frac{\cancel{5}^3 \cdot \cancel{3}^3 n^9}{\cancel{64}m^3} = \frac{135n^5}{4m}.$$

$$24. \frac{2x^2+6}{5y^3} \cdot \frac{10y^4}{3x^2+9} = \frac{\cancel{2}(x^2+3)}{\cancel{5}y^3} \cdot \frac{\cancel{10}y^4}{\cancel{3}(x^2+3)} = \frac{4y}{3}.$$

$$25. \frac{c^2-4}{x-y} \cdot \frac{3x-3y}{c^2+6c+8} = \frac{\cancel{(c+2)}(c-2)}{\cancel{x-y}} \cdot \frac{\cancel{3}(x-y)}{\cancel{(c+2)}(c+4)} = \frac{3c-6}{c+4}.$$

$$26. \frac{x^2-2x+4}{x^2-9} \cdot \frac{x^2-x-6}{x^3+8} = \frac{\cancel{x^2-2x+4}}{(x+3)(\cancel{x-3})} \cdot \frac{\cancel{(x-3)}(x+2)}{\cancel{(x+2)}(x^2-2x+4)} \\ = \frac{1}{x+3}.$$

$$27. \frac{x^2-3x+9}{2x-6} \cdot \frac{x^2-9}{x^3+27} = \frac{\cancel{x^2-3x+9}}{2(\cancel{x-3})} \cdot \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(\cancel{x^2-3x+9})} = \frac{1}{2}.$$

$$28. \frac{x^2-7x+10}{x^3-125} \cdot \frac{2x^3+10x^2+50x}{x^2-4} \\ = \frac{\cancel{(x-2)}(\cancel{x-5})}{\cancel{(x-5)}(\cancel{x^2+5x+25})} \cdot \frac{2x(\cancel{x^2+5x+25})}{(x+2)(\cancel{x-2})} \\ = \frac{2x}{x+2}.$$

$$29. \frac{x^4-16}{(x^2-4)^2} \cdot \frac{x^2-4}{4+x^2} = \frac{\cancel{(x^2+4)}(\cancel{x+2})(\cancel{x-2})}{\cancel{(x+2)}^2(\cancel{x-2})^2} \cdot \frac{\cancel{(x+2)}(\cancel{x-2})}{\cancel{x^2+4}} = 1.$$

$$30. \frac{c^2-9}{a^2-4x^2} \cdot \frac{a^2+4ax+4x^2}{ac+2cx-3a-6x} = \frac{(c+3)(\cancel{c-3})}{\cancel{(a+2x)}(a-2x)} \cdot \frac{\cancel{(a+2x)}(a+2x)}{\cancel{(c-3)}(a+2x)} \\ = \frac{c+3}{a-2x}.$$

$$31. \frac{2a^2-5ac-3c^2}{c^2-9a^2} \cdot \frac{3c+9a}{10a^2+5ac} = \frac{(a-3c)(\cancel{2a+c})}{\cancel{(c+3a)}(c-3a)} \cdot \frac{\cancel{3(c+3a)}}{\cancel{5a}(2a+c)} \\ = \frac{3(a-3c)}{5a(c-3a)}.$$

$$\begin{aligned}
 32. \quad & \frac{2x^2 + 10x + 50}{3x^3 - 12x} \cdot \frac{x^2 - 3x - 10}{4x^3 - 500} \cdot \frac{2 - x}{2} \\
 &= \frac{\cancel{2}(x^2 + 5x + 25)}{3x(x+2)\cancel{(x-2)}} \cdot \frac{\cancel{(x-5)}(x+2)}{4\cancel{(x-5)}(x^2 + 5x + 25)} \cdot \frac{\cancel{2-x}}{2} \\
 &= \frac{-1}{12x}.
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & \frac{(x^2 - 4)(x + 3)}{2x^2 + 12x + 18} \cdot \frac{x^2 - x - 6}{(x^2 - 4)^2} \cdot \frac{(x - 2)}{x + 3} \\
 &= \frac{\cancel{(x+2)}\cancel{(x-2)}(x+3)}{2(x+3)(x+3)} \cdot \frac{(x-3)\cancel{(x+2)}}{\cancel{(x+2)}^2\cancel{(x-2)}^2} \cdot \frac{\cancel{x-2}}{\cancel{x+3}} \\
 &= \frac{x-3}{2(x+3)^2}.
 \end{aligned}$$

$$\begin{aligned}
 34. \quad & \frac{4e^2 - 12ex + 9x^2}{(e^2 + ex + x^2)(3x - 2e)} \cdot (e^3 - x^3) \cdot \frac{x - e}{2e^2 - 5ex + 3x^2} \\
 &= \frac{\cancel{(2e-3x)}\cancel{(2e-3x)}}{\cancel{(e^2+ex+x^2)}(3x-2e)} \cdot \frac{\cancel{(e-x)}(e^2+ex+x^2)}{\cancel{(2e-3x)}(e-x)} \cdot \frac{x-e}{\cancel{(e-x)}} \\
 &= e - x.
 \end{aligned}$$

$$35. \quad \left(4 + \frac{3}{a^2 - 1}\right) \left(\frac{3a}{2a - 1} - 1\right) = \frac{\cancel{4a^2-1}}{\cancel{a^2-1}} \cdot \frac{\cancel{a+1}}{\cancel{2a-1}} = \frac{2a+1}{a-1}.$$

$$\begin{aligned}
 36. \quad & \left(\frac{x^2}{y^2} - 4\right) \left(\frac{2x + 4y}{x^3 - 8y^3}\right) \left(\frac{x^2 + 2xy + 4y^2}{x^2 + 4xy + 4y^2}\right) \\
 &= \frac{\cancel{(x+2y)}\cancel{(x-2y)}}{y^2} \cdot \frac{\cancel{2(x+2y)}}{\cancel{(x^2+2xy+4y^2)}\cancel{(x-2y)}} \cdot \frac{\cancel{x^2+2xy+4y^2}}{\cancel{(x+2y)}^2} \\
 &= \frac{2}{y^2}.
 \end{aligned}$$

$$\begin{aligned}
 37. \quad & \left(3x - 5 - \frac{168}{9x + 15}\right) \left(3 - \frac{4}{3 + x}\right) = \left(3x - 5 - \frac{56}{3x + 5}\right) \left(3 - \frac{4}{3 + x}\right) \\
 &= \frac{9x^2 - 81}{3x + 5} \cdot \frac{3x + 5}{3 + x} \\
 &= \frac{\cancel{9(x+3)}\cancel{(x-3)}}{\cancel{3x+5}} \cdot \frac{\cancel{3x+5}}{\cancel{x+3}} \\
 &= 9x - 27.
 \end{aligned}$$

$$\begin{aligned}
 38. \quad & \left(3x - 4 + \frac{21}{4 + x}\right) \left(\frac{1}{x + 1}\right)^2 \left(x + \frac{x - 3}{3x + 5} + 3\right) \\
 &= \frac{3x^2 + 8x + 5}{4 + x} \cdot \frac{1}{(x + 1)^2} \cdot \frac{3x^2 + 15x + 12}{3x + 5} \\
 &= \frac{\cancel{(3x+5)}\cancel{(x+1)}}{\cancel{x+4}} \cdot \frac{1}{\cancel{(x+1)}^2} \cdot \frac{\cancel{3(x+1)}\cancel{(x+4)}}{\cancel{3x+5}} \\
 &= 3.
 \end{aligned}$$

$$\begin{aligned}
 39. \quad & \left(2x - \frac{1}{2x}\right) \left(6x + \frac{3}{2x+1}\right) \left(\frac{1}{8}\right) \frac{1}{8x^3-1} \\
 &= \frac{(2x+1)(2x-1)}{2x} \cdot \frac{3(4x^2+2x+1)}{-2x+1} \cdot \frac{1}{8} \cdot \frac{1}{(2x-1)(4x^2+2x+1)} \\
 &= \frac{3}{16x}.
 \end{aligned}$$

$$\begin{aligned}
 40. \quad & \left(\frac{4}{x^2} - \frac{5}{x} + 1\right) \left(\frac{5x^4 + 5x^3}{x^2 - 11x + 28}\right) \left(1 - \frac{7x-1}{x^2-1}\right) \\
 &= \frac{4-5x+x^2}{x^2} \cdot \frac{5x^3(x+1)}{(x-7)(x-4)} \cdot \frac{x^2-7x}{x^2-1} \\
 &= \frac{(x-1)(x-4)}{x^2} \cdot \frac{5x^3(x+1)}{(x-7)(x-4)} \cdot \frac{x(x-7)}{(x+1)(x-1)} \\
 &= 5x^2.
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & \left(2b + \frac{a^2}{2b} - 2a\right) \left(a + \frac{12b^2}{a-2b} + 4b\right) \cdot \frac{2b^2}{a^3-8b^3} \\
 &= \frac{4b^2-4ab+a^2}{2b} \cdot \frac{a^2+2ab+4b^2}{a-2b} \cdot \frac{2b^2}{a^3-8b^3} \\
 &= \frac{(a-2b)(a-2b)}{\cancel{2b}} \cdot \frac{a^2+2ab+4b^2}{\cancel{a-2b}} \cdot \frac{\cancel{b}}{(a-2b)(a^2+2ab+4b^2)} = b.
 \end{aligned}$$

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$$1. \quad \frac{2}{3} \div \frac{5}{3} = \frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{5} = \frac{2}{5}.$$

$$2. \quad \frac{4}{5} \div \frac{9}{5} = \frac{4}{\cancel{5}} \cdot \frac{\cancel{5}}{9} = \frac{4}{9}.$$

$$3. \quad \frac{10}{3} \div \frac{5}{2} = \frac{\cancel{10}}{3} \cdot \frac{2}{\cancel{5}} = \frac{4}{3}.$$

$$4. \quad \frac{12}{15} \div \left(-\frac{4}{5}\right) = \frac{\cancel{12}}{\cancel{15}} \cdot -\frac{\cancel{5}}{4} = -1.$$

$$5. \quad \frac{15}{21} \div -\frac{3}{7} = \frac{\cancel{15}}{\cancel{21}} \cdot -\frac{\cancel{7}}{3} = -\frac{5}{3}.$$

$$6. \quad \frac{18}{20} \div -\frac{45}{10} = \frac{\cancel{18}}{\cancel{20}} \cdot -\frac{\cancel{10}}{\cancel{45}} = -\frac{1}{5}.$$

$$7. \quad \frac{a}{2x} \div \frac{c}{x} = \frac{a}{\cancel{2x}} \cdot \frac{\cancel{x}}{c} = \frac{a}{2c}.$$

$$8. \quad \frac{2a}{b} \div \frac{-3c}{2b} = \frac{2a}{\cancel{b}} \cdot \frac{\cancel{2b}}{-3c} = -\frac{4a}{3c}.$$

$$9. \quad \frac{x^2}{y} \div \frac{2x}{y^2} = \frac{\cancel{x^2}}{\cancel{y}} \cdot \frac{\cancel{y^2}}{\cancel{2x}} = \frac{xy}{2}.$$

$$10. \quad \frac{2m}{n} \div \frac{-m}{3n} = \frac{\cancel{2m}}{\cancel{n}} \cdot \frac{\cancel{3n}}{-\cancel{m}} = -6.$$

$$11. \quad \frac{10x^2}{14y^2} \div \frac{15x^3}{21y^4} = \frac{\cancel{10}x^2}{\cancel{14}y^2} \cdot \frac{\cancel{21}y^4}{\cancel{15}x^3} = \frac{y^2}{x}.$$

$$12. \frac{12 ab^2}{5c} \div \frac{18 a^2b}{15c^3} = \frac{\cancel{12}^2 \cancel{ab^2}^b}{\cancel{5}^1 \cancel{c}} \cdot \frac{\cancel{18}^3 \cancel{c^3}^c}{\cancel{15}^3 \cancel{a^2b}^a} = \frac{2bc^2}{a}.$$

$$13. \frac{4 a^2x}{51c} \div \frac{2 ax}{17c^3} = \frac{\cancel{4}^2 \cancel{a^2x}^a}{\cancel{51}^3 \cancel{c}} \cdot \frac{\cancel{17}^1 \cancel{c^3}^c}{\cancel{2}^1 \cancel{ax}^a} = \frac{2ac^2}{3}.$$

$$14. \left(\frac{3ab}{5c} \right)^2 \div \frac{18 a^3b}{100c^3} = \frac{\cancel{9}^3 \cancel{a^2b^2}^b}{\cancel{25}^5 \cancel{c^2}^c} \cdot \frac{\cancel{100}^4 \cancel{c^3}^c}{\cancel{18}^2 \cancel{a^3b}^a} = \frac{2bc}{a}.$$

$$15. \frac{a}{b} \div \frac{c}{b} \div \frac{a^2}{c^2} = \frac{\cancel{a}}{\cancel{b}} \cdot \frac{\cancel{b}}{\cancel{c}} \cdot \frac{\cancel{c^2}^c}{\cancel{a^2}^a} = \frac{c}{a}.$$

$$16. \frac{2x^6}{5a} \div \frac{4x^2}{15} \div \frac{(3x^2)^2}{12a^2} = \frac{\cancel{2}^1 \cancel{x^6}^4}{\cancel{5}^1 \cancel{a}} \cdot \frac{\cancel{15}^3}{\cancel{4}^2 \cancel{x^2}^2} \cdot \frac{\cancel{12}^3 \cancel{a^2}^2}{\cancel{9}^3 \cancel{x^4}^4} = 2a.$$

$$17. \frac{12 c^3d}{35e^4} \div \frac{8c^2}{7e^2d} \cdot \frac{10de}{3c^2d^3} = \frac{\cancel{12}^3 \cancel{c^3d}^c}{\cancel{35}^5 \cancel{e^4}^4} \cdot \frac{\cancel{7}^1 \cancel{e^2d}^d}{\cancel{8}^2 \cancel{c^2}^c} \cdot \frac{\cancel{10}^2 \cancel{de}^d}{\cancel{3}^1 \cancel{c^2d^3}^c} = \frac{1}{ec}.$$

$$18. \frac{18x^3}{(7ab^2)^2} \div 12x^2 \cdot \frac{(14ab^4)^2}{6x^2} = \frac{\cancel{18}^3 \cancel{x^3}^3}{\cancel{49}^7 \cancel{a^2b^4}^4} \cdot \frac{1}{\cancel{12}^3 \cancel{x^2}^2} \cdot \frac{\cancel{196}^4 \cancel{a^2b^8}^4}{\cancel{6}^2 \cancel{x^2}^2} = \frac{b^4}{x}.$$

$$19. \left(\frac{3}{5x^2} \right)^2 \div \frac{256}{(10x^2)^2} \div \frac{5x^3}{(8x^4)^2} = \frac{9}{\cancel{25}^5 \cancel{x^4}^4} \cdot \frac{\cancel{100}^4 \cancel{x^4}^4}{\cancel{256}^8} \cdot \frac{\cancel{64}^3 \cancel{x^8}^8}{\cancel{5}^1 \cancel{x^8}^8} = \frac{9x^5}{5}.$$

$$20. \frac{2a}{3b} \div \frac{(2a)^3}{12b^3} \div \frac{(7a^4)^2}{15b^5} \cdot \left(\frac{7}{5} \right)^2 = \frac{\cancel{2}^1 \cancel{a}}{\cancel{3}^1 \cancel{b}} \cdot \frac{\cancel{12}^3 \cancel{b^3}^3}{\cancel{8}^4 \cancel{a^3}^3} \cdot \frac{\cancel{15}^3 \cancel{b^5}^5}{\cancel{49}^7 \cancel{a^8}^8} \cdot \frac{\cancel{49}^7}{\cancel{25}^5} = \frac{3b^7}{5a^{10}}.$$

$$21. \frac{10-4a}{12x^2} \div \frac{5a-2a^2}{9x} = \frac{\cancel{2}^1 \cancel{(5-2a)}^1}{\cancel{12}^4 \cancel{x^2}^2} \cdot \frac{\cancel{9}^3 \cancel{x}}{a(5-2a)} = \frac{3}{2ax}.$$

$$22. \frac{(a-3)^2}{a^2-4a+3} \div \frac{a^2-9}{a^2-a} = \frac{\cancel{(a-3)}^2 \cancel{(a-3)}^2}{\cancel{(a-3)}^2 \cancel{(a-1)}^1} \cdot \frac{a(a-1)}{\cancel{(a-3)}^1 \cancel{(a+3)}^1} = \frac{a}{a+3}.$$

23. $\frac{x^2 + x - 30}{x^2 + 5x - 6} \div \frac{x^2 - 25}{x^2 - 6x + 5} = \frac{\cancel{(x+6)}(x-5)}{\cancel{(x+6)}(x-1)} \cdot \frac{(x-5)\cancel{(x-1)}}{(x+5)\cancel{(x-5)}} = \frac{x-5}{x+5}.$
24. $\frac{a^3 - 1}{a^3 - a} \div \frac{a^4 + a^3 + a^2}{a^2 + 3a + 2} = \frac{\cancel{(a-1)}(a^2 + a + 1)}{a\cancel{(a+1)}\cancel{(a-1)}} \cdot \frac{(a+2)\cancel{(a+1)}}{a^2\cancel{(a^2 + a + 1)}} = \frac{a+2}{a^3}.$
25. $(2m^2 - 4m + 8) \div \frac{m^4 + 8m}{3m} = \frac{2(m^2 - 2m + 4)}{1} \cdot \frac{3m}{m(m+2)(m^2 - 2m + 4)}$
 $= \frac{6}{m+2}.$
26. $\left(x + 1 - \frac{3}{4x}\right)\left(3 + \frac{3}{4x^2 - 1}\right) \div \frac{2x + 3}{2x + 1}$
 $= \frac{4x^2 + 4x - 3}{4x} \cdot \frac{12x^2}{4x^2 - 1} \div \frac{2x + 3}{2x + 1}$
 $= \frac{\cancel{(2x+3)}(2x-1)}{4x} \cdot \frac{3x}{\cancel{(2x+1)}(2x-1)} \cdot \frac{2x+1}{2x+3}$
 $= 3x.$
27. $\left(\frac{-c}{2d^2}\right)^2 \div \left(\frac{-d}{c^3}\right)^2 \cdot \left(\frac{-d^3}{c^2}\right)^3 \div \left(\frac{d}{-2c}\right)^2 = \frac{c^2}{\cancel{4d^4}} \cdot \frac{\cancel{c^6}}{\cancel{d^2}} \cdot \frac{\cancel{d^9}}{\cancel{c^6}} \cdot \frac{\cancel{4c^2}}{\cancel{d^2}} = -c^4d.$
28. $\left(\frac{x^2}{y} - \frac{y^2}{x}\right) \div \left(\frac{x^2 + xy + y^2}{2x^2 - 2xy}\right) \cdot \frac{1}{2(x-y)^2}$
 $= \frac{x^3 - y^3}{xy} \cdot \frac{2x^2 - 2xy}{x^2 + xy + y^2} \cdot \frac{1}{2(x-y)^2}$
 $= \frac{\cancel{(x-y)}(x^2 + xy + y^2)}{\cancel{xy}} \cdot \frac{\cancel{2x}(x-y)}{\cancel{x^2 + xy + y^2}} \cdot \frac{1}{\cancel{2}(x-y)^2} = \frac{1}{y}.$
29. $\frac{x^2 - 5xy + 6y^2}{9x^2 - 6xy + y^2} \cdot \left(3x + 5y + \frac{10y^2}{x - 2y}\right) \div \frac{x^2 - 9y^2}{3x^2 + 8xy - 3y^2}$
 $= \frac{\cancel{(x-2y)}(x-3y)}{\cancel{(3x-y)}(3x-y)} \cdot \frac{x\cancel{(3x-y)}}{x-2y} \cdot \frac{\cancel{(3x-y)}(x+3y)}{\cancel{(x+3y)}(x-3y)} = x.$
30. $\frac{(2m)^2}{m^6 + 64} \cdot \left(m^2 - 4 + \frac{16}{m^2}\right) \div \frac{1}{2m} \div \frac{4m}{4 - m^2}$
 $= \frac{\cancel{4m^2}}{(m^2 + 4)(\cancel{m^4 - 4m^2 + 16})} \cdot \frac{\cancel{m^4 - 4m^2 + 16}}{\cancel{m^2}} \cdot \frac{2m}{1} \cdot \frac{(2+m)(2-m)}{\cancel{4m}}$
 $= \frac{2(4 - m^2)}{4 + m^2}.$
31. $\frac{3x - 2}{x + 1} \cdot \left(3x + 4 + \frac{4}{3x}\right) \div \frac{9x^2 - 4}{9x^2 + 9x}$
 $= \frac{\cancel{3x-2}}{\cancel{x+1}} \cdot \frac{(3x+2)\cancel{(3x+2)}}{\cancel{3x}} \cdot \frac{\cancel{3x}(x+1)}{\cancel{(3x+2)}(3x-2)}$
 $= 9x + 6.$

$$\begin{aligned}
 32. \quad & \left(2x + 1 + \frac{1}{2x}\right) \div \frac{(8x^3 - 1)^2}{2x^2 + 2x} \cdot \left(4x - 2 + \frac{3}{x+1}\right) \\
 &= \frac{4x^2 + 2x + 1}{\cancel{2x}} \cdot \frac{\cancel{2x}(x+1)}{(2x-1)^2(4x^2 + 2x + 1)^2} \cdot \frac{4x^2 + 2x + 1}{\cancel{x+1}} \\
 &= \frac{1}{(2x-1)^2}.
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & \frac{27a^3 + 8b^3}{ax} \cdot \left(\frac{3a}{2b} \div \frac{2b}{3a}\right) \div \left[\left(3a - 2b + \frac{4b^2}{3a}\right) \cdot \frac{9ab}{x}\right] \\
 &= \frac{(3a + 2b)(9a^2 - 6ab + 4b^2)}{\cancel{ax}} \cdot \frac{\cancel{9}a^3}{4b^2} \cdot \frac{3\cancel{a}}{(9a^2 - 6ab + 4b^2)\cancel{9}ab} \\
 &= \frac{3a(3a + 2b)}{4b^3}.
 \end{aligned}$$

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$$1. \quad \frac{3 + \frac{1}{7}}{1 - \frac{1}{7}} = \frac{\cancel{22}^{\frac{11}{7}}}{\cancel{7}} \cdot \frac{\cancel{7}}{\cancel{6}_3} = \frac{11}{3}. \quad 3. \quad \frac{9 - \frac{4}{\frac{2}{5}}}{3 - \frac{2}{\frac{2}{5}}} = \frac{\cancel{221}^{\frac{17}{5}}}{\cancel{25}_5} \cdot \frac{\cancel{5}}{\cancel{13}} = \frac{17}{5}.$$

$$2. \quad \frac{\frac{6}{7} - 2}{1 - \frac{3}{7}} = \frac{\cancel{-8}^{-2}}{\cancel{7}} \cdot \frac{\cancel{7}}{\cancel{4}} = -2. \quad 4. \quad \frac{(\frac{3}{2})^2 - 16}{\frac{3}{2} + 4} = \left(\frac{9}{4} - 16\right) \div \frac{11}{2} = \frac{\cancel{-55}^{-5}}{\cancel{4}_2} \cdot \frac{\cancel{2}}{\cancel{11}} = -\frac{5}{2}.$$

$$5. \quad \frac{3^3 - (\frac{1}{2})^3}{3^2 + 3 \cdot \frac{1}{2} + (\frac{1}{2})^2} = \left(27 - \frac{1}{8}\right) \div \left(9 + \frac{3}{2} + \frac{1}{4}\right) = \frac{\cancel{215}^5}{\cancel{8}_2} \cdot \frac{\cancel{4}}{\cancel{43}} = \frac{5}{2}.$$

$$6. \quad \frac{(\frac{3}{5})^2 + \frac{3}{5} - 2}{\frac{3}{5} - 1} = \left(\frac{9}{25} + \frac{3}{5} - 2\right) \div \left(\frac{3}{5} - 1\right) = \frac{\cancel{-26}^{13}}{\cancel{25}_5} \cdot \frac{\cancel{5}}{\cancel{-2}} = \frac{13}{5}.$$

$$\begin{aligned}
 7. \quad & \frac{(\frac{2}{3})^3 - (\frac{3}{2})^3}{(\frac{2}{3})^2 + (\frac{2}{3})(\frac{3}{2}) + (\frac{3}{2})^2} = \left(\frac{8}{27} - \frac{27}{8}\right) \div \left(\frac{4}{9} + 1 + \frac{9}{4}\right) \\
 &= \frac{\cancel{-665}^{-5}}{\cancel{27 \cdot 8}_{3 \cdot 2}} \cdot \frac{\cancel{36}}{\cancel{133}} = \frac{-5}{6}.
 \end{aligned}$$

$$8. \quad \frac{(\frac{7}{2})^3 - (\frac{1}{3})^3}{(\frac{7}{2})^2 + \frac{7}{2} \cdot \frac{1}{3} + (\frac{1}{3})^2} = \frac{7}{2} - \frac{1}{3} = \frac{21}{6} - \frac{2}{6} = \frac{19}{6}.$$

$$9. \quad \frac{\frac{2}{5} + \frac{2}{5}}{\frac{2}{5}} = \frac{2}{15} + \frac{10}{3} = \frac{52}{15}.$$

$$10. \frac{\frac{c - \frac{1}{c}}{c - 1}}{c^2} = \frac{\frac{c+1}{\cancel{c^2-1}} \cdot \frac{\cancel{c}}{\cancel{c-1}}}{\cancel{c} \cdot \cancel{c-1}} = c(c+1).$$

$$12. \frac{x + \frac{x^2}{y}}{1 + \frac{x}{y}} = \frac{\frac{xy + x^2}{\cancel{y}}}{\cancel{y} \cdot \frac{x+y}{\cancel{y}}} = x.$$

$$11. \frac{2 - \frac{c^2}{2}}{\frac{2-c}{4}} = \frac{\frac{4 - \cancel{c^2}}{2}}{\frac{2}{\cancel{2}} \cdot \frac{\cancel{4}}{2-c}} = 2(2+c).$$

$$13. \frac{\frac{b^2}{5} - 5}{1 + \frac{b}{5}} = \frac{\frac{\cancel{b^2} - \cancel{25}}{5}}{\frac{5}{\cancel{5}} \cdot \frac{\cancel{b+5}}{\cancel{b+5}}} = b-5.$$

$$14. \frac{\frac{c^2}{d^2} - \frac{d^2}{c^2}}{\frac{1}{c^2} + \frac{1}{d^2}} = \frac{\frac{\cancel{c^2} - \cancel{d^2}}{\cancel{c^4} - \cancel{d^4}}}{\frac{\cancel{c^2} \cancel{d^2}}{\cancel{c^2} + \cancel{d^2}}} = c^2 - d^2.$$

$$15. \frac{\frac{a^2}{b} - \frac{b^2}{a}}{a + b + \frac{b^2}{a}} = \frac{\frac{\cancel{a^3} - \cancel{b^3}}{\cancel{ab}}}{\cancel{ab} \cdot \frac{\cancel{a^2} + \cancel{ab} + \cancel{b^2}}{\cancel{a}}} = \frac{a-b}{b}.$$

$$16. \frac{\frac{x}{y}}{\frac{z}{z}} - \frac{x}{y} = \frac{x}{yz} - \frac{xz}{y} = \frac{x}{yz} - \frac{xz^2}{yz} = \frac{x - xz^2}{yz}.$$

$$17. \frac{\frac{x}{y} + 5 + \frac{6y}{x}}{3 + \frac{2x}{y} - \frac{9y}{x}} = \frac{x^2 + 5xy + 6y^2}{xy} \cdot \frac{xy}{2x^2 + 3xy - 9y^2} \\ = \frac{(x+2y)(\cancel{x+3y})}{\cancel{xy} \cdot \cancel{(x+3y)}(2x-3y)} \cdot \frac{\cancel{xy}}{\cancel{(x+3y)}(2x-3y)} \\ = \frac{x+2y}{2x-3y}.$$

$$18. \frac{\frac{8}{a} - a^2}{1 + \frac{2}{a} + \frac{a}{2}} = \frac{\frac{\cancel{8} - \cancel{a^3}}{\cancel{a}}}{\cancel{a} \cdot \frac{-2a + 4 + \cancel{a^2}}{\cancel{2}}} = 4 - 2a.$$

$$19. \frac{1 - \frac{y}{x}}{\frac{x^2 + y^2}{x} - 2y} = \frac{\frac{\cancel{x} - \cancel{y}}{\cancel{x}}}{\cancel{x} \cdot \frac{\cancel{x^2} + \cancel{y^2} - 2xy}{x-y}} = \frac{1}{x-y}.$$

$$20. \frac{\frac{(x+3y)^2}{6xy} - 2}{\left(\frac{1}{y} - \frac{3}{x}\right)^2} = \frac{\frac{\cancel{x^2} - \cancel{6xy} + \cancel{9y^2}}{6\cancel{xy}}}{\frac{\cancel{x^2} \cancel{y^2}}{\cancel{(x-3y)^2}}} = \frac{xy}{6}.$$

$$21. \frac{\left(\frac{3a-b}{2a^3}\right)^2}{\frac{(b-3a)^2}{4a^2}} = \frac{\cancel{(3a-b)^2}}{\cancel{a^6} a^4} \cdot \frac{\cancel{4a^2}}{\cancel{(b-3a)^2}} = \frac{1}{a^4}.$$

$$22. \frac{\frac{x-2y}{2y} + \frac{x+2y}{x}}{\frac{x-2y}{x} - \frac{x+2y}{2y}} = \frac{\cancel{x^2+4y^2}}{\cancel{2xy}} \cdot \frac{\cancel{2xy}}{\cancel{-(x^2+4y^2)}} = -1.$$

$$23. \frac{\frac{1}{a} - \frac{b}{a+b}}{\frac{1}{a} + \frac{1}{b}} = \frac{a+b-ab}{a(a+b)} \cdot \frac{\cancel{ab}}{a+b} = \frac{b(a+b-ab)}{(a+b)^2}.$$

$$24. \frac{\frac{1}{x^2} - \frac{1}{xy} + \frac{1}{y^2}}{\frac{1}{x^3} + \frac{1}{y^3}} = \frac{\cancel{y^2-xy+x^2}}{\cancel{x^2y^2}} \cdot \frac{\cancel{xy}}{\cancel{x^3+y^3}} = \frac{xy}{x+y}.$$

$$25. \frac{\frac{1}{5b^2} + \frac{1}{2ab} + \frac{1}{5a^2}}{\frac{1}{2a} + \frac{1}{b}} = \frac{\cancel{2a^2+5ab+2b^2}}{\cancel{10a^2b^2}} \cdot \frac{\cancel{2ab}}{\cancel{2a+b}} = \frac{a+2b}{5ab}.$$

$$26. \frac{\frac{x}{x-1} - \frac{2x}{x-2}}{\frac{2x}{x-2} - \frac{3x}{x-3}} = \frac{-\cancel{x^2}}{(x-1)\cancel{(x-2)}} \cdot \frac{\cancel{(x-2)}(x-3)}{-\cancel{x^2}} = \frac{x-3}{x-1}.$$

$$27. \frac{\frac{2(b+d)-3(2a-c)}{2ab-bc+2ad-cd}}{\frac{2}{2a-c} - \frac{3}{b+d}} = \frac{\cancel{2b+2d-6a+3c}}{\cancel{(2a-c)(b+d)}} \cdot \frac{\cancel{(2a-c)(b+d)}}{\cancel{2b+2d-6a+3c}} = 1.$$

$$28. \frac{\left(\frac{5x+4y}{4x}\right)^2 - \frac{5y}{x}}{\frac{(5x+4y)^2}{4x} - 20y} = \frac{\cancel{(5x+4y)^2}}{\cancel{16x^2}} \cdot \frac{\cancel{4x}}{\cancel{(5x+4y)^2}} = \frac{1}{4x}.$$

$$29. \frac{2 + \frac{12-6a}{2a^2-a-6}}{2a + \frac{21}{2a+3} - 7} = \frac{\cancel{4a(a-2)}}{\cancel{(2a+3)(a-2)}} \cdot \frac{\cancel{2a+3}}{\cancel{4a(a-2)}} = \frac{1}{a-2}.$$

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$$1. \quad \frac{x}{2} + \frac{x}{3} = 10.$$

Multiplying both members by 6,
the L.C.M. of the denominators,
and canceling,

$$3x + 2x = 60.$$

$$\text{Whence} \quad x = 12.$$

$$2. \quad \frac{x}{4} + \frac{x}{8} = 3.$$

Multiplying both members by 8,
the L.C.M. of the denominators,
and canceling,

$$2x + x = 24.$$

$$\text{Whence} \quad x = 8.$$

$$3. \quad \frac{x}{3} - \frac{x}{8} = 5.$$

Multiplying both members by 24,
the L.C.M. of the denominators,
and canceling,

$$8x - 3x = 120.$$

$$\text{Whence} \quad x = 24.$$

$$4. \quad x + \frac{x}{5} = 6.$$

Multiplying both members by 5,
the L.C.M. of the denominators,
and canceling,

$$5x + x = 30.$$

$$\text{Whence} \quad x = 5.$$

$$5. \quad \frac{x}{5} + \frac{x}{3} - \frac{x}{15} = 14.$$

$$\text{L.C.M.} = 15.$$

$$3x + 5x - x = 210.$$

$$\text{Whence} \quad x = 30.$$

$$6. \quad \frac{x+5}{4} - \frac{2x+4}{9} = 1.$$

$$\text{L.C.M.} = 36.$$

$$9(x+5) - 4(2x+4) = 36.$$

$$\text{Whence} \quad x = 7.$$

$$7. \quad \frac{x-2}{2} + \frac{3x+2}{2} = 6.$$

$$\text{L.C.M.} = 2.$$

$$x-2 + 3x+2 = 12.$$

$$\text{Whence} \quad x = 3.$$

$$8. \quad \frac{x+3}{4} + \frac{4x-5}{5} = 5.$$

$$\text{L.C.M.} = 20.$$

$$5(x+3) + 4(4x-5) = 100.$$

$$\text{Whence} \quad x = 5.$$

$$9. \quad \frac{x+1}{2} + \frac{x-2}{3} - \frac{x+3}{4} = 2.$$

$$\text{L.C.M.} = 12.$$

$$6(x+1) + 4(x-2) - 3(x+3) = 24.$$

$$\text{Whence} \quad x = 5.$$

$$10. \quad \frac{2}{3}x + \frac{5}{4}x = \frac{2}{2}\frac{3}{4}.$$

$$\text{L.C.M.} = 24.$$

$$16x + 30x = 23.$$

$$\text{Whence} \quad x = \frac{1}{2}.$$

$$11. \quad \frac{4}{3}x + \frac{2}{5}x = 26.$$

$$\text{L.C.M.} = 15.$$

$$20x + 6x = 390.$$

$$\text{Whence} \quad x = 15.$$

$$12. \quad \frac{3}{2}x + \frac{4}{3}x = \frac{1}{9}7.$$

$$\text{L.C.M.} = 18.$$

$$27x + 24x = 34.$$

$$\text{Whence} \quad x = \frac{2}{3}.$$

$$13. \quad \frac{1}{x} + \frac{2}{x} = 3.$$

$$\text{L.C.M.} = x.$$

$$1 + 2 = 3x.$$

$$\text{Whence} \quad x = 1.$$

$$14. \quad \frac{3}{x} - \frac{2}{x} = 6.$$

$$\text{L.C.M.} = x.$$

$$3 - 2 = 6x.$$

$$\text{Whence} \quad x = \frac{1}{6}.$$

$$15. \quad \frac{5}{x} + \frac{4}{x} = 9.$$

$$\text{L.C.M.} = x.$$

$$5 + 4 = 9x.$$

$$\text{Whence} \quad x = 1.$$

$$16. \quad \frac{2}{x} + \frac{16}{15} = \frac{19}{3x}.$$

$$\text{L.C.M.} = 15x.$$

$$30 + 16x = 95.$$

$$\text{Whence} \quad x = \frac{65}{16}.$$

$$17. \quad \frac{2x+3}{5} - \frac{1}{3}(x-3) = 2.$$

$$\text{L.C.M.} = 15.$$

$$3(2x+3) - 5(x-3) = 30.$$

$$\text{Whence} \quad x = 6.$$

$$18. \quad \frac{5-x}{2} + \frac{2x+3}{3} = 4.$$

$$\text{L.C.M.} = 6.$$

$$3(5-x) + 2(2x+3) = 24.$$

$$\text{Whence} \quad x = 3.$$

$$19. \quad \frac{2x+13}{3} - \frac{6-x}{4} = 1.$$

$$\text{L.C.M.} = 12.$$

$$4(2x+13) - 3(6-x) = 12.$$

$$\text{Whence} \quad x = -2.$$

$$20. \quad \frac{3}{4}(x-1) + \frac{5x-7}{4} = \frac{3}{2}.$$

$$\text{L.C.M.} = 4.$$

$$3(x-1) + (5x-7) = 6.$$

$$\text{Whence} \quad x = 2.$$

$$21. \quad \frac{5x+12}{6} - \frac{4}{11}(2x+7) + \frac{1}{3} = 0.$$

$$\text{L.C.M.} = 66.$$

$$11(5x+12) - 24(2x+7) + 22 = 0.$$

$$\text{Whence} \quad x = 2.$$

$$22. \quad \frac{x-7}{6} + \frac{1}{2}\left(2 - \frac{x}{2}\right) = \frac{3x-22}{6}.$$

Removing parenthesis,

$$\frac{x-7}{6} + 1 - \frac{x}{4} = \frac{3x-22}{6}.$$

$$\text{L.C.M.} = 12.$$

$$2(x-7) + 12 - 3x = 2(3x-22).$$

$$\text{Whence} \quad x = 6.$$

$$23. \quad \frac{x}{a} + x = a + 1.$$

$$\text{L.C.M.} = a.$$

$$x + ax = (a+1)a.$$

$$(a+1)x = (a+1)a.$$

$$x = a.$$

$$24. \quad \frac{x}{a} - \frac{x}{3a} = 6.$$

$$\text{L.C.M.} = 3a.$$

$$3x - x = 18a.$$

$$\text{Whence} \quad x = 9a.$$

$$25. \quad \frac{x}{a} - \frac{2x}{5a} = \frac{3}{5b}.$$

$$\text{L.C.M.} = 5ab.$$

$$5bx - 2bx = 3a.$$

$$\text{Whence} \quad x = \frac{a}{b}.$$

26. $6 - \frac{x}{c} = \frac{1}{3}(c - 3x) + \frac{17c}{3}.$

Removing parenthesis, $6 - \frac{x}{c} = \frac{c}{3} - x + \frac{17c}{3}.$

Combining, $6 - \frac{x}{c} = 6c - x.$

$$\text{L.C.M.} = c.$$

$$6c - x = 6c^2 - cx.$$

Transposing, $cx - x = 6c^2 - 6c.$

Factoring, $x(c - 1) = 6c(c - 1).$

Whence $x = 6c.$

27. $n\left(\frac{2}{3} - \frac{n}{5}\right) - \frac{x}{3} = \frac{n}{5}(5n - 3x).$

Removing parenthesis, $\frac{2n}{3} - \frac{n^2}{5} - \frac{x}{3} = n^2 - \frac{3nx}{5}.$

$$\text{L.C.M.} = 15.$$

$$10n - 3n^2 - 5x = 15n^2 - 9nx.$$

Transposing and grouping, $(9n - 5)x = 18n^2 - 10n.$

Dividing by $9n - 5$, $x = 2n.$

28. $2x + 2a + \frac{3x + 4}{a} = \frac{2(2a^2 - 1)}{a} - 1.$

$$\text{L.C.M.} = a.$$

$$2ax + 2a^2 + 3x + 4 = 4a^2 - 2 - a.$$

$$(2a + 3)x = 2a^2 - a - 6.$$

$$x = a - 2.$$

29. $\frac{x + a}{a} - \frac{x + b}{b} = a^2(b - a).$

$$\text{L.C.M.} = ab.$$

$$b(x + a) - a(x + b) = a^3b(b - a).$$

Regrouping, $(b - a)x = a^3b(b - a).$

$$x = a^3b.$$

30. $\frac{2a - 3x}{6a} + \frac{5a - 2x}{5a} + \frac{41}{30} = 0.$

$$\text{L.C.M.} = 30a.$$

$$5(2a - 3x) + 6(5a - 2x) + 41a = 0.$$

Whence $x = 3a.$

$$31. \quad \frac{6x}{a} - \frac{1}{2} \left(\frac{2a}{3} - 12x \right) + \frac{19a}{3} = -6.$$

$$\text{Removing parenthesis,} \quad \frac{6x}{a} - \frac{a}{3} + 6x + \frac{19a}{3} = -6.$$

$$\text{L.C.M.} = 3a.$$

$$18x - a^2 + 18ax + 19a^2 = -18a.$$

Grouping,

$$18(1+a)x = -18(a+a^2).$$

Dividing,

$$x = -a.$$

32.

$$(x+5)(x-6) = x(x-\frac{5}{2}).$$

Removing parentheses,

$$x^2 - x - 30 = x^2 - \frac{5}{2}x.$$

Transposing and combining,

$$\frac{3}{2}x = 30.$$

Whence

$$x = 20.$$

33.

$$(x+7)(x+18) = x(x-\frac{1}{5}).$$

Removing parentheses,

$$x^2 + 25x + 126 = x^2 - \frac{1}{5}x.$$

Transposing and combining,

$$\frac{126}{5}x = -126.$$

$$126x = (-126)(5).$$

$$x = -5.$$

34.

$$(4x+3)(6+x) = 4x(x-\frac{1}{6}) + 45\frac{2}{3}.$$

Removing parentheses,

$$4x^2 + 27x + 18 = 4x^2 - \frac{2}{3}x + 1\frac{2}{3}.$$

Transposing and combining,

$$\frac{83x}{3} = \frac{83}{3}.$$

Whence

$$x = 1.$$

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1. Let

n = the number.

Then

$$\frac{n}{3} + \frac{n}{12} = 15.$$

Whence

$$n = 36.$$

2. Let

n = the number.

Then

$$\frac{n}{4} - \frac{n}{9} = 10.$$

Whence

$$n = 72.$$

3. Let

n = the greater number.

Then

$$38 - n = \text{the less number,}$$

and

$$\frac{n}{10} = \frac{38 - n}{9}.$$

Whence

$$n = 20; 38 - n = 18.$$

4. Let $x =$ the length of the rectangle in meters.
 Then $\frac{2}{3}x =$ the width in meters,
 and $2x + \frac{4}{3}x = 200.$
 Whence $x = 60 ; \frac{2}{3}x = 40.$
5. Let $x =$ the number.
 Then $\frac{4+x}{7} = \frac{x}{5}.$
 Whence $x = 10.$
6. Let $x =$ the first integer.
 Then $\frac{x}{2} = \frac{x+1+x+2}{5}.$
 Whence $x = 6.$
7. Let $x =$ the first integer.
 Then $\frac{x}{12} = \frac{x+2+x+4}{26}.$
 Whence $x = 36.$
8. Let $x =$ the number.
 Then $\frac{5+x}{7+x} = \frac{5}{6}.$
 Whence $x = 5.$
9. Let $x =$ the less part.
 Then $24 - x =$ the greater part,
 and $\frac{(24-x)-x}{4} = 4.$
 Whence $x = 4 ; 24 - x = 20.$
10. Let $x =$ the number.
 Then $\frac{1}{4}(3x-4) = \frac{1}{7}(5x-4).$
 Whence $x = 12.$
11. Let $x =$ the greater part.
 Then $108 - x =$ the less part,
 and $\frac{108-x}{x} = \frac{2}{7}.$
 Whence $x = 84 ; 108 - x = 24.$
12. Let $x =$ the greater number.
 Then $24 - x =$ the less number,
 and $\frac{x-(24-x)}{24} = \frac{x}{60}.$
 Whence $x = 15 ; 24 - x = 9.$
 CR

13. Let $b = \text{B's age in years.}$
 Then $\frac{5}{2}b = \text{A's age in years,}$
 and $\frac{5}{2}b + 10 = 2(b + 10).$
 Whence $b = 20; \frac{5}{2}b = 50.$
14. Let $x = \text{the husband's age in years at marriage.}$
 Then $\frac{4}{5}x = \text{the wife's age in years at marriage,}$
 and $\frac{4}{5}x + 20 = \frac{8}{9}(x + 20).$
 Whence $x = 25; \frac{4}{5}x = 20.$
15. Let $b = \text{B's age in years now.}$
 Then $b + 12 = \text{A's age in years now,}$
 and $b - 8 = \frac{5}{8}(b + 12 - 8).$
 Whence $b = 28; b + 12 = 40.$

16. Let the number of moons each planet has be represented by the initial of the planet.

Then $U + J + S + M + N = 26.$

By the conditions of the problem this may be written

$$U + (U + 5) + (2U + 2) + (U - 2) + \frac{1}{2}(U - 2) = 26.$$

Whence

$$U = 4.$$

$$J = U + 5 = 9.$$

$$S = 2U + 2 = 10.$$

$$M = U - 2 = 2.$$

$$N = \frac{1}{2}(U - 2) = 1.$$

17. The altitude of the trapezoid is 8, and one base is 12.

Let $x = \text{the other base.}$

Then
$$\frac{8(12 + x)}{2} = \frac{24 \cdot 12}{2}.$$

Whence

$$x = 24.$$

18. Let

$x = \text{distance to target in feet.}$

Then

$$\frac{x}{1925} = \text{time of flight in seconds,}$$

and

$$\frac{x}{1100} = \text{time for return of sound in seconds.}$$

Therefore

$$\frac{x}{1925} + \frac{x}{1100} = 3.$$

Whence

$$x = 2100; \frac{x}{1925} = \frac{2100}{1925} = \frac{12}{11}.$$

19. Let $x =$ velocity in feet per second.

Then $\frac{7920}{x} =$ time of flight in seconds,

and $\frac{7920}{1100} =$ time for return of sound in seconds.

Therefore $\frac{7920}{x} + \frac{7920}{1100} = \frac{47}{5}$.

Whence $x = 3600$ number of feet per second.

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2. $\frac{x-3}{3+x} = 4.$

Multiplying both members by $3+x$, the L.C.M. of the denominators, and canceling,

$$x-3 = 4(3+x).$$

Whence $x = -5.$

3. $\frac{x-5}{x-2} = \frac{1}{2}.$

Multiplying both members by $2(x-2)$, the L.C.M. of the denominators, and canceling,

$$2(x-5) = x-2.$$

Whence $x = 8.$

4. $\frac{3x+4}{2x-2} = -2.$

$$\text{L.C.M.} = 2x-2.$$

$$3x+4 = -2(2x-2).$$

Whence $x = 0.$

5. $\frac{4}{6-5x} = \frac{1}{x}.$

$$\text{L.C.M.} = x(6-5x).$$

$$4x = 6-5x.$$

Whence $x = \frac{2}{3}.$

7. $\frac{x-3}{x} = \frac{2x}{x-2}.$

$$\text{L.C.M.} = x(x-2).$$

$$(x-3)(x-2) = 2x^2.$$

Whence

$$(x-1)(x+6) = 0.$$

$$x = 1, -6.$$

8. $\frac{x-1}{x+3} = x.$

$$\text{L.C.M.} = x+3.$$

$$x-1 = x(x+3).$$

Whence

$$(x+1)^2 = 0.$$

$$x = -1, -1.$$

9. $\frac{x}{5-x} = \frac{2x}{4x-5}.$

$$\text{L.C.M.} = (5-x)(4x-5).$$

$$x(4x-5) = 2x(5-x).$$

Whence

$$3x(2x-5) = 0.$$

$$x = 0, \frac{5}{2}.$$

10. $\frac{2}{x+1} = \frac{4}{4+x}.$

$$\text{L.C.M.} = (x+1)(4+x).$$

$$2(4+x) = 4(x+1).$$

Whence $x = 2.$

11. $\frac{1}{2-x} = \frac{3}{3-x}.$

$$\text{L.C.M.} = (2-x)(3-x).$$

$$3-x = 3(2-x).$$

Whence $x = \frac{3}{2}.$

12. $\frac{x-5}{4-5x} = -6x.$

$$\text{L.C.M.} = 4-5x.$$

$$x-5 = -6x(4-5x).$$

Whence

$$5(3x-1)(2x-1) = 0.$$

$$x = \frac{1}{2}, \frac{1}{5}.$$

13.

$$\frac{x+7}{3-x} + \frac{3x}{2} = x.$$

$$\text{L.C.M.} = 2(3-x).$$

$$2(x+7) + 3x(3-x) = 2x(3-x).$$

Whence

$$(x-7)(x+2) = 0.$$

$$x = 7, -2.$$

14.

$$\frac{2}{x-6} = \frac{x-12}{x+6} + \frac{x}{x+6}.$$

$$\text{L.C.M.} = (x-6)(x+6).$$

$$2(x+6) = (x-12)(x-6) + x(x-6).$$

Whence

$$(x-3)(x-10) = 0.$$

$$x = 3, 10.$$

15.

$$\frac{x+3}{x-4} = \frac{x+9}{x-5}.$$

$$\text{L.C.M.} = (x-4)(x-5).$$

$$(x+3)(x-5) = (x+9)(x-4).$$

Whence

$$x = 3.$$

16.

$$\frac{4x-1}{x+2} = \frac{7-2x}{x+4}.$$

$$\text{L.C.M.} = (x+4)(x+2).$$

$$(4x-1)(x+4) = (7-2x)(x+2).$$

Whence

$$(x-1)(x+3) = 0.$$

$$x = 1, -3.$$

17.

$$\frac{4}{2x-3} + \frac{3x+2}{3} = x.$$

$$\text{L.C.M.} = 3(2x-3).$$

$$12 + (3x+2)(2x-3) = 3x(2x-3).$$

Whence

$$x = -\frac{3}{2}.$$

18.

$$\frac{x+6}{3-x} = \frac{5+2x}{7x-5}.$$

$$\text{L.C.M.} = (3-x)(7x-5).$$

$$(x+6)(7x-5) = (5+2x)(3-x).$$

Whence

$$9(x-1)(x+5) = 0.$$

$$x = 1, -5.$$

19.
$$\frac{3x}{8} - \frac{4}{x-4} = \frac{3x-4}{8}.$$

L.C.M. = $8(x-4).$

$$3x(x-4) - 32 = (3x-4)(x-4).$$

Whence
$$x = 12.$$

20.
$$\frac{2}{4x+1} - \frac{1}{5x-2} - \frac{1}{3x+4} = 0.$$

L.C.M. = $(4x+1)(5x-2)(3x+4).$

$$2(5x-2)(3x+4) - (4x+1)(3x+4) - (4x+1)(5x-2) = 0.$$

Whence
$$2(x-3)^2 = 0.$$

$x = 3, 3.$

21.
$$\frac{47}{220} - \frac{7}{4(x+3)} = \frac{3}{5(x+3)}.$$

L.C.M. = $220(x+3).$

$$(47)(x+3) - (7)(55) = (3)(44).$$

Whence
$$x = 8.$$

22.
$$\frac{4x}{x+3} - \frac{6}{2(x+3)} = \frac{x^2+11}{3(x+3)}.$$

Canceling,

$$\frac{4x}{x+3} - \frac{3}{x+3} = \frac{x^2+11}{3(x+3)}.$$

L.C.M. = $3(x+3).$

$$12x - 9 = x^2 + 11.$$

Whence
$$(x-2)(x-10) = 0.$$

$x = 2, 10.$

23.
$$\frac{4-x}{x-5} + \frac{7}{5} = \frac{3}{5-x}.$$

Rewriting the last fraction,

$$\frac{4-x}{x-5} + \frac{7}{5} = \frac{-3}{x-5}.$$

L.C.M. = $5(x-5).$

$$5(4-x) + 7(x-5) = -15.$$

Whence
$$x = 0.$$

24.
$$\frac{x}{x-3} + 3 = \frac{1}{3-x}.$$

Rewriting the last fraction,

$$\frac{x}{x-3} + 3 = \frac{-1}{x-3}.$$

L.C.M. = $x-3.$

$$x + 3(x-3) = -1.$$

Whence
$$x = 2.$$

25.

$$\frac{3x}{x-2} = \frac{x-3}{2-x} - x.$$

Rewriting,

$$\frac{3x}{x-2} = \frac{3-x}{x-2} - x.$$

$$\text{L.C.M.} = x-2.$$

$$3x = 3 - x - x(x-2).$$

Whence

$$(x-1)(x+3) = 0.$$

$$x = 1, -3.$$

26. From Hint,

$$3x(x+3) - 16 = x(x-3) - 2x.$$

Whence

$$x = 1, -8.$$

27.

$$\frac{1}{3-x} + \frac{2}{x+3} = \frac{6x}{x^2-9}.$$

Rewriting,

$$\frac{1}{3-x} + \frac{2}{3+x} + \frac{6x}{9-x^2} = 0.$$

$$\text{L.C.M.} = 9-x^2.$$

$$(3+x) + 2(3-x) + 6x = 0.$$

Whence

$$x = -\frac{9}{5}.$$

28.

$$\frac{2x}{x+1} + \frac{3}{x-1} + \frac{24}{1-x^2} = 0.$$

$$\frac{2x}{x+1} + \frac{3}{x-1} - \frac{24}{x^2-1} = 0.$$

$$2x(x-1) + 3(x+1) - 24 = 0.$$

Whence

$$x = 3, -\frac{7}{2}.$$

29.

$$\frac{3}{x-4} = \frac{5x-12}{x^2-16} - \frac{4}{16-x^2}.$$

$$\frac{3}{x-4} = \frac{5x-12+4}{x^2-16} = \frac{5x-8}{x^2-16}.$$

$$3(x+4) = 5x-8.$$

Whence

$$x = 10.$$

30.

$$\frac{x+2}{x-2} = \frac{10-2x^2}{4-x^2} - \frac{7}{x^2-4}.$$

$$\frac{x+2}{x-2} = \frac{-10+2x^2-7}{x^2-4} = \frac{2x^2-17}{x^2-4}.$$

$$(x+2)^2 = 2x^2-17.$$

Whence

$$x = 7, -3.$$

$$31. \quad \frac{1+x}{3-x} + \frac{x-2}{2-x} = \frac{x-1}{x^2-5x+6}.$$

$$\frac{1+x}{3-x} + \frac{x-2}{2-x} = \frac{x-1}{6-5x+x^2}.$$

$$(1+x)(2-x) + (x-2)(3-x) = x-1.$$

Whence

$$x = 1, \frac{3}{2}.$$

$$32. \quad \frac{x-4}{2x-5} + \frac{2x-15}{2x+4} = \frac{8x^2-20x-31}{4x^2-2x-20}.$$

$$(x-4)(2x+4) + (2x-15)(2x-5) = 8x^2-20x-31.$$

Whence

$$2(x+15)(x-3) = 0.$$

$$x = 3, -15.$$

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$$1. \quad .4x = 6.$$

Multiplying by 10,

$$4x = 60.$$

$$x = 15.$$

$$7. \quad .06x - 4.5 = 1.68.$$

$$6x - 450 = 168.$$

$$x = 103.$$

$$2. \quad .3x + .5 = .8.$$

Multiplying by 10,

$$3x + 5 = 8.$$

$$x = 1.$$

$$8. \quad .03x + .16 = .58.$$

$$3x + 16 = 58.$$

$$x = 14.$$

$$3. \quad .3x + 4 = .25.$$

Multiplying by 100,

$$30x + 400 = 25.$$

$$x = -12.5.$$

$$9. \quad .04x = .1x + 2.4.$$

$$4x = 10x + 240.$$

$$x = -40.$$

$$4. \quad .75 - .7x = .26.$$

$$75 - 70x = 26.$$

$$x = .7.$$

$$10. \quad .8x - 2.7 = .55x.$$

$$80x - 270 = 55x.$$

$$x = 10.8.$$

$$11. \quad .15x - .4x = 8x - 49.5.$$

$$15x - 40x = 800x - 4950.$$

$$4950 = 825x.$$

$$x = 6.$$

$$5. \quad .92 + .3x = 5.12.$$

$$92 + 30x = 512.$$

$$x = 14.$$

$$12. \quad 1.7x + 3.14 = -9.66 - 1.5x.$$

$$170x + 314 = -966 - 150x.$$

$$320x = -1280.$$

$$x = -4.$$

$$6. \quad 3.75 = 2.15 - .5x.$$

$$375 = 215 - 50x.$$

$$x = -3.2.$$

$$\begin{aligned}
 13. \quad & 3x + 7 - 1.25x = 8.845 + .52x. \\
 & 3000x + 7000 - 1250x = 8845 + 520x. \\
 & 1230x = 1845. \\
 & x = 1.5.
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & 1.7x + .17 - .03x = 4.73 + 1.1x. \\
 & 170x + 17 - 3x = 473 + 110x. \\
 & x = 8.
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & .12(2x + .5) - .2(1.5x - 2) = .4. \\
 & .24x + .06 - .3x + .4 = .4. \\
 & 24x + 6 - 30x + 40 = 40. \\
 & x = 1.
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & 6(3x - 1.1) - 8.4(.7x - 3) = 6x + .24. \\
 & 18x - 6.6 - 5.88x + 25.2 = 6x + .24. \\
 & 1800x - 660 - 588x + 2520 = 600x + 24. \\
 & 612x = -1836. \\
 & x = -3.
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{.7x}{4} - \frac{.19}{3} = \frac{.1x}{6}. \\
 \text{Clearing of decimals,} \quad & \frac{70x}{4} - \frac{19}{3} = \frac{10x}{6}. \\
 \text{Canceling,} \quad & \frac{35x}{2} - \frac{19}{3} = \frac{5x}{3}. \\
 \text{Whence} \quad & 105x - 38 = 10x. \\
 & x = \frac{2}{5} = .4.
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{.3x - 6.2}{4} - \frac{6.75 - .4x}{5} = \frac{3.5}{2}. \\
 & \frac{30x - 620}{4} - \frac{675 - 40x}{5} = \frac{350}{2}. \\
 & \frac{15x - 310}{2} - 135 + 8x = 175. \\
 & 15x - 310 - 2(135 - 8x) = 350. \\
 \text{Whence} \quad & x = 30.
 \end{aligned}$$

NOTE. Examples 17 and 18 make it plain that an equation should usually be cleared of decimals before it is cleared of fractions, since advantageous cancellations are likely to result.

$$\begin{aligned}
 19. \quad & \frac{.14x + 3.2}{6} - \frac{x - .75}{5} = \frac{2.4x - 1.5}{10}. \\
 & \frac{14x + 320}{6} - \frac{100x - 75}{5} = \frac{240x - 150}{10}. \\
 & \frac{7x + 160}{3} - (20x - 15) = 24x - 15. \\
 & 7x + 160 - 3(20x - 15) = 3(24x - 15).
 \end{aligned}$$

Whence

$$x = 2.$$

$$\begin{aligned}
 20. \text{ From Hint, } & \frac{600x - 269}{40} + \frac{150x}{24} = 3.9. \\
 & \frac{600x - 269}{40} + \frac{25x}{4} = 3.9. \\
 & 600x - 269 + 250x = 156. \\
 & x = \frac{1}{2}.
 \end{aligned}$$

$$21. \quad \frac{1.8x - 2}{1.7} = \frac{5.7 - .7x}{1.8}.$$

Multiplying the numerator and denominator of each fraction by 10,

$$\begin{aligned}
 & \frac{18x - 20}{17} = \frac{57 - 7x}{18}. \\
 & 18(18x - 20) = 17(57 - 7x). \\
 & x = 3.
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & \frac{3.2x}{.5} + \frac{4.5x}{12.5} = 1.52 + 6x. \\
 & \frac{32x}{5} + \frac{45x}{125} = 1.52 + 6x. \\
 & \frac{32x}{5} + \frac{9x}{25} = 1.52 + 6x. \\
 & 640x + 36x = 152 + 600x \text{ (cf. Ex. 18, Note).} \\
 & x = 2.
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{.3(3 - .4x)}{.16} - \frac{.5(.2x - 6)}{.8} = 5. \\
 & \frac{.9 - .12x}{.16} - \frac{.1x - 3}{.8} = 5.
 \end{aligned}$$

Multiplying the numerator and denominator of the first fraction by 100, of the second by 10,

$$\begin{aligned}
 & \frac{90 - 12x}{16} - \frac{x - 30}{8} = 5. \\
 & 90 - 12x - 2(x - 30) = 80. \\
 & x = 5.
 \end{aligned}$$

24.

$$\frac{.3(5-x)}{6.25} = \frac{1.5-10x}{14} + .56x.$$

$$\frac{30(5-x)}{625} = \frac{15-100x}{140} + .56x.$$

$$\frac{6(5-x)}{125} = \frac{3-20x}{28} + .56x.$$

$$168(5-x) = 125(3-20x) + 3500(.56x).$$

Whence

$$465 = -372x, \text{ and } x = -1.25.$$

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1. Let
Then
and

 P = the principal. $.06P$ = the interest for one year,

$$P + .06P = 371.$$

$$106P = 37,100.$$

$$P = 350.$$

2. Here

$$P + 3(.04P) = 476.$$

$$112P = 47,600.$$

$$P = 425.$$

3. If
then

 x = the number of years,

$$(x)(325)(.06) = 39.$$

$$19.50x = 39.$$

$$x = 2.$$

4. If
then

 x = the number of years,

$$(x)(480)(.065) = 156.$$

$$31.2x = 156.$$

$$x = 5.$$

6. Let
Then
and
Therefore

 x = the rate of interest.\$725 x = the interest for one year,\$2900 x = the interest for four years.

$$2900x = 145.$$

$$x = \frac{5}{100}, \text{ or } 5\%.$$

7. As in Ex. 6,

$$(250)(6)(x) = 317.50 - 250 = 67.50.$$

$$1500x = 67.50.$$

$$x = .045 = 4\frac{1}{2}\%.$$

8. As in Ex. 6, if $x =$ the number of years,
 $(300)(x)(.05) = 600 - 300 = 300.$
 $15x = 300.$
 $x = 20.$
9. As in Ex. 6, if $x =$ the number of years,
 $(500)(x)(.06) = 1500 - 500 = 1000.$
 $30x = 1000.$
 $x = 33\frac{1}{3}.$
11. Let $x =$ the number of dollars at 5%.
Then $1500 - x =$ the number of dollars at 4%,
and $.05x + .04(1500 - x) = 69.$
Whence $x = 900$; $1500 - x = 600.$
12. Let $x =$ the number of dollars at 5%.
Then $x + 100 =$ the number of dollars at 6%,
and $.05x + .06(x + 100) = 50.$
Whence $x = 400$; $x + 100 = 500.$
13. Let $x =$ the number of dollars at 4%.
Then $3600 - x =$ the number of dollars at 5%,
and $.04x = .05(3600 - x).$
Whence $x = 2000$; $3600 - x = 1600.$
14. Let $x =$ the number of dollars at 5%.
Then $240 - x =$ the number of dollars at 6%,
and $.05x + 10 = .06(240 - x).$
Whence $x = 40$; $240 - x = 200.$
15. Let $x =$ the number of dollars at 6%.
Then $4300 - x =$ the number of dollars at 5%,
and $.06x + 1.60 = .05(4300 - x).$
Whence $x = 1940$; $4300 - x = 2360.$
16. Let $x =$ the number of dollars at 5%.
Then $5360 - x =$ the number of dollars at 6%,
and $.05x - 63.40 = .06(5360 - x).$
Whence $x = 3500$; $5360 - x = 1860.$
17. Let $x =$ the number of dollars at 4%.
Then $4560 - x =$ the number of dollars at 6%,
and $.04x + .06(4560 - x) = 202.40.$
Whence $x = 3560$; $4560 - x = 1000.$

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1. $2ax - 2a^2 = 6a^2 - 2ax.$
 $4ax = 8a^2.$
 $x = 2a.$
2. $5c^2 - 4cx = c(5x - 4c).$
 $5c^2 - 4cx = 5cx - 4c^2.$
 $9c^2 = 9cx.$
 $c = x.$
3. $3a^2x - 4b^2 = b^2 - 2a^2x.$
 $5a^2x = 5b^2.$
 $x = \frac{b^2}{a^2}.$
4. $2(x + 3) - 4a = 6.$
 $2x + 6 - 4a = 6.$
 $2x = 4a.$
 $x = 2a.$
5. $3(x - 1) - 9a = 6.$
 $3x - 3 - 9a = 6.$
 $3x = 9a + 9.$
 $x = 3a + 3.$
6. $3(x - 2) - 4h = 2(x - 2).$
 $3x - 6 - 4h = 2x - 4.$
 $x = 4h + 2.$
7. $3(2x - a) = 2(x - a).$
 $6x - 3a = 2x - 2a.$
 $4x = a.$
 $x = \frac{a}{4}.$
8. $4(x - b) = 2(x + b).$
 $4x - 4b = 2x + 2b.$
 $2x = 6b.$
 $x = 3b.$
9. $4a(6x - 3b) = 3b(8 - 4a).$
 $24ax - 12ab = 24b - 12ab.$
 $24ax = 24b.$
 $x = \frac{b}{a}.$
10. $ax + bx = a^2 + ab.$
 $(a + b)x = (a + b)a.$
 $x = a.$
11. $cx + b^2 = bx + bc.$
 $cx - bx = bc - b^2.$
 $(c - b)x = (c - b)b.$
 $x = b.$
12. $ax + b^2 = a^2 - bx.$
 $ax + bx = a^2 - b^2.$
 $(a + b)x = (a - b)(a + b).$
 $x = a - b.$
13. $6ab + 15cx - 10bc - 9ax = 0.$
 $15cx - 9ax = 10bc - 6ab.$
 $(15c - 9a)x = (5c - 3a)2b.$
 $x = \frac{2b}{3}.$
14. $12 - 15a + 16x = 20ax.$
 $16x - 20ax = 15a - 12.$
 $(16 - 20a)x = 15a - 12.$
 $x = \frac{15a - 12}{16 - 20a} = -\frac{12 - 15a}{16 - 20a} = -\frac{3}{4}.$

15. $ax + bx + cx = ak + bk + ck.$
 $(a + b + c)x = (a + b + c)k.$
 $x = k.$
16. $2ax + 2a + cx + c + x + 1 = 0.$
 $(2a + c + 1)x = -(2a + c + 1).$
 $x = -1.$
17. $6ab + kx + 4a^2 = 2ax + 3bk + 2ak.$
 $kx - 2ax = (3b + 2a)k - (3b + 2a)2a.$
 $(k - 2a)x = (k - 2a)(3b + 2a).$
 $x = 3b + 2a.$
18. $5ax - 5a^2 - 10ab = 3ac + 6bc - 3cx.$
 $5ax + 3cx = 3c(a + 2b) + 5a(a + 2b).$
 $(5a + 3c)x = (5a + 3c)(a + 2b).$
 $x = a + 2b.$
19. $\frac{x}{2b} = a.$
 $x = 2ab.$
20. $\frac{3ab}{x} = b.$
 $3ab = bx.$
 $x = 3a.$
21. $\frac{c}{x} + \frac{3c}{2x} = \frac{5}{4}.$
 $4c + 6c = 5x.$
 $10c = 5x.$
 $x = 2c.$
22. $\frac{8a}{3x} + \frac{8a}{x} = \frac{3}{2} + \frac{5a}{3x}.$
 $16a + 48a = 9x + 10a.$
 $54a = 9x.$
 $x = 6a.$
23. $\frac{x}{a} + \frac{x}{b} = a + b.$
 $ax + bx = ab(a + b).$
 $x = ab.$
24. $\frac{d^2}{x} - d = \frac{a^2}{x} + a.$
 $d^2 - dx = a^2 + ax.$
 $d^2 - a^2 = (a + d)x.$
 $x = d - a.$
25. $\frac{hx}{2k} - 4k^2 = \frac{2kx}{h} - h^2.$
 $h^2x - 8k^3h = 4k^2x - 2kh^3.$
 $2kh^3 - 8k^3h = (4k^2 - h^2)x.$
 $x = -2kh.$
26. $\frac{2x}{a} + \frac{a - 4x}{3} - 3a = -4.$
 $6x + a^2 - 4ax - 9a^2 = -12a.$
 $6x - 4ax = 8a^2 - 12a.$
 $(6 - 4a)x = (4a - 6)2a.$
 $x = -2a.$

$$\begin{aligned}
 27. \quad & \frac{x}{a} + \frac{x}{c} + ac = bc + ab + \frac{x}{b}. \\
 & (bc + ab - ac)x = abc(bc - ac + ab). \\
 & x = abc.
 \end{aligned}$$

$$\begin{aligned}
 28. \quad & \frac{3a - 4x}{5a} + \frac{3a - 2x}{4a} = \frac{1}{20}. \\
 & 4(3a - 4x) + 5(3a - 2x) = a. \\
 & 26a = 26x. \\
 & x = a.
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & \frac{bx}{2} - \frac{3a}{5} \left(x - \frac{2ab}{3} \right) = ab \left(\frac{b}{2} - \frac{a}{5} \right). \\
 & \frac{bx}{2} - \frac{3ax}{5} + \frac{6a^2b}{15} = \frac{ab^2}{2} - \frac{a^2b}{5}. \\
 & 15bx - 18ax + 12a^2b = 15ab^2 - 6a^2b. \\
 & (15b - 18a)x = 15ab^2 - 18a^2b. \\
 & x = ab.
 \end{aligned}$$

$$\begin{aligned}
 30. \quad & \frac{x - a}{x - b} = \frac{b}{a}. \\
 & ax - a^2 = bx - b^2. \\
 & (a - b)x = a^2 - b^2. \\
 & x = a + b.
 \end{aligned}$$

$$\begin{aligned}
 31. \quad & \frac{b}{b(b - x)} + \frac{3}{d(b - x)} + \frac{d + 3}{2db} = 0. \\
 & 2db + 6b + (d + 3)(b - x) = 0. \\
 & 2db + 6b + bd + 3b = (d + 3)x. \\
 & x = 3b.
 \end{aligned}$$

$$\begin{aligned}
 32. \quad & \frac{c}{k(x + c)} + \frac{k}{c(x - k)} = \frac{c^2 - kc + 2k^2}{2kc(x - k)}. \\
 & 2c^2(x - k) + 2k^2(x + c) = (x + c)(c^2 - kc + 2k^2). \\
 & 2c^2x - 2c^2k + 2k^2x + 2k^2c = c^2x - kcx + 2k^2x + c^3 - kc^2 + 2k^2c. \\
 & (c^2 + kc)x = c^3 + c^2k = (c^2 + kc)c. \\
 & x = c.
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & \frac{1}{ab} + 1 - \frac{ab}{x} = \frac{1}{x}. \\
 & x + abx - a^2b^2 = ab. \\
 & (1 + ab)x = (1 + ab)ab. \\
 & x = ab.
 \end{aligned}$$

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1. $K = 2\pi RH.$
 $\frac{K}{2\pi H} = R.$
2. $A = \frac{ab}{2}.$
 $\frac{2A}{b} = a.$
3. $C = 2\pi R.$
 $\frac{C}{2\pi} = R.$
4. $d = rt.$
 (1) $\frac{d}{r} = t.$
 (2) $\frac{d}{t} = r.$
5. $\frac{a}{A} = \frac{D}{360}.$
 (1) $a = \frac{AD}{360}.$
 (2) Whence $A = \frac{360a}{D}.$
6. $\frac{D}{360} = \frac{l}{C}.$
 $CD = 360l.$
 $C = \frac{360l}{D}.$
7. $C = \frac{E}{R+r}.$
 $CR + Cr = E.$
 $Cr = E - CR.$
 $r = \frac{E - CR}{C}.$
8. $C = \frac{E}{R + nr}.$
 $CR + Cnr = E.$
 $Cnr = E - CR.$
 Whence (1) $n = \frac{E - CR}{Cr},$
 and (2) $r = \frac{E - CR}{Cn}.$
9. $C = \frac{ne}{R + nr}.$
 $CR + Cnr = ne.$
 $Cnr = ne - CR.$
 Whence (1) $r = \frac{ne - CR}{Cn}.$
 and $Cnr - ne = -CR.$
 $(e - Cr)n = CR.$
 (2) $n = \frac{CR}{e - Cr}.$
10. $C = \frac{5}{9}(F - 32).$
 $\frac{9}{5}C = F - 32.$
 $\frac{9}{5}C + 32 = F.$
11. $\frac{W_1}{W_2} = \frac{L_2}{L_1}.$
 $W_1L_1 = W_2L_2.$
 $\frac{W_1L_1}{L_2} = W_2.$
12. $A = P(1 + rt).$
 $A = P + Prt.$
 $A - P = Prt.$
 Whence $\frac{A - P}{Pr} = t,$
 and $\frac{A - P}{Pt} = r.$
13. $\frac{V_1}{V_2} = \frac{P_2}{P_1}.$
 $\frac{V_1P_1}{V_2} = P_2.$

$$14. \quad s = \frac{n(a+l)}{2}.$$

$$2s = an + ln.$$

$$\text{Whence (1)} \quad n = \frac{2s}{a+l}.$$

$$\text{and} \quad 2s - an = ln.$$

$$(2) \quad l = \frac{2s - an}{n}.$$

$$15. \quad s = \frac{rl - a}{r - 1}.$$

$$rs - s = rl - a.$$

$$\text{Whence (1)} \quad a = rl - rs + s.$$

$$(2) \quad l = \frac{rs - s + a}{r}.$$

$$(3) \quad r = \frac{s - a}{s - l}.$$

$$16. \quad \frac{D}{180} = \frac{a}{\pi}.$$

$$\frac{\pi D}{180} = a.$$

$$17. \quad V_1 = V_0(1 + .00365 t_1).$$

$$V_1 - V_0 = (V_0)(.00365 t_1).$$

$$\frac{V_1 - V_0}{.00365 V_0} = t_1.$$

$$18. \quad A = \frac{(b_1 + b_2)a}{2}.$$

$$2A = ab_1 + ab_2.$$

$$2A - ab_1 = ab_2.$$

$$b_2 = \frac{2A - ab_1}{a}.$$

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$$1. \quad l_1 W_1 = l_2 W_2 \text{ in feet and pounds.}$$

Letting subscript 1 pertain to A,

$$(4)(96) = (6)(W_2).$$

$$\text{Whence} \quad W_2 = 64 \text{ pounds.}$$

$$2. \quad l_1 W_1 = l_2 W_2 \text{ in inches and pounds.}$$

Letting subscript 1 pertain to B,

$$(80)(120) = (l_2)(100).$$

$$\text{Whence} \quad l_2 = 96 \text{ inches.}$$

$$3. \text{ Letting } x \text{ and } 9 - x \text{ be the distances from the fulcrum in feet,}$$

$$125x = 100(9 - x).$$

$$\text{Whence} \quad x = 4 \text{ feet, A's distance,}$$

$$\text{and} \quad 9 - x = 5 \text{ feet, B's distance.}$$

$$4. \text{ Letting } x \text{ and } 210 - x \text{ be the weights in pounds and reducing the distances to inches,}$$

$$45x = 60(210 - x).$$

$$\text{Whence} \quad x = 120 \text{ pounds, A's weight.}$$

$$\text{and} \quad 210 - x = 90 \text{ pounds, B's weight.}$$

$$5. \text{ Let} \quad x = \text{B's distance in feet from the fulcrum,}$$

$$\text{and} \quad y = \text{B's weight in pounds.}$$

$$\text{Then} \quad xy = (24 - x)\left(\frac{2}{3}y\right).$$

$$\text{Or dividing by } y, \quad x = 16 - \frac{2}{3}x.$$

$$\text{Whence} \quad x = \frac{48}{5} \text{ feet} = 9.6 \text{ feet.}$$

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1. Let $x =$ the less part.
 Then $240 - x =$ the greater part,
 and $\frac{240 - x}{x} = 7$.
 Whence $x = 30$; $240 - x = 210$.
2. Let $x =$ the greater part.
 Then $68 - x =$ the less part,
 and $\frac{2}{3}x = \frac{3}{4}(68 - x)$.
 Whence $x = 36$; $68 - x = 32$.
3. Let $x =$ the first part.
 Then $\frac{4}{3} - x =$ the second part,
 and $\frac{1}{3}x = \frac{1}{5}(\frac{4}{3} - x)$.
 Whence $x = \frac{1}{2}$; $\frac{4}{3} - x = \frac{5}{6}$.
4. $x = 23$; $98 - x = 75$.
5. Let $x =$ the less part.
 Then $120 - x =$ the greater part,
 and $\frac{120 - x}{x} = 5 + \frac{12}{x}$.
 Whence $x = 18$; $120 - x = 102$.
6. Let $x =$ the less number.
 Then $1797 - x =$ the greater number,
 and $\frac{1797 - x}{x} = 70 + \frac{22}{x}$.
 Whence $x = 25$; $1797 - x = 1772$.
7. Let $x =$ the smaller part.
 Then $\frac{11}{6} - x =$ the greater part,
 and $x(\frac{11}{6} - x) - \frac{1}{3} = x^2$.
 Whence $(4x - 1)(3x - 2) = 0$,
 and $x = \frac{1}{4}$; $\frac{11}{6} - x = \frac{1}{2}$;
 or $x = \frac{2}{3}$; $\frac{11}{6} - x = \frac{7}{6}$.
8. Let $x =$ the less number.
 Then $16 - x =$ the greater number,
 and $4(16 - x) - 50 = \frac{1}{2}(28 - x)$.
 Whence $x = 0$; $16 - x = 16$.
9. Let $x =$ the boy's age in years now.
 Then $x = \frac{3}{5}(x + 10)$.
 Whence $x = 15$.
 CR

10. Let $x =$ the man's age in years now.
 Then $\frac{1}{6}(x - 12) = \frac{1}{8}(x + 8).$
 Whence $x = 72.$
11. Let $n =$ the number of nickels.
 Then $60 - n =$ the number of quarters,
 and $5n + 25(60 - n) = 1100.$
 Whence $n = 20 ; 60 - n = 40.$
12. Let $d =$ the number of dimes.
 Then $23 - d =$ the number of quarters,
 and $10d + 25(23 - d) = 305.$
 Whence $d = 18 ; 23 - d = 5.$
13. Let $x =$ the certain even number.
 Then $\left(\frac{x}{2}\right)^2 + 14 = \frac{1}{4}(x + 2)(x + 4).$
 Whence $x = 8 ; x + 2 = 10 ; x + 4 = 12.$
14. Let $x =$ the width in feet.
 Then $4x =$ the length in feet,
 and $(4x - 4)(x + \frac{3}{2}) = (4x)(x) + 11.$
 Whence $x = 8\frac{1}{2} ; 4x = 34.$
15. Let $x =$ the length,
 and $\frac{2}{3}x =$ the breadth.
 Then $(2x)(\frac{2}{3}x - 14) = (x)(\frac{2}{3}x) - (\frac{1}{3}x)^2.$
 Whence $x = 36$, root 0 rejected ;
 $\frac{2}{3}x = 24.$
16. Let $x =$ the side in feet.
 Then $4x =$ the perimeter in feet,
 and $\frac{x^2}{9} =$ the area in square yards.
 Therefore $(15)\left(\frac{x^2}{9}\right) = (20)(4x).$
 Whence $x = 48$, root 0 rejected.
17. Let $x =$ the width in yards.
 Then $2x =$ the length in yards,
 and $(50)(6x) = \frac{1}{2}(15)(2x^2).$
 Whence $x = 20$, root 0 rejected ;
 $2x = 40.$

18. Let x = the width of the picture in inches.
 Then $\frac{5}{2}x$ = the length of the picture in inches.
 Therefore $(x)(\frac{5}{2}x)$ = the area of the unframed picture in square inches,

and $(x+4)(\frac{5}{2}x+4)$ = the area of the framed picture in square inches.

$$\text{But } (x+4)(\frac{5}{2}x+4) - (x)(\frac{5}{2}x) = 128.$$

$$\text{Whence } x = 8; \frac{5}{2}x = 20.$$

19. Let x = the number of dozen bought.

Then $16x$ = the expenditure in cents.

3 for 5 = 20 cents per dozen, etc.

Hence $(\frac{1}{5}x)(20) + (\frac{4}{5}x)(18)$ = the total receipts in cents.

$$\text{But } (\frac{1}{5}x)(20) + (\frac{4}{5}x)(18) - 16x = 48.$$

$$\text{Whence } x = 20.$$

$$21. \text{ As in Ex. 20, } \frac{1}{6} + \frac{1}{9} = \frac{1}{x}.$$

$$\text{Whence } x = \frac{18}{5}.$$

22. As in Ex. 20,

$$\frac{1}{6} + \frac{1}{8} + \frac{1}{8} + \frac{1}{12} = \frac{1}{x}.$$

$$\text{Whence } x = 2.$$

$$23. \text{ As in Ex. 20, } \frac{1}{3} + \frac{2}{9} = \frac{1}{x}.$$

$$\text{Whence } x = \frac{9}{5}.$$

24. Let x = the number of days by B alone.

$$\text{Then } \frac{1}{8} + \frac{1}{x} = \frac{5}{24}.$$

$$\text{Whence } x = 12.$$

25. Let x = the number of days by C alone.

$$\text{Then } \frac{4}{21} + \frac{5}{21} + \frac{1}{x} = \frac{4}{7}.$$

$$\text{Whence } x = 7.$$

26. From Hint, the part done by A in 6 days + the part done by B in 3 days = 1.

Let x = the number of days by B alone.

$$\text{Then } \frac{6}{8} + \frac{3}{x} = 1.$$

$$\text{Whence } x = 12.$$

27. As in Ex. 26, let x = the number of days A and B worked together.

Then
$$\frac{1}{4} + \frac{x}{4} + \frac{x}{5} = 1.$$

Whence
$$x = \frac{15}{9} = 1\frac{2}{3}.$$

29. A travels 48 miles; B travels 72 miles.

Let x = B's rate in miles per hour,
and $x - 4$ = A's rate in miles per hour.

Then
$$\frac{72}{x} = \text{B's time in hours,}$$

and
$$\frac{48}{x - 4} = \text{A's time in hours.}$$

But
$$\frac{72}{x} = \frac{48}{x - 4}.$$

Whence
$$x = 12; x - 4 = 8.$$

30. Let x = the rate going in miles per hour,
and $x + 5$ = the rate returning in miles per hour,

Then
$$\frac{360}{x} = \text{the number of hours going,}$$

and
$$\frac{360}{x + 5} = \text{the number of hours returning.}$$

But
$$\frac{360}{x} - 1 = \frac{360}{x + 5}.$$

Whence
$$x = 40; x + 5 = 45.$$

31. Let x = the rate going in miles per hour.

Then $x + 5$ = the rate returning in miles per hour,

and, as in Ex. 30,
$$\frac{120}{x} - 4 = \frac{120}{x + 5}.$$

Whence
$$x = 10; x + 5 = 15.$$

32. Let x = the rate of the automobile in miles per hour.

Then, noting that the automobile ran for $11\frac{1}{3} - 3\frac{1}{3} = 8$ hours,

$$(15)(11\frac{1}{3}) = (8)(x).$$

Whence
$$x = 21\frac{1}{4}.$$

34. Let x = the number of hours after A starts.

Then $x - \frac{5}{2}$ = B's time in hours.

But
$$\frac{7x}{2} + \frac{19}{2} \left(x - \frac{5}{2} \right) = 25.$$

Whence
$$x = 3\frac{3}{4}.$$

35. Let x = A's rate in miles per hour.

Then $x + 2$ = B's rate in miles per hour,

and since B traveled $\frac{5}{2}$ hours less than A, while each covered 60 miles,

$$\frac{60}{x} = \frac{60}{x+2} + \frac{5}{2}.$$

Whence
$$x = 6, \text{ root } - 8 \text{ rejected.}$$

36. From Hint, $5(4-x) = 3(4+x).$

Whence
$$x = 1.$$

37. Let x = the distance upstream in miles.

Then
$$\frac{x}{\frac{13}{2}} + \frac{x}{\frac{5}{2}} = 6.$$

Whence
$$x = 10\frac{5}{6}.$$

38. Let x = the number of hours returning.

Then $15 - x$ = the number of hours going,

and
$$(15-x) \left(\frac{16}{3} \right) = (x) \left(\frac{8}{3} \right).$$

Whence
$$x = 10.$$

39. Let x = the rate of the aëroplane in miles per hour.

Then
$$\frac{24}{x-15} + \frac{24}{x+15} = \frac{2}{3}.$$

Whence $(x-75)(x+3) = 0,$
and
$$x = 75, \text{ root } - 3 \text{ rejected.}$$

40. Let x = the number of sheep bought.

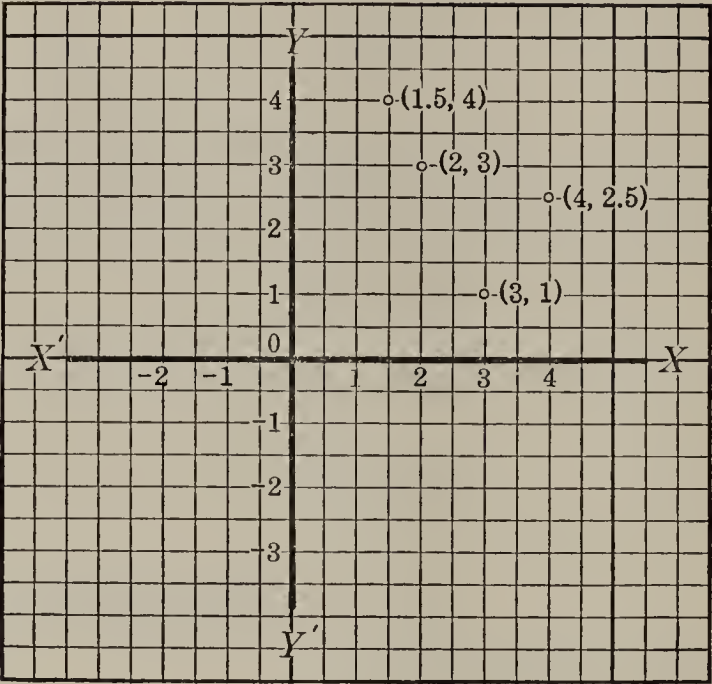
Then
$$\frac{48}{x} = \text{the price paid for each,}$$

and
$$\frac{50}{x-2} = \text{the price for which each was sold.}$$

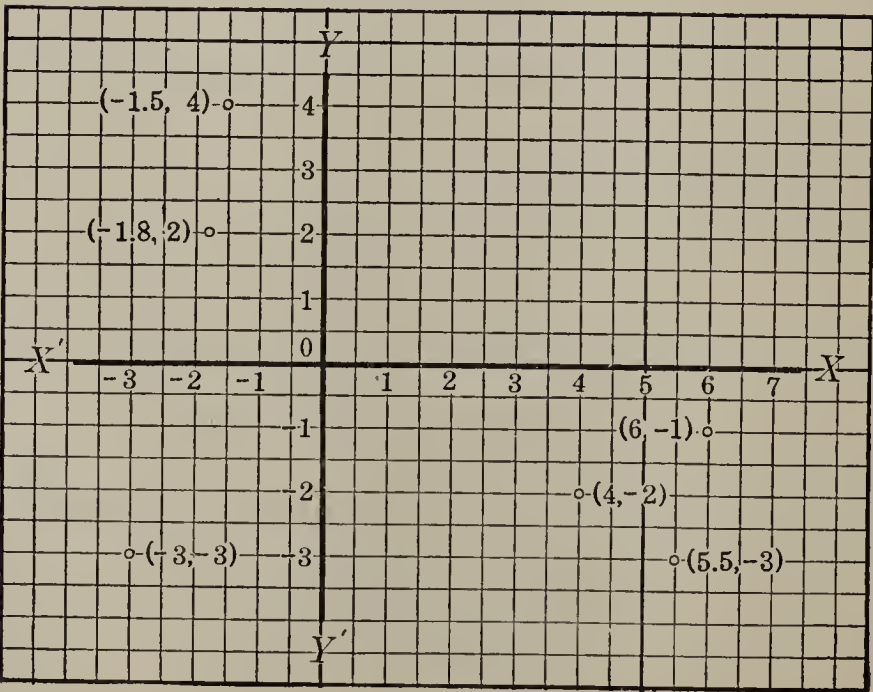
Then
$$\frac{48}{x} = \frac{50}{x-2} - 1.$$

Whence
$$x = 12, \text{ root } - 8 \text{ rejected.}$$

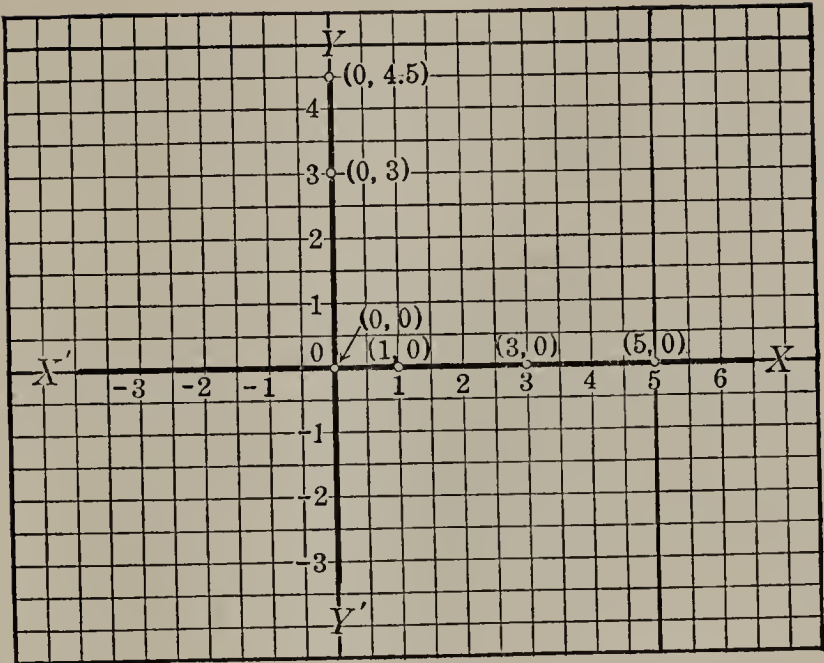
1.



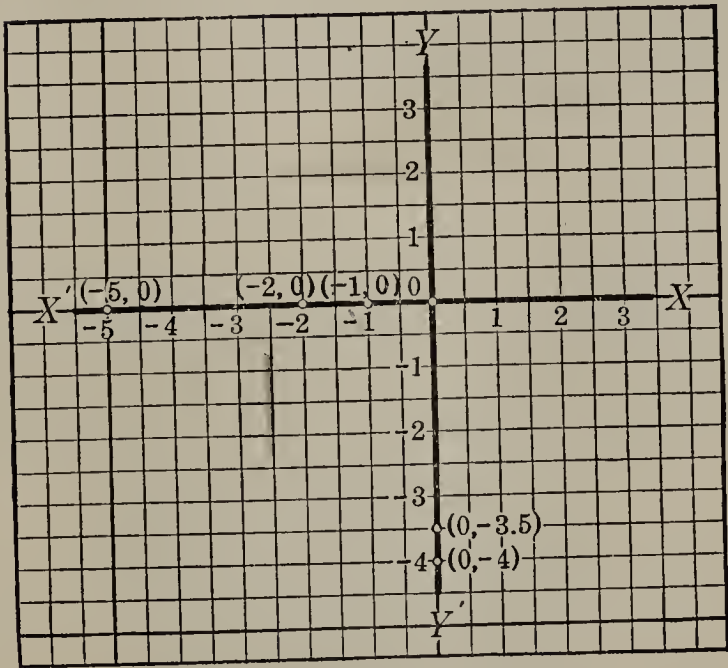
2.



3.



4.



5. If the x -distance of a point is zero, it is located on the y -axis; if both coördinates are zero, it is located at the origin.

1. For $2x + y = 8$.

If $x =$	4	2	0	5	3	1
then $y =$	0	4	8	-2	2	6

The six points are in a straight line.

Points whose x - and y -distances satisfy $2x + y = 8$ seem to lie on the straight line drawn through the points located above.

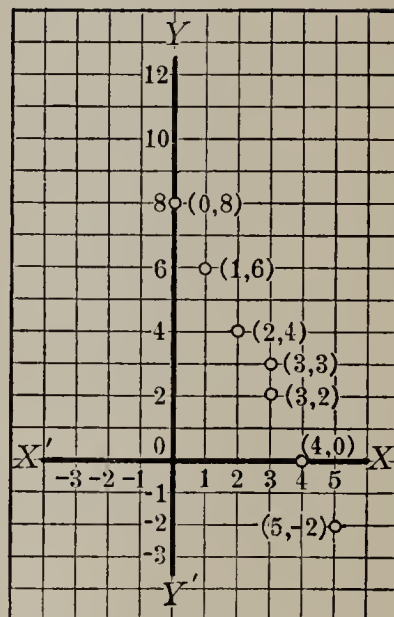
The graph of $2x + y = 8$ seems to be the straight line drawn through the points whose coördinates satisfy the equation.

The values $x = 3$, $y = 3$ do not satisfy $2x + y = 8$.

The point $(3, 3)$ is not on the graph of $2x + y = 8$.

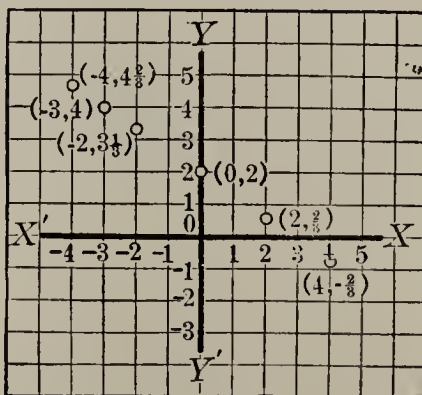
If the x - and y -distances of a point satisfy $2x + y = 8$, the point appears to be located on the graph of $2x + y = 8$.

If the x - and y -distances of a point do not satisfy $2x + y = 8$, the point is not on the graph of the equation.



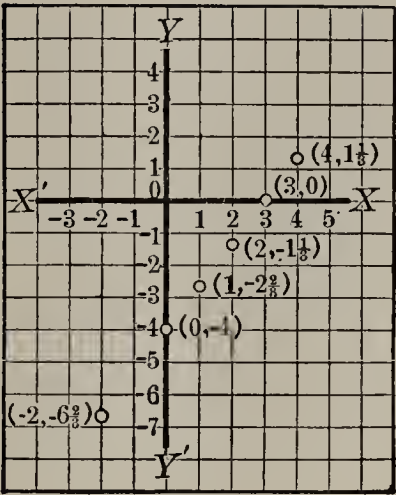
2. For $2x + 3y = 6$.

If $x =$	4	2	0	-2	-3	-4
then $y =$	$-\frac{2}{3}$	$\frac{2}{3}$	2	$3\frac{1}{3}$	4	$4\frac{2}{3}$



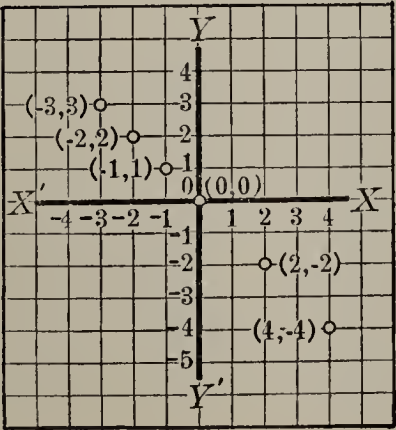
3. For $4x - 3y = 12$.

If $x =$	4	3	2	1	0	-2
then $y =$	$1\frac{1}{3}$	0	$-1\frac{1}{3}$	$-2\frac{2}{3}$	-4	$-6\frac{2}{3}$



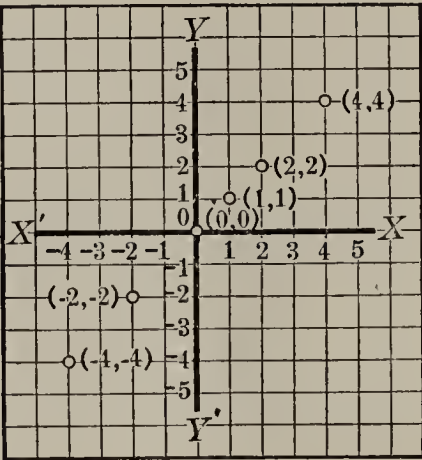
4. For $x + y = 0$.

If $x =$	4	2	0	-1	-2	-3
then $y =$	-4	-2	0	1	2	3



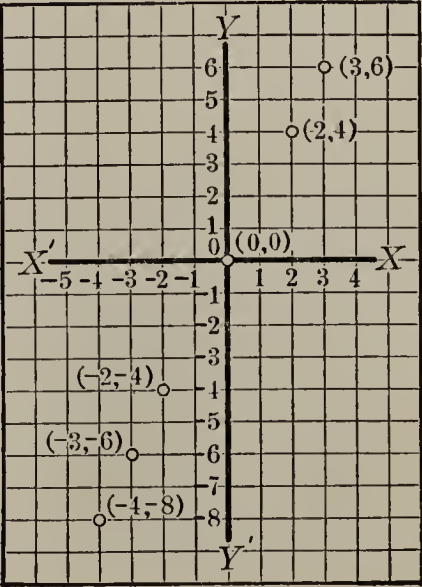
5. For $x - y = 0$.

If $x =$	4	2	1	0	- 2	- 4
then $y =$	4	2	1	0	- 2	- 4



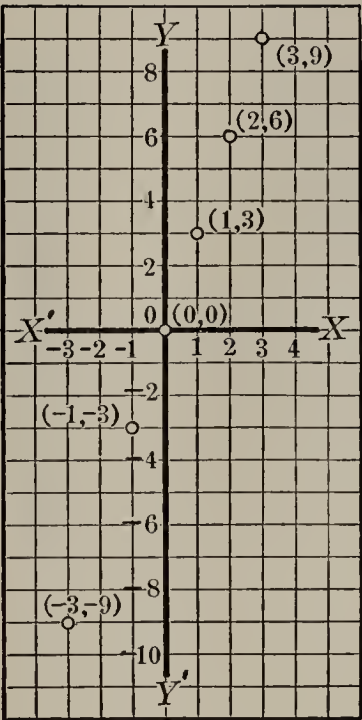
6. For $2x = y$.

If $x =$	3	2	0	- 2	- 3	- 4
then $y =$	6	4	0	- 4	- 6	- 8



7. For $y = 3x$.

If $x =$	3	2	1	0	-1	-3
then $y =$	9	6	3	0	-3	-9

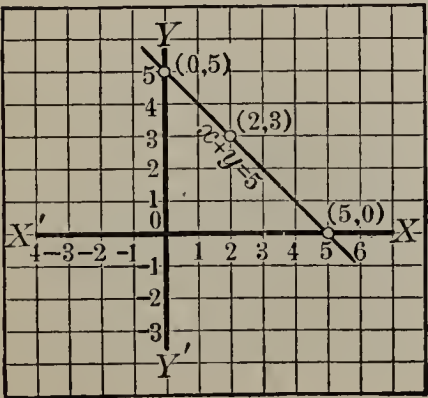


Yes, a line can be drawn through the points located in each of Exercises 2-7 inclusive.

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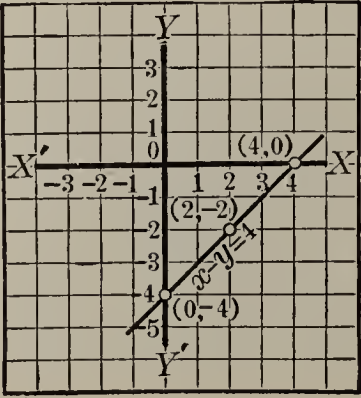
1. For $x + y = 5$.

If $x =$	5	0	2
then $y =$	0	5	3



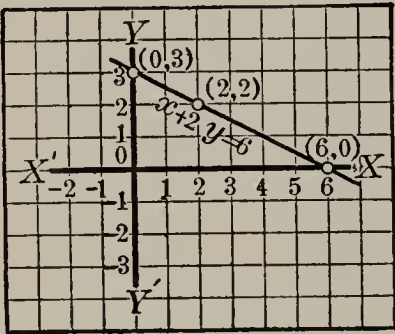
2. For $x - y = 4$.

If $x =$	0	4	2
then $y =$	-4	0	-2



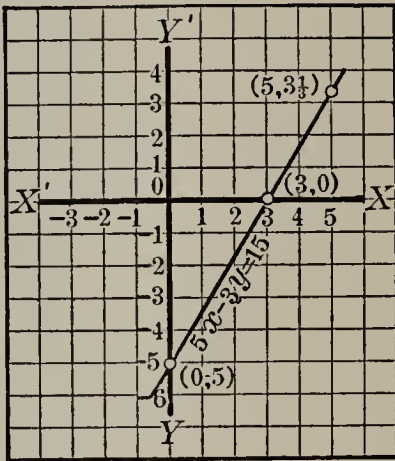
3. For $x + 2y = 6$.

If $x =$	0	6	2
then $y =$	3	0	2



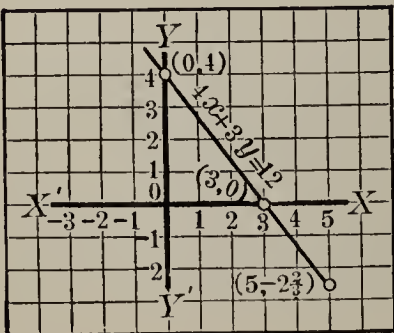
5. For $5x - 3y = 15$.

If $x =$	0	3	5
then $y =$	-5	0	$3\frac{1}{3}$



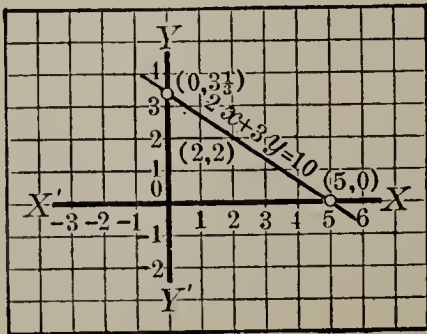
4. For $4x + 3y = 12$.

If $x =$	0	3	5
then $y =$	4	0	$-2\frac{2}{3}$



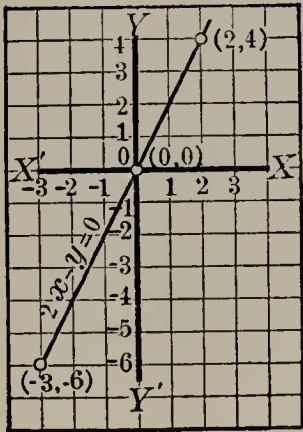
6. For $2x + 3y = 10$.

If $x =$	0	5	2
then $y =$	$3\frac{1}{3}$	0	2



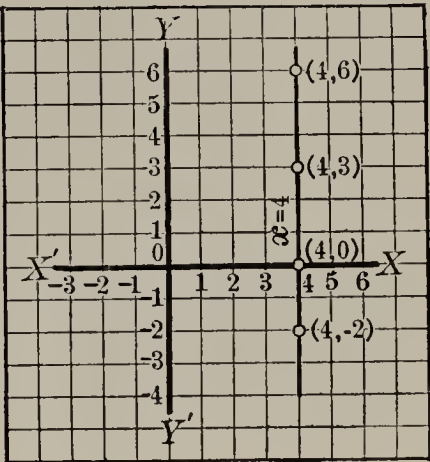
7. For $2x - y = 0$.

If $x =$	0	2	-3
then $y =$	0	4	-6



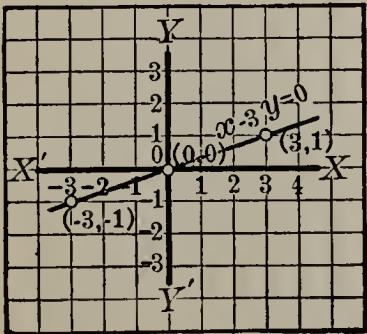
9. For $x = 4$, or $x + 0y = 4$.

If $x =$	4	4	4	4
then $y =$	3	6	0	-2



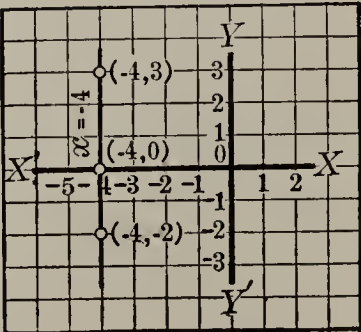
8. For $x - 3y = 0$.

If $x =$	0	3	-3
then $y =$	0	1	-1



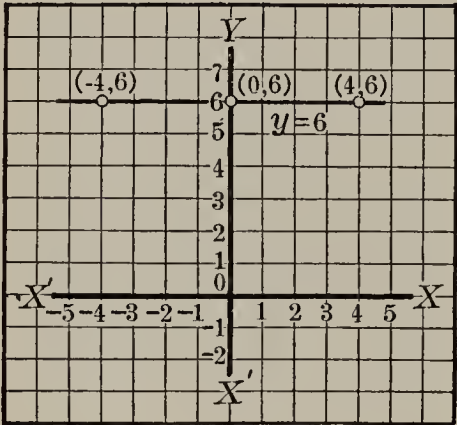
10. For $x = -4$, or $x + 0y = -4$.

If $x =$	-4	-4	-4
then $y =$	0	3	-2



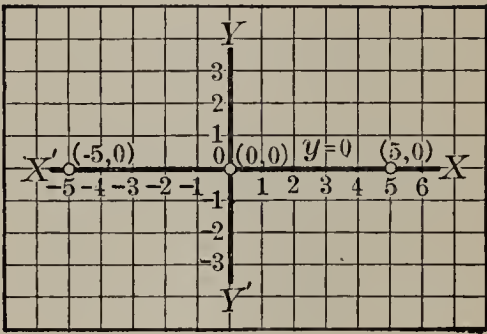
11. For $y = 6$, or $0x + y = 6$.

If $x =$	0	4	-4
then $y =$	6	6	6



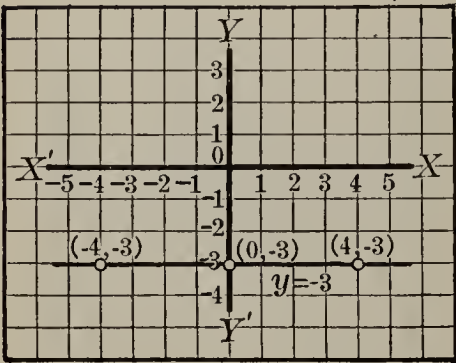
13. For $y = 0$, or $0x + y = 0$.

If $x =$	0	5	-5
then $y =$	0	0	0



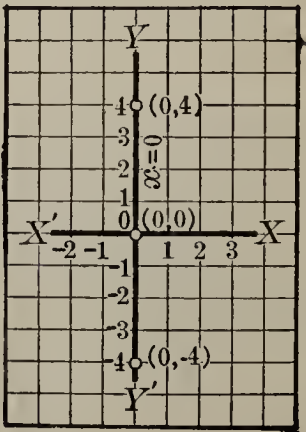
12. For $y = -3$, or $0x + y = -3$.

If $x =$	0	4	-4
then $y =$	-3	-3	-3



14. For $x = 0$, or $x + 0y = 0$.

If $x =$	0	0	0
then $y =$	0	4	-4



15. For $x = 5$, or $x + 0y = 5$.

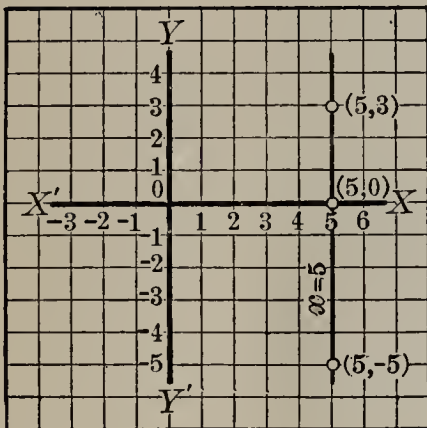
16. Yes.

17. Yes.

18. No, its values do not satisfy the equation ; no, its values do not satisfy the equation ; yes, since its values do satisfy the equation.

19. Yes, point $(6, 5)$ lies on $5x - 3y = 15$.

If $x =$	5	5	5
then $y =$	0	3	-5



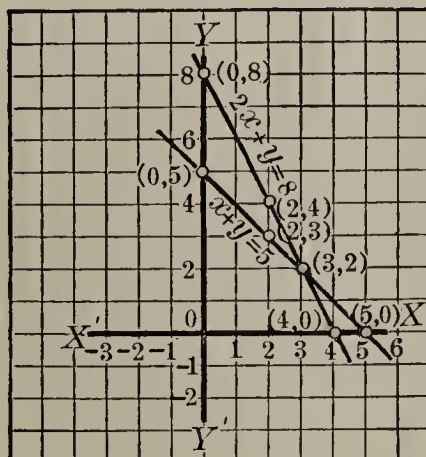
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1. For $x + y = 5$.

For $2x + y = 8$.

If $x =$	0	5	2
then $y =$	5	0	3

If $x =$	0	4	2
then $y =$	8	0	4



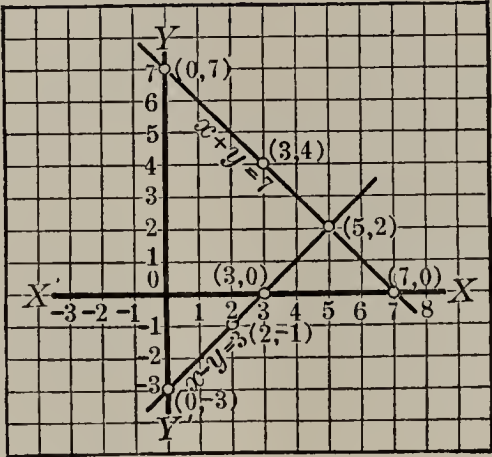
Ans. $(3, 2)$.

2. For $x + y = 7$.

For $x - y = 3$.

If $x =$	0	7	3
then $y =$	7	0	4

If $x =$	0	3	2
then $y =$	-3	0	-1



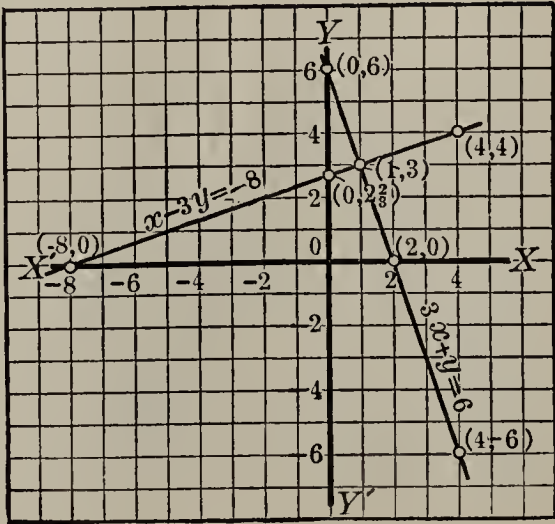
Ans. (5, 2).

3. For $3x + y = 6$.

For $x - 3y = -8$.

If $x =$	0	2	4
then $y =$	6	0	-6

If $x =$	0	-8	4
then $y =$	$2\frac{2}{3}$	0	4



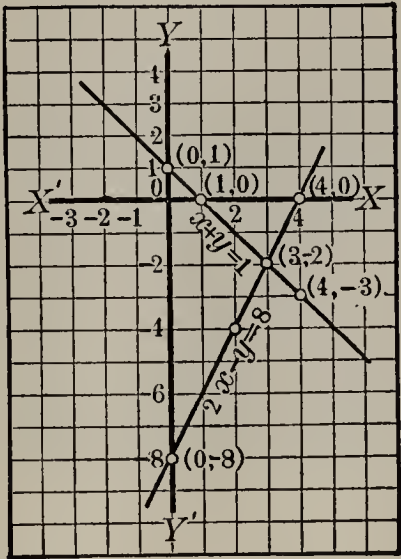
Ans. (1, 3).

4. For $x + y = 1$.

For $2x - y = 8$.

If $x =$	0	1	4
then $y =$	1	0	-3

If $x =$	0	4	2
then $y =$	-8	0	-4



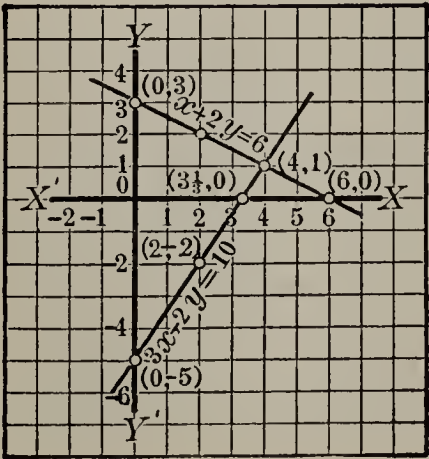
Ans. (3, -2).

5. For $3x - 2y = 10$.

For $x + 2y = 6$.

If $x =$	0	$3\frac{1}{3}$	2
then $y =$	-5	0	-2

If $x =$	0	6	2
then $y =$	3	0	2



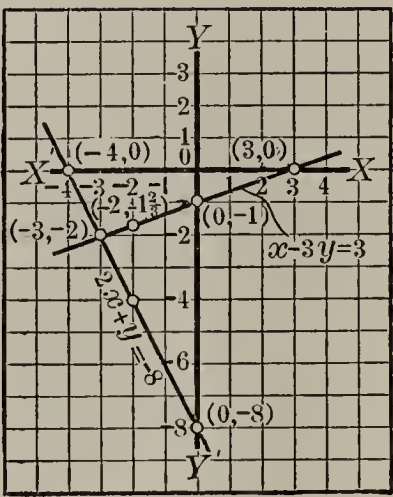
Ans. (4, 1).

6. For $x - 3y = 3$.

For $2x + y = -8$.

If $x =$	0	3	-2
then $y =$	-1	0	$-1\frac{2}{3}$

If $x =$	0	-4	-2
then $y =$	-8	0	-4



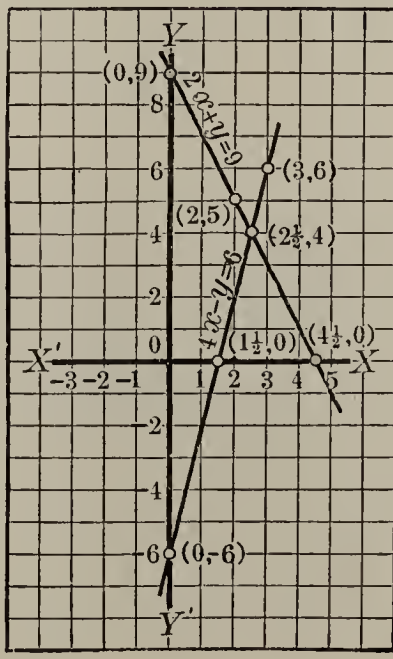
Ans. $(-3, -2)$.

7. For $4x - y = 6$.

For $2x + y = 9$.

If $x =$	0	$1\frac{1}{2}$	3
then $y =$	-6	0	6

If $x =$	0	$4\frac{1}{2}$	2
then $y =$	9	0	5



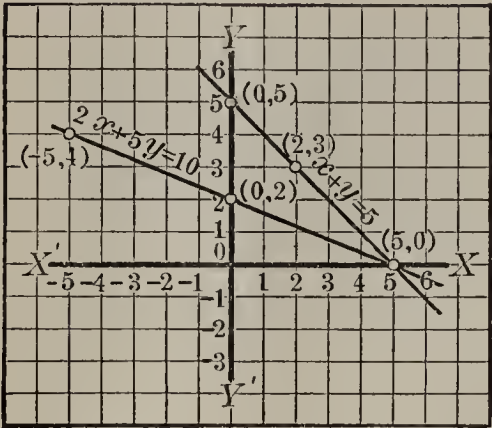
Ans. $(2\frac{1}{2}, 4)$.

8. For $2x + 5y = 10$.

For $x + y = 5$.

If $x =$	0	5	- 5
then $y =$	2	0	4

If $x =$	0	5	2
then $y =$	5	0	3



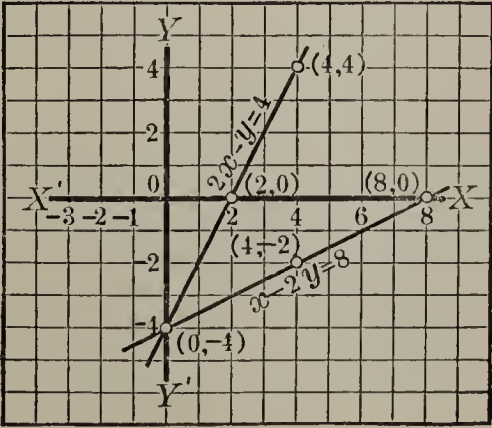
Ans. (5, 0).

9. For $x - 2y = 8$.

For $2x - y = 4$.

If $x =$	0	8	4
then $y =$	- 4	0	- 2

If $x =$	0	2	4
then $y =$	- 4	0	4



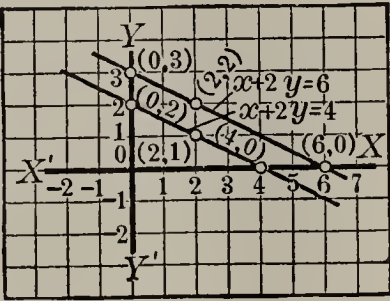
Ans. (0, - 4).

10. For $x + 2y = 4$.

For $x + 2y = 6$.

If $x =$	0	4	2
then $y =$	2	0	1

If $x =$	0	6	2
then $y =$	3	0	2



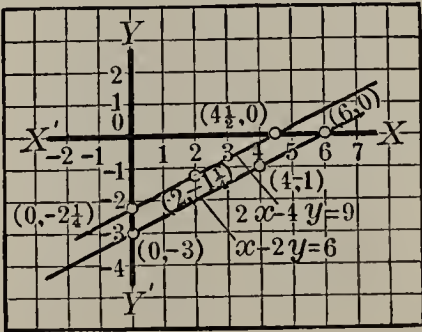
Ans. No set of roots.

11. For $x - 2y = 6$.

For $2x - 4y = 9$.

If $x =$	0	6	4
then $y =$	-3	0	-1

If $x =$	0	$4\frac{1}{2}$	2
then $y =$	$-2\frac{1}{4}$	0	$-1\frac{1}{4}$



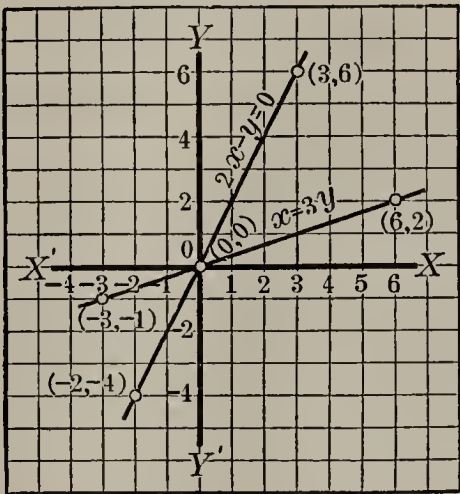
Ans. No set of roots.

12. For $2x - y = 0$.

For $x = 3y$.

If $x =$	0	3	- 2
then $y =$	0	6	- 4

If $x =$	0	6	- 3
then $y =$	0	2	- 1



Ans. (0, 0).

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1. For 1885 the curve indicates a population of 57,000,000 ; for 1905 the curve indicates a population of 84,000,000.
2. The lower rate of increase is due largely to the casualties of the Civil War.
3. For 1915 the graph indicates a population of 99,000,000 ; for 1920 the graph indicates a population of 107,000,000.
4. Decreased immigration and deaths due to the great war will probably cause a lower population figure for 1920 than that indicated by the curve.

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1. The wind velocity at midnight, August 15, was 23 miles per hour ; the wind velocity at midnight, August 16, was 76 miles per hour.
2. The wind velocity was 40 miles per hour at noon, August 16 ; at 2, 4, 6.30, 10, 12 P.M., August 17 ; at 8.30, 9.30, A.M., August 18. The wind velocity was 80 miles per hour at 10 and 11.30 P.M., August 16 ; at 12.30 and 4.30 A.M., August 17.

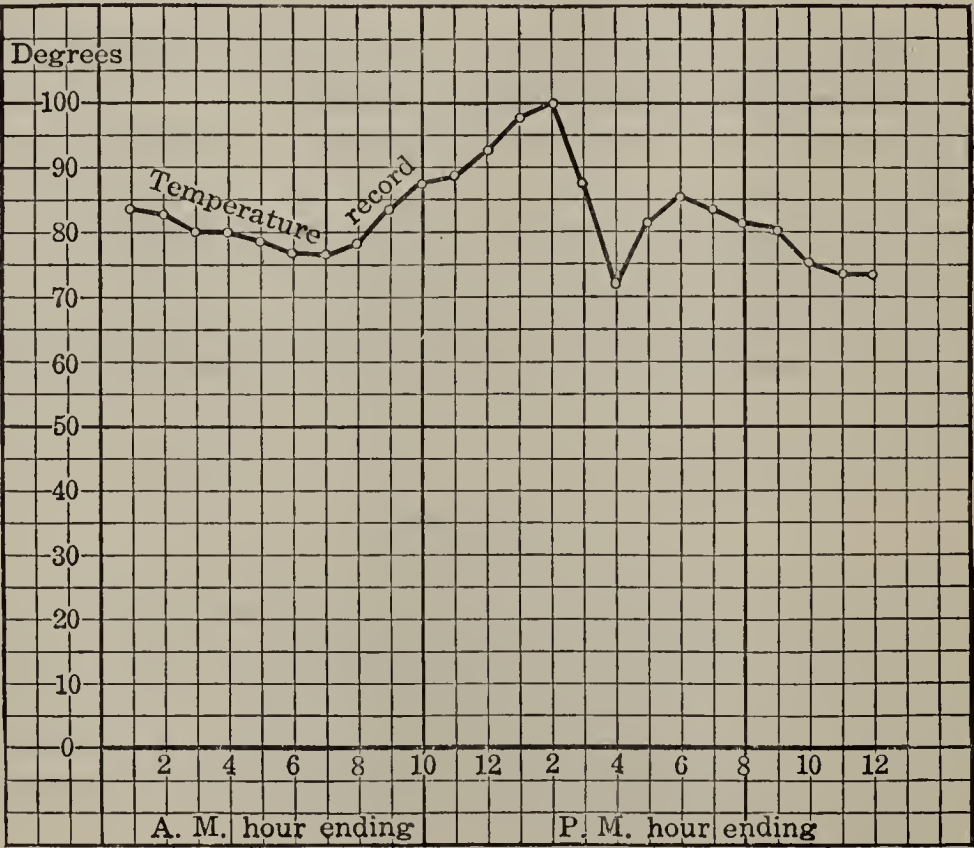
3. The barometric pressure at midnight, August 15, was 29.75 inches ; at midnight, August 16, 28.7 inches ; at 2 A.M., August 17, 28.65 inches.
4. The lowest reading of the barometer was 28.65 inches. The highest wind velocity was 91 miles per hour. These readings were recorded shortly after 2 A.M., August 17.

Page 214 (First set)

1. The graph shows a population of 67,000,000 for 1892 ; of 87,000,000 for 1907 ; of 95,000,000 for 1912.
2. Since 1901 the rate of increase in the production of wheat has practically kept pace with the rate of increase of the population.
3. Save for the last five-year interval the graph reveals a tendency toward a decrease in the number of bushels of wheat exported.
4. The graph does not indicate that under normal conditions the exportation of wheat will increase to any considerable extent. The large wheat crop of 1915 and the increased demand due to the war show an abnormal tendency for the last five-year interval.

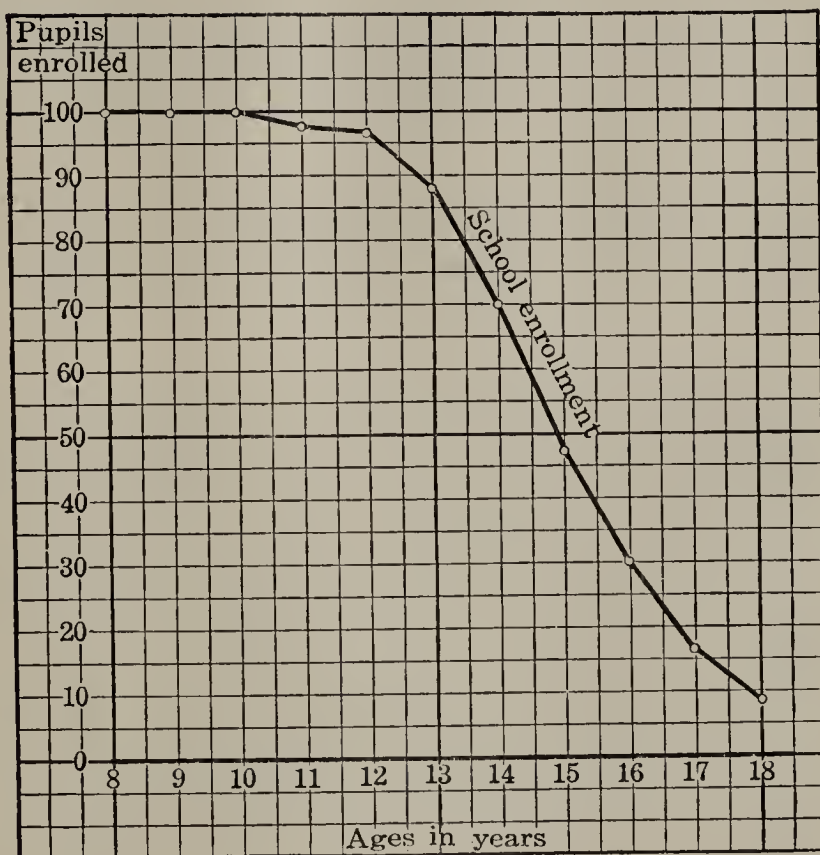
Page 214 (Second set)

1.



From the fall in temperature it appears that the storm must have occurred shortly after 2 P.M. The fall in temperature after 2 P.M. was due to atmospheric changes which took place in the storm area. The storm seems to have ceased at 4 P.M., thus allowing the sun's rays to reach the earth in the vicinity of the instrument.

2.



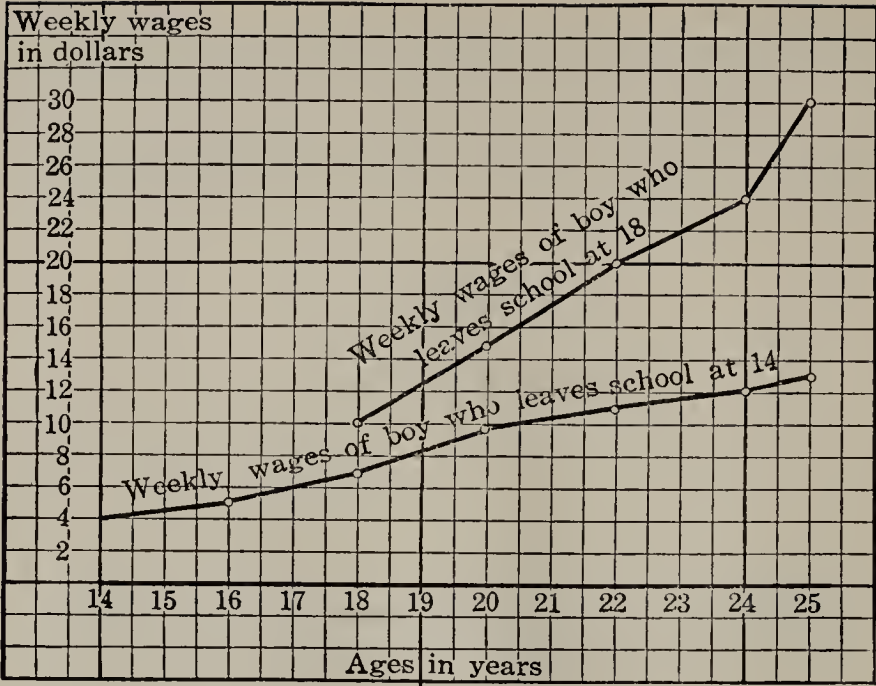
The tendency to leave school first becomes pronounced between the ages 12 and 13.

Yes, the tendency to leave school appears greater on the part of 14- and 15-year-olds.

The more decided drop in the curve between 14 and 15 shows the greater loss in school enrollment.

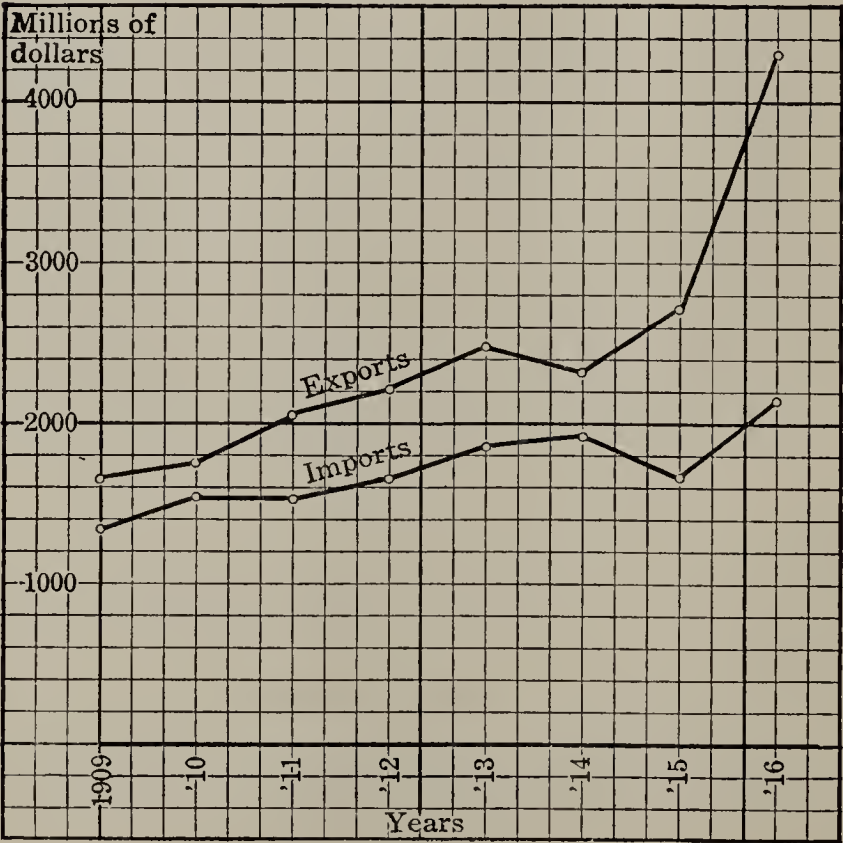
This loss is due in part to the fact that many more pupils leave school at the close of the grammar-school period than earlier.

3.



The graph shows that the boy who stays in school till he is 18 years of age receives a greater increase in weekly wages after that time than does the boy who leaves school and begins work at 14 years.

4.

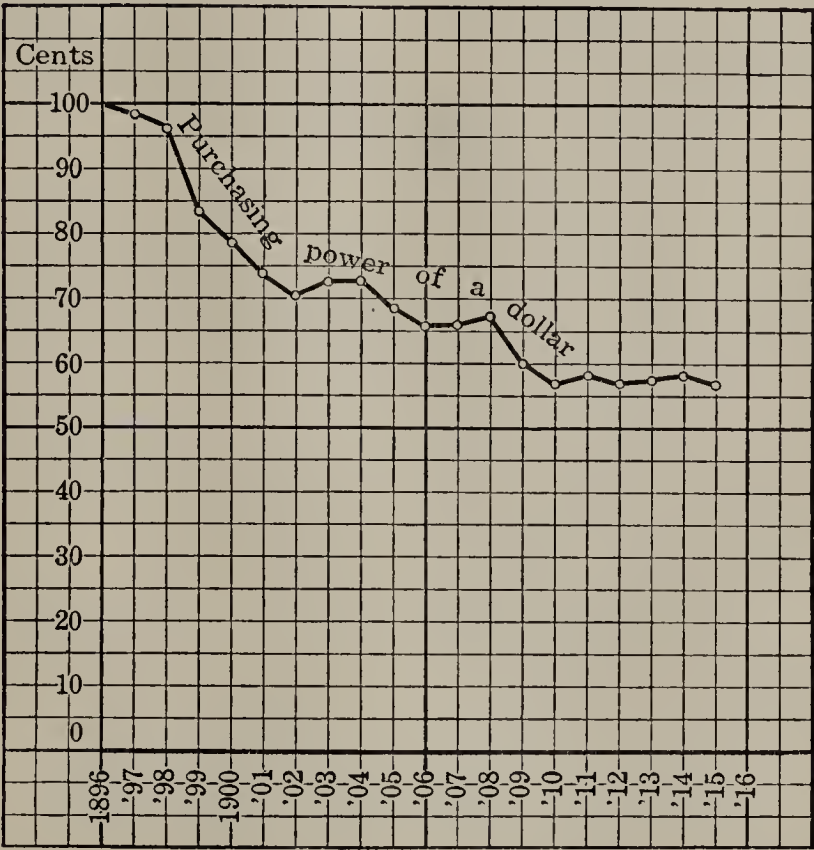


The decreased imports of 1915 were due to the fact that the United States was unable to get certain articles from certain nations at war.

The decreased exports of 1914 were due to the fact that certain nations at war were unable to get their normal amounts of merchandise from the United States.

The increased exports of 1915 are due in a large measure to the increase in the manufacture and shipping of war supplies.

5.



The purchasing power of a dollar has been almost constantly decreasing since 1896.

More merchandise could be bought for a certain sum in 1900 than in 1910. Approximately one third more could have been bought for a given sum at the earlier date.

Let x = the 1896 equivalent of \$1500 in 1910.

Then
$$\frac{1500}{x} = \frac{100}{57.8}.$$

Whence
$$x = \$867.$$

CR

Page 221

1. $x + y = 3,$ (1)
 $x - y = 1.$ (2)
 (1) + (2), $2x = 4.$
 $x = 2.$
 Substituting in (1),
 $2 + y = 3.$
 $y = 1.$
2. $x + y = 7,$ (1)
 $x - y = 1.$ (2)
 (1) + (2), $2x = 8.$
 $x = 4.$
 Substituting in (1), $y = 3.$
3. $2x + y = 9,$ (1)
 $x - y = 6.$ (2)
 (1) + (2), $3x = 15.$
 $x = 5.$
 Substituting in (1), $y = -1.$
4. $x + y = 5,$ (1)
 $x - 3y = 1.$ (2)
 (1) - (2), $4y = 4.$ (3)
 $y = 1.$ (4)
 Substituting in (1), $x = 4.$
5. $x + y = -3,$ (1)
 $x - y = 3.$ (2)
 $[(1) + (2)] \div 2,$ $x = 0,$
 and from (1), $y = -3.$
6. $x + 2y = 7,$ (1)
 $5x - 2y = 11.$ (2)
 $[(1) + (2)] \div 6,$ $x = 3,$
 and from (1), $y = 2.$
7. $2y + x = 4,$ (1)
 $3y - x = 21.$ (2)
 $[(1) + (2)] \div 5,$ $y = 5,$
 and from (1), $x = -6.$
8. $7r - s = 2,$ (1)
 $6r - s = 3.$ (2)
 (1) - (2), $r = -1,$
 and from (1), $s = -9.$
9. $3x - y = 3,$ (1)
 $5x + 2y = 16.$ (2)
 $[2(1) + (2)] \div 11,$ $x = 2,$
 and from (1), $y = 3.$
10. $4x + y = 2,$ (1)
 $x - 2y = 5.$ (2)
 $[2(1) + (2)] \div 9,$ $x = 1,$
 and from (1), $y = -2.$
11. $x + 2y = 1,$ (1)
 $3x + 10y = 4.$ (2)
 $[(2) - 3(1)] \div 4,$ $y = \frac{1}{4},$
 and from (1), $x = \frac{1}{2}.$
12. $5m - 3n = 0,$ (1)
 $15m + 12n = 75.$ (2)
 $[4(1) + (2)] \div 35,$ $m = \frac{1}{7},$
 and from (1), $n = \frac{5}{21} = \frac{2}{7}.$
13. $x - 6y = 7,$ (1)
 $12y - x = -1.$ (2)
 $[(1) + (2)] \div 6,$ $y = 1,$
 and from (1), $x = 13.$
14. $2x + 25y = 70,$ (1)
 $3x = 10y + 10.$ (2)
 Transposing (2),
 $3x - 10y = 10.$ (3)
 $[3(1) - 2(3)] \div 95,$ $y = 2,$
 and from (1), $x = 10.$
15. $4p + q = 5,$ (1)
 $p - 4q = 14.$ (2)
 $[4(1) + (2)] \div 17,$ $p = 2,$
 and from (1), $q = -3.$
16. $12x + 5y = 6,$ (1)
 $3x - 3y = 10.$ (2)
 $[(1) - 4(2)] \div 17,$ $y = -2,$
 and from (2), $x = \frac{4}{3}.$

17. $3x - 4y = 14,$ (1)
 $3y - 4x = -14.$ (2)
 $[3(1) + 4(2)] \div (-7), x = 2,$
and from (2), $y = -2.$
18. $4x + 3y = 5,$ (1)
 $9y - 8x = 0.$ (2)
 $[2(1) + (2)] \div 15, y = \frac{2}{3},$
and from (1), $x = \frac{3}{4}.$
19. $6u + 8v = 26,$ (1)
 $5u - 3v = 70.$ (2)
 $[5(1) - 6(2)] \div 58, v = -5,$
and from (1), $u = 11.$
20. $11x + 7y = 111,$ (1)
 $3y - 4x = 4.$ (2)
 $[4(1) + 11(2)] \div 61, y = 8,$
and from (1), $x = 5.$
21. $5y + 3z = 37,$ (1)
 $9z + 15y = 111.$ (2)
 $3(1) = (2).$
Hence the equations are indeterminate; any set of roots of (1), as
 $y = 2,$
 $z = 9,$
also satisfies (2).

Page 223

1. $x - 2y = 8,$ (1)
 $3x + 2y = 8.$ (2)
From (1), $x = 2y + 8.$ (3)
(2) then becomes $6y + 24 + 2y = 8.$
Whence $y = -2,$
and from (3), $x = 4.$
2. $x - 2y = -1,$ (1)
 $4x - y = 10.$ (2)
From (1), $x = 2y - 1.$ (3)
(2) then becomes $8y - 4 - y = 10.$
Whence $y = 2,$
and from (3), $x = 3.$
3. $14m - 2n = 1,$ (1)
 $n - 6m = 0.$ (2)
From (2), $n = 6m.$ (3)
(1) then becomes $14m - 12m = 1.$
Whence $m = \frac{1}{2},$
and from (3), $n = 3.$
4. $6x + 10y = 42,$ (1)
 $2y = 3x.$ (2)
From (2), $y = \frac{3}{2}x.$ (3)
(1) then becomes $6x + 15x = 42.$
Whence $x = 2,$
and from (3), $y = 3.$

5. $5x + 3y = -1,$ (1)
 $2x - 6y = 8.$ (2)
 From (2), $x = 3y + 4.$ (3)
 (1) then becomes $15y + 20 + 3y = -1.$
 Whence $y = -\frac{7}{6},$
 and from (3), $x = \frac{1}{2}.$
6. $x + y = 7,$ (1)
 $2x + 3y = 17.$ (2)
 From (1), $y = 7 - x.$ (3)
 (2) then becomes $2x + 21 - 3x = 17.$
 Whence $x = 4,$
 and from (3), $y = 3.$
7. $3x = 6y - 3,$ (1)
 $y + 4x = 5.$ (2)
 From (1), $x = 2y - 1.$ (3)
 (2) then becomes $y + 8y - 4 = 5.$
 Whence $y = 1,$
 and from (3), $x = 1.$
8. $5x + 10y = 25,$ (1)
 $5x - 3y = 9.$ (2)
 From (1), $5x = 25 - 10y.$ (3)
 (2) then becomes $25 - 10y - 3y = 9.$
 Whence $y = \frac{16}{13},$
 and from (3), $x = \frac{33}{13}.$
9. $20y - 3z = 1,$ (1)
 $z - 6y = 0.$ (2)
 From (2), $z = 6y.$ (3)
 (1) then becomes $20y - 18y = 1.$
 Whence $y = \frac{1}{2},$
 and from (3), $z = 3.$
10. $3x + 12 = 3 + y,$ (1)
 $y = x + 1.$ (2)
 (1) then becomes $3x + 12 = 3 + x + 1.$
 Whence $x = -4,$
 and from (2), $y = -3.$
11. $18 + 2k = h,$ (1)
 $h + k = -9.$ (2)
 (2) then becomes $18 + 2k + k = -9.$
 Whence $k = -9,$
 and from (2), $h = 0.$

$$12. \quad 2x = 4y + 6, \quad (1)$$

$$7x + 3y = 4. \quad (2)$$

$$\text{From (1),} \quad x = 2y + 3. \quad (3)$$

$$\text{Then (2) becomes} \quad 14y + 21 + 3y = 4.$$

$$\text{Whence} \quad y = -1,$$

$$\text{and from (3),} \quad x = 1.$$

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$$1. \quad 4x + \frac{2y}{3} = \frac{26}{3}, \quad (1)$$

$$3y - \frac{7x}{2} = -4. \quad (2)$$

$$\text{Clearing of fractions,} \quad 12x + 2y = 26, \quad (3)$$

$$-7x + 6y = -8. \quad (4)$$

$$[3(1) - (2)] \div 43, \quad x = 2,$$

$$\text{and from (4),} \quad y = 1.$$

$$2. \quad \frac{5x}{6} + \frac{y}{4} = 7, \quad (1)$$

$$\frac{2x}{3} - \frac{y}{8} = 3. \quad (2)$$

$$\text{Clearing of fractions,} \quad 20x + 6y = 168, \quad (3)$$

$$16x - 3y = 72.$$

$$[(3) + 2(4)] \div 52, \quad x = 6,$$

$$\text{and from (3),} \quad y = 8.$$

$$3. \quad x - \frac{3y}{5} = \frac{18}{5}, \quad (1)$$

$$7x + \frac{8y}{3} = -16. \quad (2)$$

$$\text{Clearing of fractions,} \quad 5x - 3y = 18, \quad (3)$$

$$21x + 8y = -48. \quad (4)$$

$$[8(3) + 3(4)] \div 103, \quad x = 0,$$

$$\text{and from (3),} \quad y = -6.$$

$$4. \quad .4x + .9y = 5.7, \quad (1)$$

$$x - \frac{y}{2} = \frac{1}{2}. \quad (2)$$

Clearing of fractions and decimals,

$$4x + 9y = 57, \quad (3)$$

$$2x - y = 1. \quad (4)$$

$$[(3) - 2(4)] \div 11, \quad y = 5,$$

$$\text{and from (4),} \quad x = 3.$$

5.

$$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}, \quad (1)$$

$$2y - 3x = 0. \quad (2)$$

From (1),

$$3y + 2x = 13. \quad (3)$$

[2(2) + 3(3)] ÷ 13,

$$y = 3,$$

and from (2),

$$x = 2.$$

6.

$$\frac{x}{3} + \frac{y}{4} = -\frac{7}{8}, \quad (1)$$

$$\frac{y}{3} - \frac{x}{4} = \frac{11}{12}. \quad (2)$$

Clearing of fractions,

$$8x + 6y = -21, \quad (3)$$

$$-3x + 4y = 11. \quad (4)$$

[2(3) - 3(4)] ÷ 25,

$$x = -3,$$

and from (4),

$$y = \frac{1}{2}.$$

7.

$$.04x + .3y = 1, \quad (1)$$

$$.5x - .25y = 4.5. \quad (2)$$

Clearing,

$$4x + 30y = 100, \quad (3)$$

$$50x - 25y = 450. \quad (4)$$

(3) ÷ 2,

$$2x + 15y = 50. \quad (5)$$

(4) ÷ 25,

$$2x - y = 18. \quad (6)$$

[(5) - (6)] ÷ 16,

$$y = 2,$$

and from (6),

$$x = 10.$$

8.

$$7x - 4y = 14, \quad (1)$$

$$\frac{2x + 7}{2} - y = 4. \quad (2)$$

From (2),

$$2x - 2y = 1. \quad (3)$$

[(1) - 2(3)] ÷ 3,

$$x = 4,$$

and from (3),

$$y = \frac{7}{2}.$$

9.

$$\frac{x}{3} + \frac{y}{4} = \frac{1}{6}, \quad (1)$$

$$\frac{x - 2}{5} - \frac{y + 1}{2} = \frac{1}{2}. \quad (2)$$

Clearing,

$$4x + 3y = 2, \quad (3)$$

$$2x - 5y = 14. \quad (4)$$

[(3) - 2(4)] ÷ 13,

$$y = -2,$$

and from (4),

$$x = 2.$$

$$10. \quad \frac{2x+6}{5} + \frac{y+4}{5} = 3, \quad (1)$$

$$\frac{7x+1}{3} - \frac{11y-4}{7} = 4. \quad (2)$$

$$\text{From (1),} \quad 2x + y = 5. \quad (3)$$

$$\text{From (2),} \quad 49x - 33y = 65. \quad (4)$$

$$[33(3) + (4)] \div 115, \quad x = 2,$$

$$\text{and from (3),} \quad y = 1.$$

$$11. \quad \frac{x+y}{2} - 8 = \frac{x-y}{3}, \quad (1)$$

$$\frac{x-y}{4} + \frac{x+y}{3} = 11. \quad (2)$$

$$\text{From (1),} \quad x + 5y = 48. \quad (3)$$

$$\text{From (2),} \quad 7x + y = 132. \quad (4)$$

$$[(3) - 5(4)] \div (-34), \quad x = 18,$$

$$\text{and from (4),} \quad y = 6.$$

$$12. \quad \frac{x}{6} - \frac{y}{4} = 1, \quad (1)$$

$$\frac{4+5x}{11} - \frac{3-2y}{5} + 4 = 0. \quad (2)$$

$$\text{From (1),} \quad 2x - 3y = 12. \quad (3)$$

$$\text{From (2),} \quad 25x + 22y = -207. \quad (4)$$

$$[25(3) - 2(4)] \div (-119), \quad y = -6,$$

$$\text{and from (3),} \quad x = -3.$$

$$13. \quad \frac{1}{x} + \frac{1}{y} = \frac{5}{6}, \quad (1)$$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{6}. \quad (2)$$

$$\text{Adding,} \quad \frac{2}{x} = 1; x = 2.$$

$$\text{Substituting in (1),} \quad \frac{1}{y} = \frac{1}{3}; y = 3.$$

$$14. \quad \frac{1}{x} + \frac{1}{y} = \frac{4}{15}, \quad (1)$$

$$\frac{3}{x} + \frac{2}{y} = \frac{19}{15}. \quad (2)$$

$$[2(1) - (2)], \quad -\frac{1}{x} = -\frac{11}{15}; x = \frac{15}{11},$$

$$\text{and from (1),} \quad \frac{1}{y} = -\frac{7}{15}; y = -\frac{15}{7}.$$

$$15. \quad \frac{6}{x} + \frac{7}{y} + \frac{3}{2} = 0, \quad (1)$$

$$\frac{7}{x} - \frac{6}{y} = \frac{16}{3}. \quad (2)$$

$$[6(1) + 7(2)] \div 85, \quad \frac{1}{x} = \frac{1}{3}; x = 3,$$

$$\text{and from (1),} \quad \frac{7}{y} = -\frac{7}{2}; y = -2.$$

$$16. \quad \frac{5}{x} + 12 = 17, \quad (1)$$

$$\frac{2}{x} - 3y = 0. \quad (2)$$

$$\text{From (1),} \quad \frac{1}{x} = 1; x = 1.$$

$$\text{Then (2) becomes} \quad 3y = 2; y = \frac{2}{3}.$$

$$17. \quad \frac{x-y}{2} = \frac{25}{6} - \frac{x+y}{3}, \quad (1)$$

$$\frac{x+y-9}{2} + \frac{x-y+6}{3} = 0. \quad (2)$$

$$\text{From (1),} \quad 5x - y = 25.$$

$$\text{From (2),} \quad 5x + y = 15.$$

$$\text{Adding, etc.,} \quad x = 4,$$

$$\text{and from (2),} \quad y = -5.$$

$$18. \quad \frac{x-2}{5} - \frac{y-10}{4} = \frac{10-x}{3}, \quad (1)$$

$$\frac{y+2}{3} - \frac{2x+y}{16} = \frac{x+13}{8}. \quad (2)$$

$$\text{From (1),} \quad 32x - 15y = 74. \quad (3)$$

$$\text{From (2),} \quad -12x + 13y = 46. \quad (4)$$

$$[3(3) + 8(4)] \div 59, \quad y = 10,$$

$$\text{and from (3),} \quad x = 7.$$

$$19. \quad \frac{x+2}{7} + 8 - 2x = \frac{x-y}{4}, \quad (1)$$

$$2y - \frac{3x-2y}{3} = 3x+4. \quad (2)$$

$$\text{From (1),} \quad -59x + 7y = -232. \quad (3)$$

$$\text{From (2),} \quad -12x + 8y = 12. \quad (4)$$

$$[8(3) - 7(4)] \div (-388), \quad x = 5,$$

$$\text{and from (4),} \quad y = 9.$$

20. From Hint,

$$x + 9y = 28. \quad (4)$$

$$8x - y = 5. \quad (5)$$

From (5),

$$y = 8x - 5. \quad (6)$$

(4) then becomes

$$x + 72x - 45 = 28.$$

Whence

$$x = 1,$$

and from (6),

$$y = 3.$$

21.

$$\frac{4x + y}{6x + y} = \frac{2}{5}, \quad (1)$$

$$x + \frac{10y}{3} = \frac{71}{3}. \quad (2)$$

From (1),

$$8x + 3y = 0. \quad (3)$$

From (2),

$$3x + 10y = 71. \quad (4)$$

From (3),

$$y = -\frac{8}{3}x. \quad (5)$$

(4) then becomes

$$3x - \frac{80}{3}x = 71.$$

Whence

$$x = -3,$$

and from (5),

$$y = 8.$$

22.

$$\frac{4x}{3y + 2} = 4, \quad (1)$$

$$\frac{2x}{4x - 3y + 1} = \frac{1}{4}. \quad (2)$$

From (1),

$$4x - 12y = 8,$$

or

$$x = 3y + 2. \quad (3)$$

From (2),

$$4x + 3y = 1,$$

or

$$4(3y + 2) + 3y = 1.$$

Whence

$$y = -\frac{7}{13},$$

and from (3),

$$x = \frac{3}{5}.$$

23.

$$\frac{3x - 4y + 2}{x - y} = 3, \quad (1)$$

$$\frac{x + y}{x} = \frac{5}{3}. \quad (2)$$

From (1),

$$3x - 4y + 2 = 3x - 3y,$$

or

$$y = 2.$$

(2) then becomes

$$3(x + 2) = 5x,$$

or

$$x = 3.$$

24.

$$\frac{x - 3}{y + 1} = \frac{x - 6}{y - 2}, \quad (1)$$

$$\frac{x + y - 5}{x - y + 1} = 3. \quad (2)$$

From (1), $(x - 3)(y - 2) = (x - 6)(y + 1),$
or $-3x + 3y = -12.$

Whence $x = y + 4. \quad (3)$

From (2), $x + y - 5 = 3(x - y + 1),$
 $-2x + 4y = 8,$

or using (3), $-2y - 8 + 4y = 8.$

Whence $y = 8,$
and from (3), $x = 12.$

25. $\frac{5x + 3y - 5}{3} = y + \frac{5}{3}, \quad (1)$

$\frac{25x}{10x + 2y} = 1. \quad (2)$

From (1), $5x = 10; x = 2.$
(2) then becomes $50 = 20 + 2y,$
 $y = 15.$

26. $\frac{r + 5}{r + 1} = \frac{s + 2}{s - 2}, \quad (1)$

$\frac{11r + 4}{r + s + 1} = 4. \quad (2)$

From (1), $-4r + 4s = 12. \quad (3)$

From (2), $7r - 4s = 0. \quad (4)$
[(3) + (4)] $\div 3,$ $r = 4,$
and from (4), $s = 7.$

27. $\frac{2}{x + y + 2} = \frac{1}{3x + 5y - 5}, \quad (1)$

$\frac{x}{4} + \frac{y}{2} = 1. \quad (2)$

From (1), $5x + 9y = 12. \quad (3)$

From (2), $x + 2y = 4. \quad (4)$
 $5(4) - (3),$ $y = 8,$
and from (4), $x = -12.$

28. $\frac{4}{x - 1} = \frac{3}{1 - y}, \quad (1)$

$\frac{5}{2y - 5} = \frac{7}{2x - 39}. \quad (2)$

From (1), $3x + 4y = 7. \quad (3)$

From (2), $10x - 14y = 160, \quad (4)$
or $5x - 7y = 80. \quad (5)$

$[5(3) - 3(5)] \div 41,$ $y = -5,$
and from (3), $x = 9.$

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1. Let x and y represent the numbers.
 Then $x + y = 36$,
 and $x - y = 20$.
 Whence $x = 28$; $y = 8$.
2. Let $x =$ the greater number,
 and $y =$ the less.
 Then $\frac{x}{y} = 12$,
 and $x + y = 39$.
 Whence $y = 3$; $x = 36$.
3. Let $x =$ the greater number,
 and $y =$ the less.
 Then $x - y = 28$,
 and $\frac{x}{y} = 5$.
 Whence $y = 7$; $x = 35$.
4. Let $\frac{x}{y}$ represent the fraction.
 Then $\frac{x + 6}{y + 12} = \frac{2}{3}$,
 and $\frac{x}{y} = \frac{3}{4}$.
 Whence $x = 18$; $y = 24$.
 Therefore $\frac{x}{y} = \frac{18}{24}$.
5. Let $n =$ the number of nickels,
 and $q =$ the number of quarters.
 Then $n + q = 63$,
 and $5n + 25q = 835$.
 Whence $n = 37$; $q = 26$.
6. Let $\frac{x}{y} =$ the fraction.
 Then $x + y = 11$,
 and $\frac{x - \frac{2}{3}}{y + \frac{4}{3}} = \frac{1}{4}$.
 Whence $x = 3$; $y = 8$.
 Therefore $\frac{x}{y} = \frac{3}{8}$.

7. Let $\frac{x}{y}$ = the fraction.
 Then $y - x = 11$,
 and $\frac{x + \frac{1}{2}}{y - \frac{3}{4}} = \frac{6}{19}$.
 Whence $x = 4$; $y = 15$.
 $\frac{x}{y} = \frac{4}{15}$.
8. Let x = the greater weight in pounds,
 and y = the less weight in pounds.
 Then $8x = 12y$,
 and $6(x + 8) = 12y$.
 Whence $x = 24$; $y = 16$.
9. Let x = the first weight in pounds,
 and y = the second weight in pounds.
 Then $10x = 8y$,
 and $10(x - 4) = 6y$.
 Whence $x = 16$; $y = 20$.
10. Let a and b be the ages in years now.
 Then $a + 20 = 2(b + 20)$,
 and $a - 10 = 8(b - 10)$.
 Whence $b = 15$; $a = 50$.
11. Let l and w be the dimensions in feet.
 Then $l = 2w + 8$,
 and $2(l + w) = 184$.
 Whence $w = 28$; $l = 64$.
12. Let x = the number of dollars at 6%,
 and y = the number of dollars at 5%.
 Then $x + y = 1500$,
 and $.06x + .05y = 81$.
 Whence $x = 600$; $y = 900$.
13. Let x = the number of dollars at $4\frac{1}{2}\%$,
 and y = the number of dollars at $3\frac{1}{2}\%$.
 Then $x + y = 3000$,
 and $.045x = 2(.035y) - 3$.
 Whence $x = 1800$; $y = 1200$.

14. Let x = the number of dollars at 4%,
 and y = the number of dollars at 6%.
 Then $x + y = 5000$,
 and $4(.04x) = 3(.06y) - 16$.
 Whence $x = 2600$; $y = 2400$.
15. Let r and m be the values in cents.
 Then $5r = 10m - 5$,
 and $12m = 4r + 100$.
 Whence $m = 24$; $r = 47$.
16. Let x = the value in cents on Aug. 1, 1914,
 and y = the value in cents on Jan. 1, 1916.
 Then $6y = 5x$,
 and $10x - 10y = 40$.
 Whence $y = 20$; $x = 24$.
18. Let x = the tens' digit,
 and y = the units' digit.
 Then $x + y = 10$,
 and $10x + y + 54 = 10y + x$.
 Whence $x = 2$; $y = 8$. The number is 28.
19. As in Ex. 18, $x = \frac{1}{2}y$.
 $10x + y + 36 = 10y + x$.
 Whence $x = 4$; $y = 8$. The number is 48.
20. As in Ex. 18, $\frac{10x + y}{x + y} = 4$.
 $2(10x + y) - 9 = 10y + x$.
 Whence $x = 3$; $y = 6$. The number is 36.
21. As in Ex. 18, $\frac{10x + y + 4}{x + y} = 3$.
 $\frac{2(10x + y)}{x} = 29$.
 Whence $x = 2$; $y = 9$. The number is 29.
22. As in Ex. 18, $\frac{10x + y}{x + y} = 7$.
 $\frac{10y + x}{x + y + 3} = 3$.
 Whence $x = 6$; $y = 3$. The number is 63.
23. $\frac{1}{6}$; $\frac{1}{7a}$; $\frac{3}{2}$; $\frac{4x}{3}$; $\frac{2}{5}$; $\frac{5}{17}$.

24. Let
and

x = the less number,
 y = the greater number.

Then $\frac{1}{x}$ will be greater than $\frac{1}{y}$.

Therefore $\frac{1}{x} + \frac{1}{y} = \frac{11}{28}$,

and $\frac{1}{x} - \frac{1}{y} = \frac{3}{28}$.

Whence $x = 4$; $y = 7$.

25. Let
and

x = the greater number,
 y = the less.

Then $\frac{1}{y}$ will be greater than $\frac{1}{x}$.

Therefore $\frac{1}{y} - \frac{1}{x} = \frac{1}{24}$,

and $\frac{x}{y} = \frac{16}{15}$.

Whence $x = \frac{8}{5}$; $y = \frac{3}{2}$.

26. Let

x = the original weight in the first pan
in grams,

and y = the original weight in the second
pan in grams.

Then $x - 5 = \frac{1}{2}(y + 5)$,

and $x + 15 = y - 15$.

Whence $x = 45$; $y = 75$.

27. Let
and

a = the number of dollars A had at first,
 b = the number of dollars B had at first.

Then $2(a - 30) = b + 30$,

and $\frac{1}{5}(a - 30 + 150) = b + 30 - 150$.

Whence $a = 130$; $b = 170$.

28. Let

f = the circumference of the fore wheel
in feet,

and r = the circumference of the rear wheel
in feet.

Then $f = r - \frac{3}{2}$,

and $\frac{286}{f} = \frac{325}{r}$.

Whence $f = 11$; $r = 12\frac{1}{2}$.

29. Let a = the number of days required by A alone,
and b = the number of days required by B alone.

Then
$$\frac{1}{a} + \frac{1}{b} = \frac{5}{36},$$

and
$$\frac{3}{a} + \frac{3}{b} + \frac{7}{a} = 1.$$

Whence
$$a = 12; b = 18.$$

30. From Hint,
$$\frac{10}{x + y} = 2,$$

and
$$\frac{10}{x - y} = 4.$$

Whence
$$x = 1\frac{5}{4}; y = \frac{5}{4}.$$

31. Let x = the rate of boat in still water in miles per hour,
and y = the rate of current in miles per hour.

Then
$$\frac{45}{x + y} = 3,$$

and
$$\frac{15}{x - y} = 3.$$

Whence
$$x = 10; y = 5.$$

32. Let x = the rate of current in miles per hour,
and y = the distance from B to A in miles.

Then
$$\frac{y}{1\frac{1}{2} + x} = 12,$$

and
$$\frac{\frac{y}{2}}{1\frac{1}{2} - x} = 9.$$

Whence
$$x = 1\frac{1}{2}; y = 108.$$

33. Let x = the rate of current in miles per hour,
and y = the distance from A to C in miles.

Then
$$\frac{y}{12 + x} = 7,$$

and
$$\frac{y - 36}{12 - x} = 5.$$

Whence
$$x = 1; y = 91.$$

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$$1. \quad x + y = 3a, \quad (1)$$

$$x - y = a. \quad (2)$$

$$[(1) + (2)] \div 2, \quad x = 2a.$$

$$\text{From (1),} \quad y = a.$$

$$2. \quad 5x + 4y = 17a, \quad (1)$$

$$8x + y = 11a. \quad (2)$$

$$[4(2) - (1)] \div 27, \quad x = a.$$

$$\text{From (2),} \quad y = 3a.$$

$$3. \quad 3x - y = 10b, \quad (1)$$

$$4x + 9y = 3b. \quad (2)$$

$$\text{From (1),} \quad y = 3x - 10b. \quad (3)$$

$$\text{In (2),} \quad 4x + 27x - 90b = 3b.$$

$$\text{Whence} \quad x = 3b,$$

$$\text{and from (3),} \quad y = -b.$$

$$4. \quad 5x - 4y = 10a - 4, \quad (1)$$

$$x - 2ay = 0. \quad (2)$$

$$\text{From (2),} \quad x = 2ay. \quad (3)$$

$$\text{In (1),} \quad 10ay - 4y = 10a - 4.$$

$$\text{Whence} \quad y = 1,$$

$$\text{and in (3),} \quad x = 2a.$$

$$5. \quad 3p + 4q - a = p - q + 22a, \quad (1)$$

$$p + a - q = 3p - 2q + 8a. \quad (2)$$

$$\text{From (1),} \quad 2p + 5q = 23a. \quad (3)$$

$$\text{From (2),} \quad -2p + q = 7a. \quad (4)$$

$$[(3) + (4)] \div 6, \quad q = 5a,$$

$$\text{and from (3),} \quad p = -a.$$

$$6. \quad 3x + 4y = 6c, \quad (1)$$

$$\frac{x}{c} - \frac{3y}{4c} = 2. \quad (2)$$

$$\text{From (2),} \quad 4x - 3y = 8c. \quad (3)$$

$$[3(1) + 4(3)] \div 25, \quad x = 2c,$$

$$\text{and from (1),} \quad y = 0.$$

$$7. \quad \frac{2x}{5} + \frac{2y}{3} = \frac{6a}{5}, \quad (1)$$

$$\frac{y}{2} + \frac{x}{2} = a. \quad (2)$$

$$\text{From (1),} \quad 6x + 10y = 18a. \quad (3)$$

From (2), $x + y = 2a.$
 Whence $y = 2a - x.$ (4)
 Then in (3), $6x + 20a - 10x = 18a.$

Whence $x = \frac{a}{2},$

and in (4), $y = \frac{3a}{2}.$

8. $7.5m + 3n = 6b,$ (1)

$.25m + .5n = 0.$ (2)

From (1), $75m + 30n = 60b,$
 or $5m + 2n = 4b.$ (3)

From (2), $m = -2n.$ (4)

Then in (3), $-10n + 2n = 4b.$

Whence $n = -\frac{1}{2}b,$
 and in (4), $m = b.$

9. $3x - 3y - 4a = x - b,$ (1)

$x + y + a + b = 2(a + b).$ (2)

From (1), $2x - 3y = 4a - b.$ (3)

From (2), $x + y = a + b.$ (4)

$[(3) + 3(4)] \div 5,$ $x = \frac{7a + 2b}{5},$

and from (4), $y = \frac{3b - 2a}{5}.$

10. $\frac{x}{2a} - \frac{2y}{a} = -10,$ (1)

$\frac{3x}{a} - \frac{7y}{4a} = \frac{3}{2}.$ (2)

From (1), $x - 4y = -20a.$

Whence $x = 4y - 20a.$ (3)

From (2), $12x - 7y = 6a,$
 and using (3), $48y - 240a - 7y = 6a.$

Whence $y = 6a,$
 and from (3), $x = 4a.$

11. $x + y = a,$

$x - y = b.$

Adding, $x = \frac{a + b}{2}.$

Subtracting, $y = \frac{a - b}{2}.$

12.

$$ax + 3y = 3 + 6a, \quad (1)$$

$$a^2x + y = 5a. \quad (2)$$

$$a(1) - (2),$$

or

$$3ay - y = -2a + 6a^2,$$

$$(3a - 1)y = (3a - 1)2a.$$

Whence

$$y = 2a,$$

and from (2),

$$x = \frac{3}{a}.$$

13.

$$cx + y = 3, \quad (1)$$

$$c(y - 3) = x. \quad (2)$$

From (1),

$$y = 3 - cx. \quad (3)$$

In (2),

$$c(-cx) = x.$$

Whence

$$x = 0,$$

and from (3),

$$y = 3.$$

14.

$$4x - 3y = 12(a - b), \quad (1)$$

$$3x - 2y = 9a - 8b. \quad (2)$$

$$3(2) - 2(1),$$

and from (2),

$$x = 3a,$$

$$y = 4b.$$

15.

$$4x + 2y = a, \quad (1)$$

$$2x + 4y = b. \quad (2)$$

$$[2(2) - (1)] \div 6,$$

$$y = \frac{2b - a}{6},$$

and from (2),

$$x = \frac{2a - b}{6}.$$

16.

$$2x - ay = b, \quad (1)$$

$$8x - by = 4a. \quad (2)$$

$$4(1) - (2),$$

$$(b - 4a)y = 4(b - a).$$

$$y = \frac{4(b - a)}{b - 4a},$$

and from (1),

$$x = \frac{b^2 - 4a^2}{2(b - 4a)}.$$

17.

$$\frac{1}{x} - \frac{1}{y} = a, \quad (1)$$

$$\frac{3}{x} - \frac{2}{y} = b. \quad (2)$$

$$3(1) - (2),$$

$$-\frac{1}{y} = 3a - b. \quad (3)$$

$$y = \frac{1}{b - 3a}.$$

Substituting from (3) in (1), $\frac{1}{x} = b - 2a.$

$$x = \frac{1}{b - 2a}.$$

$$18. \quad \frac{a}{x} + \frac{b}{y} = c, \quad (1)$$

$$\frac{a}{x} - \frac{b}{y} = d. \quad (2)$$

Adding, $\frac{2a}{x} = c + d.$

$$x = \frac{2a}{c + d}.$$

$[(1) - (2)] \div 2,$ $\frac{b}{y} = \frac{c - d}{2}.$

$$y = \frac{2b}{c - d}.$$

$$19. \quad \begin{aligned} ax + by &= a + b, \\ ax - by &= a - b. \end{aligned}$$

Adding, etc., $x = \frac{2a}{2a} = 1.$

Subtracting, etc., $y = \frac{2b}{2b} = 1.$

$$20. \quad ax - by = c. \quad (1)$$

$$3ax - 2by = 4c. \quad (2)$$

$[(2) - 3(1)] \div b,$ $y = \frac{c}{b},$

and from (1), $x = \frac{2c}{a}.$

$$21. \quad ax + by = c, \quad (1)$$

$$hx + ky = m. \quad (2)$$

$h(1) - a(2),$ $(bh - ak)y = ch - am.$

$$y = \frac{ch - am}{bh - ak}.$$

$k(1) - b(2),$ $(ak - bh)x = ck - bm.$

$$x = \frac{ck - bm}{ak - bh}.$$

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1. Solving,

$$x = \frac{20}{a-2}.$$

2. Let

 x = the decrease of the base in feet.

Then

$$\frac{(a+h)(b-x)}{2} = \frac{ab}{2}.$$

Whence

$$x = \frac{bh}{a+h}.$$

3. Let
and x = the greater number, y = the less.

Then

$$x + y = s,$$

and

$$x - y = d.$$

Whence

$$x = \frac{s+d}{2}; y = \frac{s-d}{2}.$$

4. Let
and x = the first number, y = the second.

Then

$$x = ay,$$

and

$$x - y = b.$$

Whence

$$x = \frac{ab}{a-1}; y = \frac{b}{a-1}.$$

5. Let
and x = the first number, y = the second.

Then

$$x + y = b,$$

and

$$\frac{x}{y} = a.$$

Whence

$$x = \frac{ab}{a+1}; y = \frac{b}{a+1}.$$

6. Solving,

$$n = a + 2b; d = a + b.$$

Then

$$\frac{n}{d} = \frac{a+2b}{a+b}.$$

7. Let
and n = the numerator, d = the denominator.

Then

$$\frac{n+1}{d} = x, \quad \text{and} \quad \frac{n}{d-2} = y.$$

Whence, solving for n and d , $n = \frac{y-2xy}{x-y}; d = \frac{1-2y}{x-y}.$

8. Let
and

n = the numerator,
 d = the denominator.

Then

$$\frac{n}{d} = b,$$

and

$$\frac{n+2}{d} = c.$$

Whence

$$n = \frac{2b}{c-b}; d = \frac{2}{c-b}.$$

9. Let
and

x = the distance, in feet, of first boy,
 y = the distance, in feet, of second boy.

Then
and

$$x + y = l,$$

$$ax = by.$$

Whence

$$x = \frac{bl}{a+b}; y = \frac{al}{a+b}.$$

10. Let
and

x = the number originally at 80 cents,
 y = the number originally at \$1.10.

Then
and

$$80x + 110y = 100h,$$

$$110x + 80y = 100k.$$

Whence

$$x = \frac{110k - 80h}{57};$$

$$y = \frac{110h - 80k}{57}.$$

11. Let
and

x = the cost in cents of first book,
 y = the cost in cents of second book.

Then
and

$$x + y = 100c,$$

$$x - y = d.$$

Whence

$$x = \frac{100c + d}{2}; y = \frac{100c - d}{2}.$$

12. Let
and

a = the number of dollars A has,
 b = the number of dollars B has.

Then
and

$$a + b = k,$$

$$a - h = b + h.$$

Whence

$$a = \frac{k + 2h}{2}; b = \frac{k - 2h}{2}.$$

13. Let
and

a = the number of dollars A has,
 b = the number of dollars B has.

Then
and

$$a - h = b + h,$$

$$2(b - k) = a + k.$$

Whence

$$a = 4h + 3k; b = 2h + 3k.$$

14. Let $a =$ the number of dollars A has,
 and $b =$ the number of dollars B has.
 Then $a - 10 = b + 10 - h$,
 and $3(b - k) = a + k$.
 Whence $a = \frac{60 + 4k - 3h}{2}$;
 $b = \frac{20 + 4k - h}{2}$.

15. Let $a =$ the number of dollars A has,
 and $b =$ the number of dollars B has.
 Then $a + b = 40$,
 and $a - h + k = b + h - k$.
 Whence $a = 20 + h - k$; $b = 20 - h + k$.

16. Let $a =$ the number of dollars A has,
 and $b =$ the number of dollars B has.
 Then $a - r = \frac{1}{2}(b + r)$,
 and $b + r - 8 = \frac{3}{4}(a - r + 8)$.
 Whence $a = \frac{5r + 56}{5}$; $b = \frac{112 - 5r}{5}$.

17. Let $x =$ the number of dollars at $a\%$,
 and $b =$ the number of dollars at $b\%$.
 Then $x + y = 1000$,
 and $\frac{ax}{100} + \frac{by}{100} = c$.
 Whence $x = \frac{1000b - 100c}{b - a}$;
 $y = \frac{1000a - 100c}{a - b}$.

18. Let $p =$ the number of dollars at 5% ,
 and $q =$ the number of dollars at 4% .
 Then $p + q = x$,
 and $.05p + .04q = y$.
 Solving for p and q , $p = 100y - 4x$,
 $q = 5x - 100y$.

19. Solving, $a = \frac{4c}{3}$; $b = 4c$.

20. Let a = the number of days required by A alone,
 and b = the number of days required by B alone.
 Then $ah = b$,
 and $\frac{1}{a} + \frac{1}{b} = \frac{1}{4}$.
 Whence $a = \frac{4}{h}(h + 1)$; $b = 4(h + 1)$.

21. Let a = the number of days required by A alone,
 and b = the number of days required by B alone.
 Then $\frac{1}{a} + \frac{1}{b} = \frac{1}{d}$,
 and $\frac{6}{a} = \frac{2}{3}$.
 Whence $a = 9$; $b = \frac{9d}{9 - d}$.

22. Let a = the number of days required by A alone,
 and b = the number of days required by B alone.
 Then $\frac{1}{a} + \frac{1}{b} = \frac{1}{5}$,
 and $\frac{k}{a} = \frac{2}{5}$.
 Whence $a = \frac{5k}{2}$; $b = \frac{5k}{k - 2}$.

23. Let a = the number of days required by A alone,
 and b = the number of days required by B alone.
 Then $\frac{1}{a} + \frac{1}{b} = \frac{1}{n}$,
 and $2a = b$.
 Whence $a = \frac{3}{2}n$; $b = 3n$.

24. Let a = the number of days required by A alone,
 and b = the number of days required by B alone.
 Then $\frac{1}{a} + \frac{1}{b} = \frac{1}{p}$.
 and $qa = b$.
 Whence $a = \frac{p}{q}(q + 1)$; $b = p(q + 1)$.

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(In the exercises solved below the given equations may be combined in other orders than the one shown.)

1. $x + y + z = 9$, (1) $(1) - 3(2)$, $13B - 7C = -35$. (4)
 $x - y - z = -3$, (2) $(3) - 2(2)$, $11B - C = -5$. (5)
 $x + y - z = 5$. (3) $[(4) - 7(5)] \div -64$, $B = 0$.
 $[(1) + (2)] \div 2$, $x = 3$. (4) Then from (5), $C = 5$,
 $[(1) - (3)] \div 2$, $z = 2$. (5) and from (2), $A = -1$.
 Substituting from (4) and (5) in (1), $y = 4$.
2. $x + y + z = 3$, (1) $(1) + 2(2)$, $7x + 4y = 3$. (4)
 $2x + y - z = 6$, (2) $3(2) + 2(3)$, $16x + 13y = 3$. (5)
 $x - y - 2z = 0$. (3) $[13(4) - 4(5)] \div 27$, $x = 1$.
 $(1) + (2)$, $3x + 2y = 9$. (4) Then from (4), $y = -1$,
 $2(1) + (3)$, $3x + y = 6$. (5) and from (1), $z = 1$.
 $(4) - (5)$, $y = 3$.
 Then from (5), $x = 1$,
 and from (1), $z = -1$.
3. $x + y + z = 0$, (1) $3(2) - 2(1)$, $8k + 7m = -3$. (4)
 $3x + 2y + 3z = -1$, (2) $5(2) - 4(3)$, $14k + m = -39$. (5)
 $x - y - 2z = -8$. (3) $[7(5) - (4)] \div 90$, $k = -3$.
 $(1) + (3)$, $2x - z = -8$. (4) Then from (5), $m = 3$,
 $(2) - 2(1)$, $x + z = -1$. (5) and from (2), $h = 2$.
 $[(4) + (5)] \div 3$, $x = -3$.
 Then from (5), $z = 2$,
 and from (1), $y = 1$.
4. $3A + B - C = -8$, (1) $(1) - (3)$, $y + 2z = 25$. (4)
 $A - 4B + 2C = 9$, (2) $[(4) - (2)] \div 6$, $z = 5$.
 $2A + 3B + 3C = 13$. (3) Then from (4), $y = 15$.
 and from (3), $x = -38$.
5. $3x - 2y + 4z = 9$, (1)
 $2x + 3y - 2z = -3$, (2)
 $5x + 2y + 3z = 6$. (3)
6. $6h + 5k + 4m = 9$, (1)
 $4h + 6k + 5m = 5$, (2)
 $5h + 4k + 6m = 16$. (3)
7. $x + 3y + 2z = 17$, (1)
 $y - 4z = -5$, (2)
 $x + 2y = -8$. (3)

$$8. \quad 3x - y - 2z = -2, \quad (1)$$

$$6x + z = 4, \quad (2)$$

$$3y - 4z = -11. \quad (3)$$

$$(2) - 2(1), \quad 2y + 5z = 8. \quad (4)$$

$$[3(4) - 2(3)] \div 23, \quad z = 2.$$

$$\text{Then from (4),} \quad y = -1,$$

$$\text{and from (1),} \quad x = \frac{1}{3}.$$

$$9. \quad x + y = 1, \quad (1)$$

$$y + z = 3, \quad (2)$$

$$z + x = 8. \quad (3)$$

$$(1) - (2), \quad x - z = -2. \quad (4)$$

$$[(3) + (4)] \div 2, \quad x = 3.$$

$$\text{Then from (1),} \quad y = -2,$$

$$\text{and from (3),} \quad z = 5.$$

$$10. \quad 2p_1 - 3p_2 = 4, \quad (1)$$

$$3p_1 + p_3 = 5, \quad (2)$$

$$p_2 - 2p_3 = 2. \quad (3)$$

$$2(2) + (3), \quad 6p_1 + p_2 = 12. \quad (4)$$

$$[(1) + 3(4)] \div 20, \quad p_1 = 2.$$

$$\text{Then from (2),} \quad p_3 = -1,$$

$$\text{and from (3),} \quad p_2 = 0.$$

$$11. \quad 5x - 7y + 4z = 3, \quad (1)$$

$$6x + 3y - 5z = -3, \quad (2)$$

$$4x + 6y + 3z = 25. \quad (3)$$

$$5(1) + 4(2), \quad 49x - 23y = 3. \quad (4)$$

$$4(3) - 3(1), \quad x + 45y = 91. \quad (5)$$

$$49(5), \quad 49x + 2205y = 4459. \quad (6)$$

$$[(6) - (4)] \div 2228, \quad y = 2.$$

$$\text{Then from (4),} \quad x = 1,$$

$$\text{and from (3),} \quad z = 3.$$

$$12. \quad 3A_1 + 2A_2 + 4A_3 = 9, \quad (1)$$

$$A_1 - A_2 - 2A_3 = 3, \quad (2)$$

$$A_1 - 3A_2 + 2A_3 = 2. \quad (3)$$

$$(1) - 3(2), \quad 5A_2 + 10A_3 = 0,$$

$$\text{or} \quad A_2 + 2A_3 = 0. \quad (4)$$

$$(2) - (3), \quad 2A_2 - 4A_3 = 1. \quad (5)$$

$$[(5) + 2(4)] \div 4, \quad A_2 = \frac{1}{4}.$$

$$\text{Then from (4),} \quad A_3 = -\frac{1}{8},$$

$$\text{and from (2),} \quad A_1 = 3.$$

$$13. \quad \frac{1}{x} + \frac{1}{y} - \frac{1}{z} = 1, \quad (1)$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{2}{3}, \quad (2)$$

$$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 0. \quad (3)$$

$$[(2) - (1)] \div 2, \quad \frac{1}{z} = -\frac{1}{6}, z = -6.$$

$$[(1) + (3)] \div 2, \quad \frac{1}{x} = \frac{1}{2}, x = 2.$$

$$[(2) - (3)] \div 2, \quad \frac{1}{y} = \frac{1}{3}, y = 3.$$

$$14. \quad \frac{2}{p} + \frac{10}{q} - \frac{3}{r} = -3, \quad (1)$$

$$\frac{4}{p} + \frac{5}{q} + \frac{6}{r} = 15, \quad (2)$$

$$\frac{1}{p} + \frac{5}{q} - \frac{1}{r} = -\frac{1}{2}. \quad (3)$$

$$2(1) - (2), \quad \frac{15}{q} - \frac{12}{r} = -21. \quad (4)$$

$$(1) - 2(3), \quad -\frac{1}{r} = -2, r = \frac{1}{2}.$$

$$\text{Then in (4),} \quad \frac{1}{q} = \frac{1}{5}, q = 5,$$

$$\text{and in (3),} \quad \frac{1}{p} = \frac{1}{2}, p = 2.$$

$$15. \quad \frac{2}{x} + \frac{3}{z} = \frac{34}{3}. \quad (1)$$

$$\frac{3}{x} + \frac{4}{y} = 24, \quad (2)$$

$$\frac{4}{z} + \frac{5}{y} = 28. \quad (3)$$

$$3(1) - 2(2), \quad \frac{9}{z} - \frac{8}{y} = -14. \quad (4)$$

$$[9(3) - 4(4)] \div 77, \quad \frac{1}{y} = 4, y = \frac{1}{4}.$$

$$\text{Then from (2),} \quad \frac{1}{x} = \frac{8}{3}, x = \frac{3}{8},$$

$$\text{and from (1),} \quad \frac{1}{z} = 2, z = \frac{1}{2}.$$

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1. Let x , y , and z represent the three numbers respectively.

$$\text{Then } x + y = 54.$$

$$y + z = 65.$$

$$x + z = 59.$$

$$\text{Whence } x = 24, y = 30, z = 35.$$

2. Let x , y , and z represent the three numbers respectively.

$$\text{Then } x + y + z = 70.$$

$$x + y = 45.$$

$$\frac{x}{y} = 5.$$

$$\text{Whence } x = 37\frac{1}{2}, y = 7\frac{1}{2}, z = 25.$$

3. Let x , y , and z represent the three sides in feet.

$$\text{Then } x + y + z = 60.$$

$$x = y.$$

$$z = x + 6.$$

$$\text{Whence } x = 18, y = 18, z = 24.$$

4. Let x , y , and z represent the three sides in feet.

$$\text{Then } x + y = 53.$$

$$x - y = 19.$$

$$x + y + z = 62.$$

$$\text{Whence } x = 36, y = 17, z = 9.$$

5. Let x , y , and z represent the three sides in feet.

$$\text{Then } x + y = 41.$$

$$y + z = 46.$$

$$x + z = 57.$$

$$\text{Whence } x = 26, y = 15, z = 31.$$

6. Let x and y equal the first two numbers, and let z equal the third.

$$\text{Then } x + y + z = 24.$$

$$\frac{x}{y} = 3.$$

$$\frac{x + y}{z} = 5.$$

$$\text{Whence } x = 15, y = 5, z = 4.$$

7. Let x , y , and z represent the number of degrees in the three angles respectively.

$$\text{Then } x = y.$$

$$x + y = z.$$

$$x + y + z = 180.$$

$$\text{Whence } x = 45, y = 45, z = 90.$$

8. We have at once

$$A - 12 = B.$$

$$B - 21 = C.$$

$$A + B + C = 180.$$

$$\text{Whence } A = 75, B = 63, C = 42.$$

9. Let x , y , and z represent the number of degrees in the three angles respectively.

$$\text{Then } x + y = z + 30.$$

$$z = 15(x - y).$$

$$x + y + z = 180.$$

$$\text{Whence } x = 55, y = 50, z = 75.$$

10. Let a , b , and c represent the number of days required by A, B, and C respectively.

$$\text{Then } \frac{1}{a} + \frac{1}{b} = \frac{1}{6}.$$

$$\frac{1}{a} + \frac{1}{c} = \frac{1}{8}.$$

$$\frac{1}{b} + \frac{1}{c} = \frac{1}{12}.$$

$$\text{Whence } a = 4\frac{8}{5}, b = 16, c = 48.$$

11. Let a , b , and c represent the number of hours required by the pipes respectively.

$$\text{Then } \frac{1}{a} + \frac{1}{b} = \frac{1}{6};$$

$$\frac{1}{a} + \frac{1}{c} = \frac{1}{8}.$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{4}.$$

$$\text{Whence } a = 24, b = 8, c = 12.$$

12. Let $x =$ the hundreds' digit,
 and $y =$ the tens' digit,
 $z =$ the units' digit.
 Then $x + y + z = 19.$
 $z - y = 3.$
 $495 + 100x + 10y + z = 100z + 10y + x.$
 Whence $x = 4, y = 6, z = 9.$

13. As in Ex. 12,
 $100x + 10y + z - 54 = 100x + 10z + y.$
 $100x + 10y + z + 360 = 100y + 10x + z.$
 $x + y + z = 11.$
 Whence $x = 3, y = 7, z = 1.$

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$$1. \quad \begin{array}{r} a^2 + 6a + 9 \overline{)a + 3} \\ a^2 \\ \hline 2a + 3 \overline{)6a + 9} \\ 6a + 9 \\ \hline \end{array}$$

$$3. \quad \begin{array}{r} x^4 - 8x^2 + 16 \overline{)x^2 - 4} \\ x^4 \\ \hline 2x^2 - 4 \overline{)-8x^2 + 16} \\ -8x^2 + 16 \\ \hline \end{array}$$

$$2. \quad \begin{array}{r} x^2 + 10ax + 25a^2 \overline{)x + 5a} \\ x^2 \\ \hline 2x + 5a \overline{)10ax + 25a^2} \\ 10ax + 25a^2 \\ \hline \end{array}$$

$$4. \quad \begin{array}{r} a^4 + 4a^2x^2 + 4x^4 \overline{)a^2 + 2x^2} \\ a^4 \\ \hline 2a^2 + 2x^2 \overline{)4a^2x^2 + 4x^4} \\ 4a^2x^2 + 4x^4 \\ \hline \end{array}$$

$$5. \quad \begin{array}{r} a^4 + 2a^3 + 3a^2 + 2a + 1 \overline{)a^2 + a + 1} \\ a^4 \\ \hline 2a^2 + a \overline{)2a^3 + 3a^2} \\ 2a^3 + a^2 \\ \hline 2a^2 + 2a + 1 \overline{)2a^2 + 2a + 1} \\ 2a^2 + 2a + 1 \\ \hline \end{array}$$

$$6. \quad \begin{array}{r} x^4 - 8x^3 + 24x^2 - 32x + 16 \overline{)x^2 - 4x + 4} \\ x^4 \\ \hline 2x^2 - 4x \overline{)-8x^3 + 24x^2} \\ -8x^3 + 16x^2 \\ \hline 2x^2 - 8x + 4 \overline{)8x^2 - 32x + 16} \\ 8x^2 - 32x + 16 \\ \hline \end{array}$$

$$7. \quad \begin{array}{r} x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 \overline{) x^2 + 2xy + y^2} \\ x^4 \end{array}$$

$$\begin{array}{r} 2x^2 + 2xy \overline{) 4x^3y + 6x^2y^2} \\ \underline{4x^3y + 4x^2y^2} \\ 2x^2 + 4xy + y^2 \overline{) 2x^2y^2 + 4xy^3 + y^4} \\ \underline{2x^2y^2 + 4xy^3 + y^4} \end{array}$$

$$8. \quad \begin{array}{r} c^4 - 10c^3 + 21c^2 + 20c + 4 \overline{) c^2 - 5c - 2} \\ c^4 \end{array}$$

$$\begin{array}{r} 2c^2 - 5c \overline{) -10c^3 + 21c^2} \\ \underline{-10c^3 + 25c^2} \\ 2c^2 - 10c - 2 \overline{) -4c^2 + 20c + 4} \\ \underline{-4c^2 + 20c + 4} \end{array}$$

$$9. \quad \begin{array}{r} n^6 - 6n^4 + 10n^3 + 9n^2 - 30n + 25 \overline{) n^3 - 3n + 5} \\ n^6 \end{array}$$

$$\begin{array}{r} 2n^3 - 3n \overline{) -6n^4 + 10n^3 + 9n^2} \\ \underline{-6n^4 \qquad \qquad + 9n^2} \\ 2n^3 - 6n + 5 \overline{) 10n^3 \qquad - 30n + 25} \\ \underline{10n^3 \qquad - 30n + 25} \end{array}$$

$$10. \quad \begin{array}{r} c^4 - 6c^3 + 13c^2 - 12c + 4 \overline{) c^2 - 3c + 2} \\ c^4 \end{array}$$

$$\begin{array}{r} 2c^2 - 3c \overline{) -6c^3 + 13c^2} \\ \underline{-6c^3 + 9c^2} \\ 2c^2 - 6c + 2 \overline{) 4c^2 - 12c + 4} \\ \underline{4c^2 - 12c + 4} \end{array}$$

$$11. \quad \begin{array}{r} 4a^6 + 12a^5 + 5a^4 - 22a^3 - 23a^2 + 8a + 16 \overline{) 2a^3 + 3a^2 - a - 4} \\ 4a^6 \end{array}$$

$$\begin{array}{r} 4a^3 + 3a^2 \overline{) +12a^5 + 5a^4} \\ \underline{+12a^5 + 9a^4} \\ 4a^3 + 6a^2 - a \overline{) -4a^4 - 22a^3 - 23a^2} \\ \underline{-4a^4 - 6a^3 + a^2} \\ 4a^3 + 6a^2 - 2a - 4 \overline{) -16a^3 - 24a^2 + 8a + 16} \\ \underline{-16a^3 - 24a^2 + 8a + 16} \end{array}$$

$$12. \quad \begin{array}{r} c^4 - 4c^3a + 6c^2a^2 - 4ca^3 + a^4 \overline{) c^2 - 2ca + a^2} \\ c^4 \end{array}$$

$$\begin{array}{r} 2c^2 - 2ca \overline{) -4c^3a + 6c^2a^2} \\ \underline{-4c^3a + 4c^2a^2} \\ 2c^2 - 4ca + a^2 \overline{) 2c^2a^2 - 4ca^3 + a^4} \\ \underline{2c^2a^2 - 4ca^3 + a^4} \end{array}$$

$$\begin{array}{r}
 13. \quad 25y^4 + 30xy^3 - 11x^2y^2 - 12x^3y + 4x^4 \mid \underline{5y^2 + 3xy - 2x^2} \\
 \underline{25y^4} \\
 10y^2 + 3xy \mid \begin{array}{l} 30xy^3 - 11x^2y^2 \\ 30xy^3 + 9x^2y^2 \end{array} \\
 \hline
 10y^2 + 6xy - 2x^2 \mid \begin{array}{l} -20x^2y^2 - 12x^3y + 4x^4 \\ -20x^2y^2 - 12x^3y + 4x^4 \end{array}
 \end{array}$$

$$\begin{array}{r}
 14. \quad 9a^6 - 36a^4x - 24a^3x^2 + 36a^2x^2 + 48ax^3 + 16x^4 \mid \underline{3a^3 - 6ax - 4x^2} \\
 \underline{9a^6} \\
 6a^3 - 6ax \mid \begin{array}{l} -36a^4x - 24a^3x^2 \\ -36a^4x \qquad \qquad + 36a^2x^2 \end{array} \\
 \hline
 6a^3 - 12ax - 4x^2 \mid \begin{array}{l} -24a^3x^2 \qquad \qquad + 48ax^3 + 16x^4 \\ -24a^3x^2 \qquad \qquad + 48ax^3 + 16x^4 \end{array}
 \end{array}$$

$$\begin{array}{r}
 15. \quad 9c^4 - 12abc^3 - 2a^2b^2c^2 + 4a^3b^3c + a^4b^4 \mid \underline{3c^2 - 2abc - a^2b^2} \\
 \underline{9c^4} \\
 6c^2 - 2abc \mid \begin{array}{l} -12abc^3 - 2a^2b^2c^2 \\ -12abc^3 + 4a^2b^2c^2 \end{array} \\
 \hline
 6c^2 - 4abc - a^2b^2 \mid \begin{array}{l} -6a^2b^2c^2 + 4a^3b^3c + a^4b^4 \\ -6a^2b^2c^2 + 4a^3b^3c + a^4b^4 \end{array}
 \end{array}$$

$$\begin{array}{r}
 16. \quad c^6 - 4xc^3 + 2a^2xc^3 + a^4x^2 - 4a^2x^2 + 4x^2 \mid \underline{c^3 - 2x + a^2x} \\
 \underline{c^6} \\
 2c^3 - 2x \mid \begin{array}{l} -4xc^3 + 2a^2xc^3 \\ -4xc^3 \qquad \qquad \qquad + 4x^2 \end{array} \\
 \hline
 2c^3 - 4x + a^2x \mid \begin{array}{l} +2a^2xc^3 + a^4x^2 - 4a^2x^2 \\ +2a^2xc^3 + a^4x^2 - 4a^2x^2 \end{array}
 \end{array}$$

$$\begin{array}{r}
 17. \quad 4 - \frac{4c^2}{5} + \frac{c^4}{25} \mid \underline{2 - \frac{c^2}{5}} \\
 \underline{4} \\
 4 - \frac{c^2}{5} \mid \begin{array}{l} -\frac{4c^2}{5} + \frac{c^4}{25} \\ -\frac{4c^2}{5} + \frac{c^4}{25} \end{array}
 \end{array}$$

$$\begin{array}{r}
 18. \quad \frac{9}{a^2} - 3 + \frac{a^2}{4} \mid \underline{\frac{3}{a} - \frac{a}{2}} \\
 \underline{\frac{9}{a^2}} \\
 \frac{6}{a} - \frac{a}{2} \mid \begin{array}{l} -3 + \frac{a^2}{4} \\ -3 + \frac{a^2}{4} \end{array}
 \end{array}$$

$$\begin{array}{r}
 19. \quad x^4 - 4x^3 + 5x^2 - 2x + \frac{1}{4} \mid \underline{x^2 - 2x + \frac{1}{2}} \\
 \underline{x^4} \\
 2x^2 - 2x \mid \begin{array}{l} -4x^3 + 5x^2 \\ -4x^3 + 4x^2 \end{array} \\
 \hline
 2x^2 - 4x + \frac{1}{2} \mid \begin{array}{l} x^2 - 2x + \frac{1}{4} \\ x^2 - 2x + \frac{1}{4} \end{array}
 \end{array}$$

20.
$$\begin{array}{r} a^4 + 4a^3 + 4a^2 - \frac{4a^2}{3} - \frac{8a}{3} + \frac{4}{9} \bigg| a^2 + 2a - \frac{2}{3} \\ \underline{a^4} \\ 2a^2 + 2a \quad \bigg| \begin{array}{l} 4a^3 + 4a^2 \\ 4a^3 + 4a^2 \end{array} \\ \hline 2a^2 + 4a - \frac{2}{3} \quad \bigg| \begin{array}{l} -\frac{4a^2}{3} - \frac{8a}{3} + \frac{4}{9} \\ -\frac{4a^2}{3} - \frac{8a}{3} + \frac{4}{9} \end{array} \\ \hline \end{array}$$

21.
$$\begin{array}{r} x^4 - x^3 + \frac{x^2}{4} + \frac{2x^2}{3} - \frac{x}{3} + \frac{1}{9} \bigg| x^2 - \frac{x}{2} + \frac{1}{3} \\ \underline{x^4} \\ 2x^2 - \frac{x}{2} \quad \bigg| \begin{array}{l} -x^3 + \frac{x^2}{4} \\ -x^3 + \frac{x^2}{4} \end{array} \\ \hline 2x^2 - x + \frac{1}{3} \quad \bigg| \begin{array}{l} \frac{2x^2}{3} - \frac{x}{3} + \frac{1}{9} \\ \frac{2x^2}{3} - \frac{x}{3} + \frac{1}{9} \end{array} \\ \hline \end{array}$$

22.
$$\begin{array}{r} \frac{25m^4}{4} + \frac{10m^3}{3} - \frac{127m^2}{18} - 2m + \frac{9}{4} \bigg| \frac{5m^2}{2} + \frac{2m}{3} - \frac{3}{2} \\ \underline{\frac{25m^4}{4}} \\ 5m^2 + \frac{2m}{3} \quad \bigg| \begin{array}{l} \frac{10m^3}{3} \\ \frac{10m^3}{3} + \frac{4m^2}{9} \end{array} \\ \hline 5m^2 + \frac{4m}{3} - \frac{3}{2} \quad \bigg| \begin{array}{l} -\frac{15m^2}{2} - 2m + \frac{9}{4} \\ -\frac{15m^2}{2} - 2m + \frac{9}{4} \end{array} \\ \hline \end{array}$$

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1.
$$\begin{array}{r} 44'89 \big| 67 \\ \underline{36} \\ 127 \big| \begin{array}{l} 8\ 89 \\ 8\ 89 \end{array} \end{array}$$
2.
$$\begin{array}{r} 51'84 \big| 72 \\ \underline{49} \\ 142 \big| \begin{array}{l} 2\ 84 \\ 2\ 84 \end{array} \end{array}$$
3.
$$\begin{array}{r} 53'29 \big| 73 \\ \underline{49} \\ 143 \big| \begin{array}{l} 4\ 29 \\ 4\ 29 \end{array} \end{array}$$
4.
$$\begin{array}{r} 62'41 \big| 79 \\ \underline{49} \\ 149 \big| \begin{array}{l} 13\ 41 \\ 13\ 41 \end{array} \end{array}$$

5. 92'16 | 96
 81
186 | 11 16
 | 11 16

6. 1'61'29 | 127
 1
22 | 61
 | 44
247 | 1729
 | 1729

7. 2'46'49 | 157
 1
25 | 1 46
 | 1 25
307 | 21 49
 | 21 49

8. 4'34'36 | 208.413
 4
408 | 34 36
 | 32 64
4164 | 1 7200
 | 1 6656
41681 | 54400
 | 41681
416823 | 1271900
 | 1250469

12. 32'83'29 | 573
 25
107 | 7 83
 | 7 49
1143 | 34 29
 | 34 29

9. 5'33'61 | 231
 4
43 | 133
 | 129
461 | 461
 | 461

13. 63 16
 63 16
 189 96
 378 16
 3969 256
 256
 42'25 | 65
 36
125 | 625
 | 625

10. 16'56'80 | 407.038
 16
807 | 56 80
 | 56 49
81403 | 310000
 | 244209
814068 | 6579100
 | 6512544

Area = $\frac{63 \times 16}{2} = 504.$

11. 22'37'29 | 473
 16
87 | 637
 | 609
943 | 2829
 | 2829

14. 48 55
 48 55
 384 275
 192 275
 2304 3025
 3025
 53'29 | 73
 49
143 | 429
 | 429

Area = $\frac{55 \times 48}{2} = 1320.$

15.

104	153
<u>104</u>	<u>153</u>
416	459
104	765
<u>10816</u>	<u>153</u>
	23409
	10816
	<u>3'42'25</u> 185
	1
28	<u>2 42</u>
	2 24
365	<u>18 25</u>
	18 25

Area = $\frac{104 \times 153}{2} = 7956$.

18.

2.57	.32
<u>2.57</u>	<u>.32</u>
1799	64
1285	96
<u>514</u>	<u>.1024</u>
6.6049	
<u>.1024</u>	
6.5025	Area = $\frac{2.55 \times .32}{2}$
4	= .408.
45 <u>250</u>	
225	
505 <u>2525</u>	
2525	

$$\begin{array}{r}
 16. \quad 645 \qquad \qquad \qquad 812 \\
 \underline{645} \qquad \qquad \qquad \underline{812} \\
 3225 \qquad \qquad \qquad 1624 \\
 2580 \qquad \qquad \qquad 812 \\
 \underline{3870} \qquad \qquad \qquad \underline{6496} \\
 416025 \qquad \qquad \qquad 659344 \\
 \qquad \qquad \qquad \underline{416025} \\
 \text{Area} = \frac{645 \times 812}{2} = 261870.
 \end{array}$$

$$\begin{array}{r}
 19. \quad 2.05 \qquad \qquad \qquad 1.87 \\
 \underline{2.05} \qquad \qquad \qquad \underline{1.87} \\
 1025 \qquad \qquad \qquad 1309 \\
 410 \qquad \qquad \qquad 1496 \\
 \underline{4.2025} \qquad \qquad \qquad \underline{187} \\
 3.4969 \qquad \qquad \qquad 3.4969 \\
 \hline
 .7056 \overline{) .84} \\
 \underline{.64} \qquad \qquad \qquad \text{Area} = \frac{1.87 \times .84}{2} \\
 164 \overline{) 656} \qquad \qquad \qquad = .7854. \\
 \underline{656}
 \end{array}$$

$$\begin{array}{r}
 17. \quad 109 \qquad \qquad \qquad 91 \\
 \quad \underline{109} \qquad \qquad \qquad \underline{91} \\
 \quad 981 \qquad \qquad \qquad 91 \\
 \underline{109} \qquad \qquad \qquad 819 \\
 11881 \qquad \qquad \qquad 8281 \\
 \quad 8281 \\
 \boxed{\begin{array}{r} 3600 \\ 36 \end{array}} \begin{array}{l} 60 \\ \hline \end{array} \\
 \quad \underline{\quad} \quad \quad \quad 00
 \end{array}
 \qquad \qquad \qquad
 \begin{array}{l}
 \text{Area} = \frac{60 \times 91}{2} \\
 = 2730.
 \end{array}$$

20. $6.4271 \overline{) 2.5351}$

4

45 $\overline{) 242}$

225

503 $\overline{) 1771}$

1509

5065 $\overline{) 26200}$

25325

50701 $\overline{) 87500}$

50701

21. $884.3 \overline{)29.7371}$

$$\begin{array}{r} 4 \\ 49 \overline{)484} \\ \underline{441} \\ 587 \overline{)4330} \\ \underline{4109} \\ 5943 \overline{)22100} \\ \underline{17829} \\ 59467 \overline{)427100} \\ \underline{416269} \\ 594741 \overline{)1083100} \\ \underline{594741} \end{array}$$

22. $.0869 \overline{).2947}$

$$\begin{array}{r} 4 \\ 49 \overline{)469} \\ \underline{441} \\ 584 \overline{)2800} \\ \underline{2336} \\ 5887 \overline{)46400} \\ \underline{41209} \end{array}$$

23. $.00'32'10' \overline{).0566}$

$$\begin{array}{r} 25 \\ 106 \overline{)710} \\ \underline{636} \\ 1126 \overline{)7400} \\ \underline{6756} \end{array}$$

24. $3.00 \overline{)1.732}$

$$\begin{array}{r} 1 \\ 27 \overline{)200} \\ \underline{189} \\ 343 \overline{)1100} \\ \underline{1029} \\ 3462 \overline{)7100} \\ \underline{6924} \end{array}$$

25. $6. \overline{)2.449}$

$$\begin{array}{r} 4 \\ 44 \overline{)200} \\ \underline{176} \\ 484 \overline{)2400} \\ \underline{1936} \\ 4889 \overline{)46400} \\ \underline{44001} \end{array}$$

26. $2.6 \overline{)1.612}$

$$\begin{array}{r} 1 \\ 26 \overline{)160} \\ \underline{156} \\ 321 \overline{)400} \\ \underline{321} \\ 3222 \overline{)7900} \\ \underline{6444} \end{array}$$

27. $2\frac{3}{7} = 2.42'85'71 \overline{)1.558}$

$$\begin{array}{r} 1 \\ 25 \overline{)142} \\ \underline{125} \\ 305 \overline{)1785} \\ \underline{1525} \\ 3108 \overline{)26071} \\ \underline{24864} \end{array}$$

28. $\frac{3}{17} = .17'64'70 \overline{).420}$

$$\begin{array}{r} 16 \\ 82 \overline{)164} \\ \underline{164} \\ 840 \overline{)7000} \end{array}$$

29. $\sqrt{(195)^2 + (28)^2}$
 $= \sqrt{38025 + 784}$
 $= \sqrt{3'88'09} \overline{)197}$

$$\begin{array}{r} 1 \\ 29 \overline{)288} \\ \underline{261} \\ 387 \overline{)2709} \\ \underline{2709} \end{array}$$

$$\begin{aligned}
 30. \quad & \sqrt{(409)^2 - (391)^2} \\
 &= \sqrt{167281 - 152881} \\
 &= \sqrt{1'44'00} \\
 &= 120, \text{ side.}
 \end{aligned}$$

$$\text{Area} = 391 \times 120 = 46920.$$

$$\begin{aligned}
 31. \quad & \sqrt{(533)^2 - (92)^2} \\
 &= \sqrt{284089 - 8464} \\
 &= \sqrt{2756'25} \quad \underline{525} \\
 &\quad \begin{array}{r} 25 \\ 102 \overline{) 256} \\ \underline{204} \\ 1045 \overline{) 5225} \\ \underline{5225} \end{array}
 \end{aligned}$$

$$\text{Perimeter} = 2 \times 92 + 2 \times 525 = 1234.$$

$$\begin{aligned}
 32. \quad & 2l + 2w = 46. \\
 & l - w = 7. \\
 & \therefore l = 15 \text{ and } w = 8. \\
 & \sqrt{(15)^2 + 8^2} = \sqrt{225 + 64} \\
 &= \sqrt{289} \\
 &= 17 \text{ yards.}
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & s^2 + s^2 = (74)^2 = 5476. \\
 & s = \sqrt{27'38} \quad \underline{52.32} \\
 &\quad \begin{array}{r} 25 \\ 102 \overline{) 238} \\ \underline{204} \\ 1043 \overline{) 3400} \\ \underline{3129} \\ 10462 \overline{) 27100} \\ \underline{20924} \end{array}
 \end{aligned}$$

$$\begin{aligned}
 34. \quad & \sqrt{(52)^2 + (52)^2} \\
 &= \sqrt{2704 + 2704} \\
 &= \sqrt{54'08} \quad \underline{73.53} \\
 &\quad \begin{array}{r} 49 \\ 143 \overline{) 508} \\ \underline{429} \\ 1465 \overline{) 7900} \\ \underline{7325} \\ 14703 \overline{) 57500} \\ \underline{44109} \end{array}
 \end{aligned}$$

$$\begin{aligned}
 35. \quad & (2x)^2 - x^2 = (10)^2. \\
 & 3x^2 = 100. \\
 & x^2 = \frac{100}{3}. \\
 & x = \sqrt{33.3333}.
 \end{aligned}$$

$$33.3333 \quad \underline{5.77}$$

$$\begin{array}{r} 25 \\ 107 \overline{) 833} \\ \underline{749} \\ 1147 \overline{) 8433} \\ \underline{8029} \end{array}$$

$$\text{Hypotenuse} = 5.77 \times 2 = 11.54.$$

$$\begin{aligned}
 36. \quad & (3x)^2 - x^2 = 16^2. \\
 & 8x^2 = 256. \\
 & x = \sqrt{32}. \\
 & \quad = 5.656. \\
 & 3x = 16.96.
 \end{aligned}$$

$$\begin{aligned}
 37. \quad & x^2 + (2.4x)^2 = (52)^2. \\
 & x^2 + 5.76x^2 = 2704. \\
 & 6.76x^2 = 2704. \\
 & x^2 = 400. \\
 & x = 20, \text{ width.}
 \end{aligned}$$

$$\text{Length} = 20 \times 2.4 = 48.$$

$$\begin{aligned}
 38. \quad & x^2 + \left(\frac{3x}{4}\right)^2 = (100)^2. \\
 & \frac{25x^2}{16} = 10,000. \\
 & x^2 = 6400. \\
 & x = 80, \text{ length.} \\
 & \frac{3x}{4} = 60, \text{ width.}
 \end{aligned}$$

$$\text{Area} = 80 \times 60 = 4800.$$

$$\begin{aligned}
 39. \quad & (2x)^2 - x^2 = (10)^2. \\
 & 3x^2 = 100. \\
 & x = \sqrt{\frac{100}{3}} \\
 & \quad = 5.77.
 \end{aligned}$$

$$\begin{aligned}
 40. \quad \overline{BD}^2 + (3)^2 &= 6^2. \\
 \overline{BD}^2 &= 36 - 9 \\
 &= 27. \\
 BD &= \sqrt{27} \\
 &= 5.196. \\
 \text{Area} &= \frac{6 \times 5.196}{2} \\
 &= 15.58.
 \end{aligned}$$

$$\begin{aligned}
 41. \quad AC &= BC \\
 &= 8. \\
 \overline{BD}^2 + (4)^2 &= (8)^2. \\
 \overline{BD}^2 &= 64 - 16 \\
 &= 48. \\
 BD &= \sqrt{48} \\
 &= 6.928. \\
 \text{Area} &= \frac{6.928 \times 8}{2} \\
 &= 27.71.
 \end{aligned}$$

$$\begin{aligned}
 42. \quad \text{Let } AB &= x. \\
 \text{Then } (14)^2 + \left(\frac{x}{2}\right)^2 &= x^2. \\
 784 + x^2 &= 4x^2. \\
 x &= \sqrt{784} \\
 &= 28. \\
 \text{Area} &= \frac{14 \times 28}{2} \\
 &= 196.
 \end{aligned}$$

$$\begin{aligned}
 43. \quad s &= 45 \div 3 \\
 &= 15. \\
 (15)^2 - \left(\frac{1}{2}s\right)^2 &= a^2. \\
 \therefore a &= \sqrt{225 - \frac{225}{4}} \\
 &= 15.
 \end{aligned}$$

$$\begin{aligned}
 44. \quad s^2 - \left(\frac{s}{2}\right)^2 &= (25)^2. \\
 3s^2 &= 2500. \\
 s &= \sqrt{\frac{2500}{3}} \\
 &= 28.87.
 \end{aligned}$$

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$$\begin{aligned}
 1. \quad a^{\frac{3}{2}}. & \quad 5. \quad 20a^{\frac{1}{2}}x. & \quad 9. \quad 6x^{\frac{4}{3}}. & \quad 13. \quad 12x^2 \cdot a^{\frac{1}{3}}x = 12x^3a^{\frac{1}{3}}. \\
 2. \quad a^{\frac{1}{2}}x^2. & \quad 6. \quad a^{\frac{2}{3}}. & \quad 10. \quad 12a^{\frac{1}{3}}x. & \quad 14. \quad cd^{\frac{3}{2}}e^{\frac{3}{2}}. \\
 3. \quad 3 \cdot 2^{\frac{1}{2}}x^{\frac{5}{2}}. & \quad 7. \quad a^{\frac{1}{3}}x^{\frac{4}{3}}. & \quad 11. \quad 2a^{\frac{1}{2}}x^{\frac{3}{4}}. & \quad 15. \quad 3a^{\frac{3}{2}}x^{\frac{2}{3}}. \\
 4. \quad 3x^{\frac{1}{2}}. & \quad 8. \quad 2^{\frac{4}{3}}x^{\frac{2}{3}}. & \quad 12. \quad 8x^{\frac{1}{4}}. & \quad 16. \quad x^{\frac{a}{n}} \cdot y^{\frac{b}{n}}. \\
 17. \quad x^{\frac{a}{n}} \cdot x^{\frac{3a}{n}} &= x^{\frac{4a}{n}}. & \quad 19. \quad 6 \cdot \frac{1}{3} = 2. \\
 18. \quad (-32)^{\frac{4}{5}} \cdot (-64)^{\frac{1}{3}} &= 16 \cdot (-4) = -64. & \quad 20. \quad 3 \cdot 3 = 9. \\
 & & \quad 21. \quad \frac{1}{2} \cdot 4 = 2.
 \end{aligned}$$

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$$\begin{aligned}
 1. \quad \sqrt{8} &= \sqrt{4 \cdot 2} = 2\sqrt{2}. & \quad 8. \quad \sqrt{63} &= \sqrt{9 \cdot 7} = 3\sqrt{7}. \\
 2. \quad \sqrt{12} &= \sqrt{4 \cdot 3} = 2\sqrt{3}. & \quad 9. \quad \sqrt{98} &= \sqrt{49 \cdot 2} = 7\sqrt{2}. \\
 3. \quad \sqrt{18} &= \sqrt{9 \cdot 2} = 3\sqrt{2}. & \quad 10. \quad 2\sqrt{72} &= 2\sqrt{36 \cdot 2} = 12\sqrt{2}. \\
 4. \quad \sqrt{20} &= \sqrt{4 \cdot 5} = 2\sqrt{5}. & \quad 11. \quad 3\sqrt{80} &= 3\sqrt{16 \cdot 5} = 12\sqrt{5}. \\
 5. \quad \sqrt{45} &= \sqrt{9 \cdot 5} = 3\sqrt{5}. & \quad 12. \quad 5\sqrt{128} &= 5\sqrt{64 \cdot 2} = 40\sqrt{2}. \\
 6. \quad \sqrt{50} &= \sqrt{25 \cdot 2} = 5\sqrt{2}. & \quad 13. \quad \sqrt{147} &= \sqrt{49 \cdot 3} = 7\sqrt{3}. \\
 7. \quad \sqrt{75} &= \sqrt{25 \cdot 3} = 5\sqrt{3}. & \quad 14. \quad 3\sqrt{162} &= 3\sqrt{81 \cdot 2} = 27\sqrt{2}.
 \end{aligned}$$

15. $10\sqrt{175} = 10\sqrt{25 \cdot 7} = 50\sqrt{7}.$
16. $\sqrt{192} = \sqrt{64 \cdot 3} = 8\sqrt{3}.$
17. $\sqrt{243} = \sqrt{81 \cdot 3} = 9\sqrt{3}.$
18. $\sqrt{245} = \sqrt{49 \cdot 5} = 7\sqrt{5}.$
19. $\sqrt{343} = \sqrt{49 \cdot 7} = 7\sqrt{7}.$
20. $\sqrt{363} = \sqrt{121 \cdot 3} = 11\sqrt{3}.$
21. $2\sqrt{720} = 2\sqrt{144 \cdot 5} = 24\sqrt{5}.$
22. $\sqrt{1250} = \sqrt{625 \cdot 2} = 25\sqrt{2}.$
23. $\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = 2\sqrt[3]{2}.$
24. $\sqrt[3]{24} = \sqrt[3]{8 \cdot 3} = 2\sqrt[3]{3}.$
25. $2\sqrt[3]{40} = 2\sqrt[3]{8 \cdot 5} = 4\sqrt[3]{5}.$
26. $3\sqrt[3]{56} = 3\sqrt[3]{8 \cdot 7} = 6\sqrt[3]{7}.$
27. $4\sqrt[3]{72} = 4\sqrt[3]{8 \cdot 9} = 8\sqrt[3]{9}.$
28. $5\sqrt[3]{96} = 5\sqrt[3]{8 \cdot 12} = 10\sqrt[3]{12}.$
29. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = 3\sqrt[3]{2}.$
30. $\sqrt[3]{81} = \sqrt[3]{27 \cdot 3} = 3\sqrt[3]{3}.$
31. $\sqrt[3]{135} = \sqrt[3]{27 \cdot 5} = 3\sqrt[3]{5}.$
32. $3\sqrt[3]{189} = 3\sqrt[3]{27 \cdot 7} = 9\sqrt[3]{7}.$
33. $7\sqrt[3]{128} = 7\sqrt[3]{64 \cdot 2} = 28\sqrt[3]{2}.$
34. $\sqrt[3]{192} = \sqrt[3]{64 \cdot 3} = 4\sqrt[3]{3}.$
35. $\sqrt[3]{250} = \sqrt[3]{125 \cdot 2} = 5\sqrt[3]{2}.$
36. $8\sqrt[3]{375} = 8\sqrt[3]{125 \cdot 3} = 40\sqrt[3]{3}.$
37. $\sqrt[3]{448} = \sqrt[3]{64 \cdot 7} = 4\sqrt[3]{7}.$
38. $2\sqrt[3]{625} = 2\sqrt[3]{125 \cdot 5} = 10\sqrt[3]{5}.$
39. $\sqrt{a^3} = \sqrt{a^2 \cdot a} = a\sqrt{a}.$
40. $\sqrt{a^5} = \sqrt{a^4 \cdot a} = a^2\sqrt{a}.$
41. $\sqrt{4x^3} = \sqrt{4x^2 \cdot x} = 2x\sqrt{x}.$
42. $\sqrt{8x^2} = \sqrt{4x^2 \cdot 2} = 2x\sqrt{2}.$
43. $\sqrt{8x^3} = \sqrt{4x^2 \cdot 2x} = 2x\sqrt{2x}.$
44. $\sqrt{a^3x^2} = \sqrt{a^2x^2 \cdot a} = ax\sqrt{a}.$
45. $2\sqrt{x^3a} = 2\sqrt{x^2 \cdot xa} = 2x\sqrt{ax}.$
46. $3\sqrt{8a^3x} = 3\sqrt{4a^2 \cdot 2ax} = 6a\sqrt{2ax}.$
47. $5x\sqrt{4a^5x^3} = 5x\sqrt{4a^4x^2 \cdot ax} = 10a^2x^2\sqrt{ax}.$
48. $\sqrt[3]{a^4} = \sqrt[3]{a^3 \cdot a} = a\sqrt[3]{a}.$
49. $\sqrt[3]{a^5} = \sqrt[3]{a^3 \cdot a^2} = a\sqrt[3]{a^2}.$
50. $2a\sqrt[3]{x^7} = 2a\sqrt[3]{x^6 \cdot x} = 2ax^2\sqrt[3]{x}.$
51. $\sqrt[3]{8x^4} = \sqrt[3]{8x^3 \cdot x} = 2x\sqrt[3]{x}.$
52. $\sqrt[3]{27x^5} = \sqrt[3]{27x^3 \cdot x^2} = 3x\sqrt[3]{x^2}.$
53. $3x\sqrt[3]{4a^3x} = 3x\sqrt[3]{a^3 \cdot 4x} = 3ax\sqrt[3]{4x}.$
54. $5x\sqrt[3]{4a^2x^3} = 5x\sqrt[3]{x^3 \cdot 4a^2} = 5x^2\sqrt[3]{4a^2}.$
55. $\sqrt[3]{27a^3x} = 3a\sqrt[3]{x}.$
56. $\sqrt[3]{56a^4x} = \sqrt[3]{8a^3 \cdot 7ax} = 2a\sqrt[3]{7ax}.$
58. $\sqrt{4 - 4\sqrt{3}} = \sqrt{4(1 - \sqrt{3})} = 2\sqrt{1 - \sqrt{3}}.$
59. $\sqrt{18 + 9\sqrt{3}} = \sqrt{9(2 + \sqrt{3})} = 3\sqrt{2 + \sqrt{3}}.$
60. $\sqrt{100 - 25\sqrt{5}} = \sqrt{25(4 - \sqrt{5})} = 5\sqrt{4 - \sqrt{5}}.$
61. $\sqrt{9\sqrt{2} - 27} = \sqrt{9(\sqrt{2} - 3)} = 3\sqrt{\sqrt{2} - 3}.$
62. $\sqrt{R^2 - R^2\sqrt{3}} = \sqrt{R^2(1 - \sqrt{3})} = R\sqrt{1 - \sqrt{3}}.$
63. $\sqrt{R^2 + 4R^2\sqrt{5}} = \sqrt{R^2(1 + 4\sqrt{5})} = R\sqrt{1 + 4\sqrt{5}}.$
64. $\sqrt{5^2\sqrt{3} - 25^2} = \sqrt{25(\sqrt{3} - 25)} = 5\sqrt{\sqrt{3} - 25}.$
65. $\sqrt{a^2x^2 + a^3\sqrt{x}} = \sqrt{a^2(x^2 + a\sqrt{x})} = a\sqrt{x^2 + a\sqrt{x}}.$

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1. $\sqrt{\frac{3}{4}} = \sqrt{\frac{1}{4} \cdot 3} = \frac{1}{2} \sqrt{3}.$
2. $\sqrt{\frac{5}{9}} = \sqrt{\frac{1}{9} \cdot 5} = \frac{1}{3} \sqrt{5}.$
3. $\sqrt{\frac{3}{16}} = \sqrt{\frac{1}{16} \cdot 3} = \frac{1}{4} \sqrt{3}.$
4. $\sqrt{\frac{1}{2}} = \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{4} \cdot 2} = \frac{1}{2} \sqrt{2}.$
5. $\sqrt{\frac{2}{3}} = \sqrt{\frac{6}{9}} = \sqrt{\frac{1}{9} \cdot 6} = \frac{1}{3} \sqrt{6}.$
6. $\sqrt{\frac{3}{2}} = \sqrt{\frac{6}{4}} = \sqrt{\frac{1}{4} \cdot 6} = \frac{1}{2} \sqrt{6}.$
7. $\sqrt{\frac{3}{5}} = \sqrt{\frac{15}{25}} = \sqrt{\frac{1}{25} \cdot 15} = \frac{1}{5} \sqrt{15}.$
8. $\sqrt{\frac{5}{3}} = \sqrt{\frac{15}{9}} = \sqrt{\frac{1}{9} \cdot 15} = \frac{1}{3} \sqrt{15}.$
9. $\sqrt{\frac{4}{7}} = \sqrt{\frac{28}{49}} = \sqrt{\frac{4}{49} \cdot 7} = \frac{2}{7} \sqrt{7}.$
10. $\sqrt{\frac{7}{5}} = \sqrt{\frac{35}{25}} = \sqrt{\frac{1}{25} \cdot 35} = \frac{1}{5} \sqrt{35}.$
11. $\sqrt{\frac{5}{8}} = \sqrt{\frac{10}{16}} = \sqrt{\frac{1}{16} \cdot 10} = \frac{1}{4} \sqrt{10}.$
12. $\sqrt{\frac{5}{2}} = \sqrt{\frac{10}{4}} = \sqrt{\frac{1}{4} \cdot 10} = \frac{1}{2} \sqrt{10}.$
13. $3 \sqrt{\frac{3}{7}} = 3 \sqrt{\frac{21}{49}} = 3 \sqrt{\frac{1}{49} \cdot 21} = \frac{3}{7} \sqrt{21}.$
14. $2 \sqrt{\frac{2}{5}} = 2 \sqrt{\frac{10}{25}} = 2 \sqrt{\frac{1}{25} \cdot 10} = \frac{2}{5} \sqrt{10}.$
15. $\sqrt{\frac{8}{5}} = \sqrt{\frac{40}{25}} = \sqrt{\frac{4}{25} \cdot 10} = \frac{2}{5} \sqrt{10}.$
16. $\sqrt{\frac{5}{12}} = \sqrt{\frac{15}{36}} = \sqrt{\frac{1}{36} \cdot 15} = \frac{1}{6} \sqrt{15}.$
17. $3 \sqrt{\frac{7}{12}} = 3 \sqrt{\frac{21}{36}} = 3 \sqrt{\frac{1}{36} \cdot 21} = \frac{1}{2} \sqrt{21}.$
18. $2 \sqrt{\frac{5}{18}} = 2 \sqrt{\frac{10}{36}} = 2 \sqrt{\frac{1}{36} \cdot 10} = \frac{1}{3} \sqrt{10}.$
19. $3a \sqrt{\frac{3}{11}} = 3a \sqrt{\frac{33}{121}} = 3a \sqrt{\frac{1}{121} \cdot 33} = \frac{3a}{11} \sqrt{33}.$
20. $3a^2 \sqrt{\frac{3}{32}} = 3a^2 \sqrt{\frac{6}{64}} = 3a^2 \sqrt{\frac{1}{64} \cdot 6} = \frac{3a^2}{8} \sqrt{6}.$
21. $\sqrt{\frac{3}{4}} = \sqrt{\frac{1}{4} \cdot 3} = \frac{1}{2} \sqrt{3}.$
22. $\sqrt{1 - (\frac{1}{3})^2} = \sqrt{\frac{8}{9}} = \sqrt{\frac{4}{9} \cdot 2} = \frac{2}{3} \sqrt{2}.$
23. $\sqrt{9 - (\frac{3}{2})^2} = \sqrt{\frac{27}{4}} = \sqrt{\frac{9}{4} \cdot 3} = \frac{3}{2} \sqrt{3}.$
24. $\sqrt{25 - (\frac{5}{2})^2} = \sqrt{\frac{75}{4}} = \sqrt{\frac{25}{4} \cdot 3} = \frac{5}{2} \sqrt{3}.$
25. $\sqrt{2 + (\frac{3}{8})^2} = \sqrt{\frac{137}{64}} = \frac{1}{8} \sqrt{137}.$
26. $\sqrt{81 - (\frac{9}{2})^2} = \sqrt{\frac{243}{4}} = \sqrt{\frac{81}{4} \cdot 3} = \frac{9}{2} \sqrt{3}.$
27. $\sqrt{121 - (\frac{11}{2})^2} = \sqrt{\frac{363}{4}} = \sqrt{\frac{121}{4} \cdot 3} = \frac{11}{2} \sqrt{3}.$
28. $\sqrt{169 - (\frac{13}{2})^2} = \sqrt{\frac{507}{4}} = \sqrt{\frac{169}{4} \cdot 3} = \frac{13}{2} \sqrt{3}.$
29. $\sqrt{\frac{R}{3}} = \sqrt{\frac{3R}{9}} = \frac{1}{3} \sqrt{3R}.$
30. $\sqrt{\frac{R^2}{3}} = \sqrt{\frac{3R^2}{9}} = \frac{R}{3} \sqrt{3}.$
31. $\sqrt{\frac{3x^2}{4}} = \frac{x}{2} \sqrt{3}.$
32. $\sqrt{R^2 + R^2} = \sqrt{2R^2} = R \sqrt{2}.$

33. $\sqrt{R^2 - \left(\frac{R}{2}\right)^2} = \sqrt{\frac{3R^2}{4}} = \frac{R}{2}\sqrt{3}.$
34. $\sqrt{R^2 - \left(\frac{R}{4}\right)^2} = \sqrt{\frac{15R^2}{16}} = \frac{R}{4}\sqrt{15}.$
35. $\sqrt{\frac{3R^2}{4} - \frac{R^2}{4}}\sqrt{2} = \sqrt{\frac{R^2}{4}(3 - \sqrt{2})} = \frac{R}{2}\sqrt{3 - \sqrt{2}}.$
36. $\sqrt{\frac{R^2}{4} + \frac{R^2x}{2}} = \sqrt{\frac{R^2}{4}(1 + 2x)} = \frac{R}{2}\sqrt{1 + 2x}.$
37. $\sqrt{\frac{R^2}{4} + \frac{R^2}{2}}\sqrt{2} = \sqrt{\frac{R^2}{4}(1 + 2\sqrt{2})} = \frac{R}{2}\sqrt{1 + 2\sqrt{2}}.$
38. $\sqrt{R^2\sqrt{2} - \frac{R^2}{2}} = \sqrt{\frac{4R^2\sqrt{2} - 2R^2}{4}} = \sqrt{\frac{R^2}{4}(4\sqrt{2} - 2)} = \frac{R}{2}\sqrt{4\sqrt{2} - 2}.$
39. $\sqrt[3]{R^3 - \left(\frac{R}{2}\right)^3} = \sqrt[3]{\frac{7R^3}{8}} = \frac{R}{2}\sqrt[3]{7}.$
40. $\sqrt[3]{16 + 8\sqrt{2}} = \sqrt[3]{8(2 + \sqrt{2})} = 2\sqrt[3]{2 + \sqrt{2}}.$
41. $\sqrt[3]{81 + 3\sqrt{243}} = \sqrt[3]{81 + 27\sqrt{3}} = \sqrt[3]{27(3 + \sqrt{3})} = 3\sqrt[3]{3 + \sqrt{3}}.$

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2. $\sqrt{8^2 - 4^2} = \sqrt{48} = 4\sqrt{3}.$
 Area = $\frac{4 \cdot 4\sqrt{3}}{2} = 8\sqrt{3}.$
3. $\sqrt{R^2 - \left(\frac{R}{2}\right)^2} = \sqrt{\frac{3R^2}{4}} = \frac{R}{2}\sqrt{3}.$
 Area = $\frac{\left(\frac{R}{2}\sqrt{3} \cdot \frac{R}{2}\right)}{2} = \frac{R^2}{8}\sqrt{3}.$
4. $\sqrt{12^2 + 12^2} = \sqrt{288} = 12\sqrt{2}.$
5. $s^2 + s^2 = (12)^2.$
 $\therefore s = 6\sqrt{2}.$
6. $s^2 + s^2 = 4R^2.$
 $\therefore s = R\sqrt{2}.$
7. $x = \sqrt{36 + 36} = 6\sqrt{2}.$
8. $h^2 = 8^2 + 8^2 = 128.$
 $\therefore h = 8\sqrt{2}.$
 $h^2 = (13)^2 + (13)^2 = 338.$
 $\therefore h = 13\sqrt{2}.$
9. $x^2 = 50.$
 $\therefore x = 5\sqrt{2}.$
10. $x^2 + x^2 = 169.$
 $\therefore x = \frac{13}{2}\sqrt{2}.$
11. $s^2 = (20)^2 - (10)^2 = 300.$
 $\therefore s = 10\sqrt{3}.$
12. $s^2 = (12)^2 - (6)^2 = 108.$
 $s = 6\sqrt{3}.$
13. $s^2 = (20)^2 - (10)^2 = 300.$
 $s = 10\sqrt{3}, h = 20.$
14. $s^2 = (24)^2 - (12)^2 = 432.$
 $\therefore s = 12\sqrt{3}, h = 24.$
15. $a^2 = (12)^2 - (6)^2 = 108.$
 $a = 6\sqrt{3}.$
 Area = $\frac{12 \cdot 6\sqrt{3}}{2} = 36\sqrt{3}.$

$$16. \quad a^2 = S^2 - \left(\frac{S}{2}\right)^2 = \frac{3S^2}{4}.$$

$$a = \frac{S}{2} \sqrt{3}.$$

$$\text{Area} = \frac{S \cdot \frac{S}{2} \sqrt{3}}{2} = \frac{S^2}{4} \sqrt{3}.$$

$$17. \quad s^2 - \left(\frac{s}{2}\right)^2 = 100.$$

$$\frac{3s^2}{4} = 100.$$

$$\therefore s = \frac{20}{3} \sqrt{3}.$$

$$\text{Area} = \frac{\frac{20}{3} \sqrt{3} \cdot 10}{2} = \frac{100}{3} \sqrt{3}.$$

$$18. \quad s^2 + s^2 = (20)^2 = 400.$$

$$\therefore s = 10 \sqrt{2}.$$

$$19. \quad \text{Area} = \frac{s \cdot s}{2} = 50.$$

$$\therefore s = 10.$$

$$h^2 = (10)^2 + (10)^2.$$

$$h = 10 \sqrt{2}.$$

$$20. \quad h^2 = \left(\frac{R}{2}\right)^2 + \left(\frac{3R}{2}\right)^2 = \frac{10R^2}{4}.$$

$$\therefore h = \frac{R}{2} \sqrt{10}.$$

$$21. \quad h^2 = R^2 - \left(\frac{R}{5}\right)^2 = \frac{24R^2}{25}.$$

$$\therefore h = \frac{2R}{5} \sqrt{6}.$$

$$22. \quad x^2 + x^2 = R^2.$$

$$\therefore x = \frac{R}{2} \sqrt{2}.$$

$$23. \quad x = \frac{R}{2}.$$

$$\therefore y^2 = R^2 - \left(\frac{R}{2}\right)^2 = \frac{3R^2}{4}.$$

$$y = \frac{R}{2} \sqrt{3}.$$

$$24. \quad h^2 = R^2 + \left[\frac{R}{2}(a-1)\right]^2.$$

$$= \frac{R^2}{4}(a^2 - 2a + 5).$$

$$\therefore h = \frac{R}{2} \sqrt{a^2 - 2a + 5}.$$

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$$1. \quad \sqrt{27} + \sqrt{12} = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}.$$

$$2. \quad \sqrt{45} - \sqrt{20} = 3\sqrt{5} - 2\sqrt{5} = \sqrt{5}.$$

$$3. \quad 2\sqrt{200} - 3\sqrt{8} = 20\sqrt{2} - 6\sqrt{2} = 14\sqrt{2}.$$

$$4. \quad \sqrt{28} + \sqrt{63} = 2\sqrt{7} + 3\sqrt{7} = 5\sqrt{7}.$$

$$5. \quad 7\sqrt{18} - \sqrt{98} = 21\sqrt{2} - 7\sqrt{2} = 14\sqrt{2}.$$

$$6. \quad \sqrt{75} - \sqrt{27} + 2\sqrt{48} = 5\sqrt{3} - 3\sqrt{3} + 8\sqrt{3} = 10\sqrt{3}.$$

$$7. \quad \sqrt{2} + \sqrt{\frac{1}{2}} = \sqrt{2} + \frac{1}{2}\sqrt{2} = \frac{3}{2}\sqrt{2}.$$

$$8. \quad \sqrt{3} - \sqrt{\frac{1}{3}} = \sqrt{3} - \frac{1}{3}\sqrt{3} = \frac{2}{3}\sqrt{3}.$$

$$9. \quad \frac{3}{2} + \sqrt{\frac{3}{4}} = \frac{3}{2} + \frac{1}{2}\sqrt{3}.$$

$$10. \quad \frac{2}{3} + \sqrt{\frac{4}{9}} = \frac{2}{3} + \frac{2}{3}\sqrt{3}.$$

$$11. \quad \frac{R}{2} + \sqrt{\frac{9R^2}{2}} = \frac{R}{2} + \frac{3R}{2}\sqrt{2}.$$

$$12. \quad \frac{3R}{2} - \sqrt{\frac{9R^2}{2}} = \frac{3R}{2} - \frac{3R}{2}\sqrt{2}.$$

$$13. \quad R - \sqrt{\frac{3R^2}{4}} = R - \frac{R}{2}\sqrt{3}.$$

$$14. \quad 5\sqrt{\frac{1}{2}} - \frac{3}{2}\sqrt{2} = \frac{5}{2}\sqrt{2} - \frac{3}{2}\sqrt{2} = \sqrt{2}.$$

$$15. \quad 8\sqrt{\frac{9}{2}} - \sqrt{72} = 12\sqrt{2} - 6\sqrt{2} = 6\sqrt{2}.$$

16. $\sqrt{\frac{1}{3}} + 2\sqrt{\frac{4}{3}} - 3\sqrt{\frac{2}{3}} = \frac{1}{3}\sqrt{3} + \frac{4}{3}\sqrt{3} - \sqrt{6} = \frac{5}{3}\sqrt{3} - \sqrt{6}.$
17. $\sqrt{\frac{2}{5}} - \sqrt{\frac{9}{10}} + 2\sqrt{10} = \frac{1}{5}\sqrt{10} - \frac{3}{10}\sqrt{10} + 2\sqrt{10} = \frac{19}{10}\sqrt{10}.$
18. $\sqrt{\frac{3}{10}} - \sqrt{120} - 2\sqrt{\frac{6}{5}} = \frac{1}{10}\sqrt{30} - 2\sqrt{30} - \frac{2}{5}\sqrt{30} = -\frac{23}{10}\sqrt{30}.$
19. $\sqrt{\frac{5}{18}} + 2\sqrt{\frac{32}{5}} - \sqrt{\frac{10}{9}} = \frac{1}{6}\sqrt{10} + \frac{8}{5}\sqrt{10} - \frac{1}{3}\sqrt{10} = \frac{43}{30}\sqrt{10}.$
20. $\sqrt[3]{56} + 2\sqrt[3]{189} = 2\sqrt[3]{7} + 6\sqrt[3]{7} = 8\sqrt[3]{7}.$
21. $2\sqrt[3]{320} - \sqrt{50} = 8\sqrt[3]{5} - 5\sqrt{2}.$
22. $\sqrt[4]{25} - \sqrt{20} = \sqrt{5} - 2\sqrt{5} = -\sqrt{5}.$
23. $\sqrt[4]{9} + \sqrt{12} = \sqrt[4]{3^2} + 2\sqrt{3} = \sqrt{3} + 2\sqrt{3} = 3\sqrt{3}.$
24. $\sqrt[4]{32} + 5\sqrt[4]{162} = 2\sqrt[4]{2} + 15\sqrt[4]{2} = 17\sqrt[4]{2}.$
25. $2x\sqrt[3]{54x} - 3\sqrt[3]{16x^4} + \sqrt[6]{4x^2} = 6x\sqrt[3]{2x} - 6x\sqrt[3]{2x} + \sqrt[3]{2x} = \sqrt[3]{2x}.$
26. $\sqrt[3]{81x^7} + x\sqrt[3]{375x^4} - \sqrt[12]{16x^4} = 3x^2\sqrt[3]{3x} + 5x^2\sqrt[3]{3x} - \sqrt[3]{2x}$
 $= 8x^2\sqrt[3]{3x} - \sqrt[3]{2x}.$
27. $\sqrt{a^3bc} - a\sqrt{abc} + ac\sqrt{\frac{b}{ac}} = a\sqrt{abc} - a\sqrt{abc} + \sqrt{abc} = \sqrt{abc}.$
28. $rs\sqrt[3]{rs} + \sqrt[3]{\frac{1}{r^2s^2}} - 2\sqrt[3]{r^4s^4} = rs\sqrt[3]{rs} + \frac{1}{rs}\sqrt[3]{rs} - 2rs\sqrt[3]{rs}$
 $= \frac{\sqrt[3]{rs}}{rs} - rs\sqrt[3]{rs}.$
29. $\sqrt{a^3 + 4a^2 + 4a} - \sqrt{a^3} - \frac{2}{a^3}\sqrt{a^7} = (a+2)\sqrt{a} - a\sqrt{a} - 2\sqrt{a} = 0.$
30. $\sqrt{3x^2 - 18x + 27} - \sqrt{27(x^2 + 2x + 1)} = (x-3)\sqrt{3} - (3x+3)\sqrt{3}$
 $= -2x\sqrt{3} - 6\sqrt{3}.$

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1. $\sqrt{5} \cdot \sqrt{6} = \sqrt{30}.$
2. $\sqrt{7} \cdot \sqrt{7} = 7.$
3. $2\sqrt{3} \cdot 3\sqrt{2} = 6\sqrt{6}.$
4. $3\sqrt{2} \cdot 5\sqrt{2} = 30.$
5. $\sqrt{2} \cdot \sqrt{8} = 4.$
6. $\sqrt{3} \cdot \sqrt{27} = 9.$
7. $5^{\frac{1}{2}} \cdot 20^{\frac{1}{2}} = (100)^{\frac{1}{2}} = 10.$
8. $18^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (144)^{\frac{1}{2}} = 12.$
9. $\sqrt{\frac{12}{5}} \cdot \sqrt{75} = \sqrt{36} = 6.$
10. $\sqrt{\frac{2}{3}} \cdot \sqrt{\frac{27}{8}} = \frac{3}{2}.$
11. $\sqrt{11} \cdot \sqrt{\frac{1}{11}} = 1.$
12. $a^{\frac{1}{2}} \cdot (bc)^{\frac{1}{2}} = \sqrt{abc}.$
13. $R\sqrt{2} \cdot R\sqrt{3} = R^2\sqrt{6}.$
14. $\left(\sqrt{\frac{R}{2}} - \sqrt{2}\right)^2 = \frac{R}{2} - \sqrt{2}.$
15. $\frac{R}{2}\sqrt{2} \cdot R\sqrt{3} = \frac{R^2}{2}\sqrt{6}.$
16. $2\sqrt{x} \cdot \sqrt{4x^3} = 4x^2.$
17. $2\sqrt{rs} \cdot 7\sqrt{r^3s^3t^2} = 14r^2s^2t.$
18. $\sqrt{75a} \cdot (45a)^{\frac{1}{2}} = 15a\sqrt{15}.$
19. $\sqrt{2u} \cdot \sqrt{4v} \cdot \sqrt{6uv} = 4uv\sqrt{3}.$
20. $5\sqrt{3m} \cdot 5\sqrt{3m} = 75m.$
21. $(3\sqrt{3x})^2 = 27x.$
22. $\sqrt[3]{16} \cdot \sqrt[3]{4} = 4.$
23. $\sqrt[3]{4} \cdot \sqrt[3]{12} = 2\sqrt[3]{6}.$
24. $(100)^{\frac{1}{3}} \cdot (30)^{\frac{1}{3}} = (3000)^{\frac{1}{3}} = 10\sqrt[3]{3}.$

25. $\sqrt[4]{8} \cdot \sqrt[4]{32} = \sqrt[4]{256} = 4.$
 26. $5 \sqrt[3]{2a} \cdot 3 \sqrt[3]{16a} = 30 \sqrt[3]{4a^2}.$
 27. $(\sqrt{R} - \sqrt{2})^2 = R - \sqrt{2}.$
 28. $\left(\frac{R}{2} \sqrt{2 - \sqrt{5}}\right)^2 = \frac{R^2}{2} - \frac{R^2}{4} \sqrt{5}.$
 29. $\sqrt{\frac{a}{x}} \cdot \sqrt{\frac{4x}{a}} = 2.$
 30. $\sqrt{\frac{5x}{a}} \cdot \sqrt{\frac{a^3}{5x^2}} = \frac{a}{x} \sqrt{x}.$
 32. $7 \sqrt{3} = \sqrt{49} \sqrt{3} = \sqrt{147}.$
 33. $10 \sqrt{5} = \sqrt{100} \sqrt{5} = \sqrt{500}.$
 34. $8 \sqrt{\frac{1}{2}} = \sqrt{64} \sqrt{\frac{1}{2}} = \sqrt{32}.$
 35. $12 \sqrt{\frac{5}{3}} = \sqrt{144} \cdot \sqrt{\frac{5}{3}} = \sqrt{240}.$
 36. $a \sqrt{x} = \sqrt{a^2} \sqrt{x} = \sqrt{a^2 x}.$
 37. $2a \sqrt{c} = \sqrt{4a^2} \sqrt{c} = \sqrt{4a^2 c}.$
 38. $\sqrt[3]{27} \cdot \sqrt[3]{2} = \sqrt[3]{54}.$
 39. $5 \sqrt[3]{2} = \sqrt[3]{125} \sqrt[3]{2} = \sqrt[3]{250}.$
 40. $2 \sqrt[3]{3} = \sqrt[3]{8} \sqrt[3]{3} = \sqrt[3]{24}.$
 41. $2 \sqrt[3]{4} = \sqrt[3]{8} \cdot \sqrt[3]{4} = \sqrt[3]{32}.$
 42. $3 \sqrt[3]{\frac{1}{3}} = \sqrt[3]{27} \sqrt[3]{\frac{1}{3}} = \sqrt[3]{9}.$
 43. $a \sqrt{\frac{1}{a}} = \sqrt[3]{a^3} \sqrt[3]{\frac{1}{a}} = \sqrt[3]{a^2}.$
 44. $(a+x) \sqrt[3]{\frac{1}{a+x}} = \sqrt[3]{(a+x)^3} \sqrt[3]{\frac{1}{a+x}} = \sqrt[3]{(a+x)^2}.$

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1.
$$\begin{array}{r} \sqrt{2} - 3 \\ 3\sqrt{2} + 5 \\ + 6 - 9\sqrt{2} \\ + 5\sqrt{2} - 15 \\ \hline -4\sqrt{2} - 9 \end{array}$$
2.
$$\begin{array}{r} 2\sqrt{5} - 4 \\ 3\sqrt{5} + 3 \\ 30 - 12\sqrt{5} \\ + 6\sqrt{5} - 12 \\ \hline 18 - 6\sqrt{5} \end{array}$$
3.
$$\begin{array}{r} 4\sqrt{7} - 3 \\ 4\sqrt{7} - 3 \\ 112 - 12\sqrt{7} \\ - 12\sqrt{7} + 9 \\ \hline 121 - 24\sqrt{7} \end{array}$$
4.
$$\begin{array}{r} 2\sqrt{7} - 3\sqrt{2} \\ 2\sqrt{7} - 3\sqrt{2} \\ 28 - 6\sqrt{14} \\ - 6\sqrt{14} + 18 \\ \hline 46 - 12\sqrt{14} \end{array}$$
5.
$$\begin{array}{r} 3\sqrt{2} + \sqrt{3} \\ -\sqrt{2} + 2\sqrt{3} \\ \hline -6 - \sqrt{6} \\ + 6\sqrt{6} + 6 \\ \hline 5\sqrt{6} \end{array}$$
6.
$$\begin{array}{r} 2\sqrt{5} - 3\sqrt{2} \\ 3\sqrt{5} + 2\sqrt{2} \\ 30 - 9\sqrt{10} \\ + 4\sqrt{10} - 12 \\ \hline 18 - 5\sqrt{10} \end{array}$$
7.
$$\begin{array}{r} \sqrt{5} - \sqrt{3} + \sqrt{2} \\ \sqrt{5} - \sqrt{3} + \sqrt{2} \\ \hline 5 - \sqrt{15} + \sqrt{10} \\ + 3 - \sqrt{15} - \sqrt{6} \\ + 2 + \sqrt{10} - \sqrt{6} \\ \hline 10 - 2\sqrt{15} + 2\sqrt{10} - 2\sqrt{6} \end{array}$$
8.
$$\begin{array}{r} 3\sqrt{2} + 2\sqrt{3} + \sqrt{30} \\ \sqrt{2} + \sqrt{3} - \sqrt{5} \\ \hline 6 + 2\sqrt{6} + 2\sqrt{15} \\ 6 + 3\sqrt{6} + 3\sqrt{10} \\ - 5\sqrt{6} - 2\sqrt{15} - 3\sqrt{10} \\ \hline 12 \end{array}$$
9.
$$\begin{array}{r} R\sqrt{2} - 2 \\ R\sqrt{2} - 3 \\ \hline 2R^2 - 2R\sqrt{2} \\ - 3R\sqrt{2} + 6 \\ \hline 2R^2 - 5R\sqrt{2} + 6 \end{array}$$
10.
$$\begin{array}{r} R\sqrt{3} - R\sqrt{2} \\ R\sqrt{3} + 2R\sqrt{2} \\ \hline 3R^2 - R^2\sqrt{6} \\ - 4R^2 + 2R^2\sqrt{6} \\ \hline -R^2 + R^2\sqrt{6} \end{array}$$

$$11. R - \frac{R}{2}\sqrt{2}$$

$$R - \frac{R}{2}\sqrt{2}$$

$$\frac{R^2 - \frac{R^2}{2}\sqrt{2}}{}$$

$$- \frac{R^2}{2}\sqrt{2} + \frac{R^2}{2}$$

$$\frac{3R^2}{2} - R^2\sqrt{2}$$

$$12. \frac{R}{2} + \frac{R}{2}\sqrt{3}$$

$$\frac{R}{2} + \frac{R}{2}\sqrt{3}$$

$$\frac{R^2}{4} + \frac{R^2}{4}\sqrt{3}$$

$$+ \frac{R^2}{4}\sqrt{3} + \frac{3R^2}{4}$$

$$R^2 + \frac{R^2}{2}\sqrt{3}$$

$$13. \frac{R}{2}(\sqrt{5} - 1)^2 = \frac{R}{2}(6 - 2\sqrt{5}) = 3R - R\sqrt{5}.$$

$$14. \left(\frac{R}{2}\sqrt{5} - \frac{R}{2}\right)^2 = \frac{3R^2}{2} - \frac{R^2}{2}\sqrt{5}.$$

$$R^2 - \left(\frac{3R^2}{2} - \frac{R^2}{2}\sqrt{5}\right) = -\frac{R^2}{2} + \frac{R^2}{2}\sqrt{5}.$$

$$15. R^2 - \left(\frac{R}{2} - \frac{R}{2}\sqrt{2}\right)^2 = R^2 - \left(\frac{R^2}{4} - \frac{R^2}{2}\sqrt{2} + \frac{R^2}{2}\right) = \frac{R^2}{4} + \frac{R^2}{2}\sqrt{2}$$

$$16. \sqrt{R^2 - \left(\frac{R}{2} - \frac{R}{2}\sqrt{3}\right)^2} = \sqrt{R^2 - \left(\frac{R^2}{4} - \frac{R^2}{2}\sqrt{3} + \frac{3R^2}{4}\right)}$$

$$= \sqrt{\frac{R^2}{2}\sqrt{3}} = \frac{R}{2}\sqrt{2\sqrt{3}}.$$

$$17. \sqrt{R^2 - \left(\frac{R\sqrt{5} - R}{4}\right)^2} = \sqrt{R^2 - \frac{5R^2 - 2R^2\sqrt{5} + R^2}{16}}$$

$$= \sqrt{\frac{10R^2 + 2R^2\sqrt{5}}{16}} = \frac{R}{4}\sqrt{10 + 2\sqrt{5}}.$$

$$18. l = \sqrt{R^2 - \left(R - \frac{R}{2}\sqrt{3}\right)^2} = \sqrt{R^2 - R^2 + R^2\sqrt{3} - \frac{3R^2}{4}}$$

$$= \frac{R}{2}\sqrt{4\sqrt{3} - 3}.$$

$$19. l = \sqrt{R^2 - \left[\frac{R}{2}(\sqrt{5} - 1)\right]^2} = \sqrt{R^2 - \frac{5R^2}{4} + \frac{2R^2\sqrt{5}}{4} - \frac{R^2}{4}}$$

$$= \frac{R}{2}\sqrt{2\sqrt{5} - 2}.$$

$$\begin{aligned}
 20. \quad h &= \sqrt{(R\sqrt{2+\sqrt{2}})^2 + \left(\frac{R}{2}\sqrt{2-\sqrt{2}}\right)^2} \\
 &= \sqrt{2R^2 + R^2\sqrt{2} + \frac{2R^2}{4} - \frac{R^2}{4}\sqrt{2}} \\
 &= \frac{R}{2}\sqrt{10 + 3\sqrt{2}}.
 \end{aligned}$$

$$\text{Area} = \frac{1}{2} \left(R\sqrt{2+\sqrt{2}} \right) \left(\frac{R}{2}\sqrt{2-\sqrt{2}} \right) = \frac{R^2}{4} \sqrt{4-2} = \frac{R^2}{4} \sqrt{2}.$$

$$21. \quad (2 + 2\sqrt{3})^2 - 4(2 + 2\sqrt{3}) = 4 + 8\sqrt{3} + 12 - 8 - 8\sqrt{3} = 8.$$

$2 + 2\sqrt{3}$ is a root.

$$\begin{aligned}
 22. \quad \left(\frac{7}{2} - \frac{1}{2}\sqrt{41}\right)^2 - 7\left(\frac{7}{2} - \frac{1}{2}\sqrt{41}\right) + 2 \\
 = \frac{49}{4} - \frac{7}{2}\sqrt{41} + \frac{41}{4} - \frac{49}{2} + \frac{7}{2}\sqrt{41} + 2 \\
 = \frac{90}{4} - \frac{98}{4} + 2 = 0. \quad \text{Yes.}
 \end{aligned}$$

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$$\begin{array}{lll}
 1. \quad \sqrt{2x-7} = 3. & 2. \quad \sqrt{3x-5} = 5. & 3. \quad \sqrt{x-6} = \sqrt{5}. \\
 2x-7 = 9. & 3x-5 = 25. & x-6 = 5. \\
 2x = 16. & 3x = 30. & x = 11. \\
 x = 8. & x = 10. & \text{Check. } \sqrt{11-6} = \sqrt{5}. \\
 \text{Check. } \sqrt{16-7} = 3. & \text{Check. } \sqrt{30-5} = 5. & \sqrt{5} = \sqrt{5}. \\
 3 = 3. & 5 = 5. &
 \end{array}$$

$$\begin{array}{ll}
 4. \quad \sqrt{2x+3} = \sqrt{7}. & 6. \quad \sqrt{x-3} - 6 = 0. \\
 2x+3 = 7. & \sqrt{x-3} = 6. \\
 2x = 4. & x-3 = 36. \\
 x = 2. & x = 39.
 \end{array}$$

$$\begin{array}{ll}
 \text{Check. } \sqrt{4+3} = \sqrt{7}. & \text{Check. } \sqrt{39-3} - 6 = 0. \\
 \sqrt{7} = \sqrt{7}. & \sqrt{36} - 6 = 0. \\
 & 6 - 6 = 0.
 \end{array}$$

$$\begin{aligned}
 5. \quad 2\sqrt{x-1} &= \sqrt{6}. \\
 4x-4 &= 6. \\
 4x &= 10. \\
 x &= \frac{5}{2}.
 \end{aligned}$$

$$\begin{aligned}
 \text{Check. } 2\sqrt{\frac{5}{2}-1} &= \sqrt{6}. \\
 2\sqrt{\frac{3}{2}} &= \sqrt{6}. \\
 \sqrt{4 \cdot \frac{3}{2}} &= \sqrt{6}. \\
 \sqrt{6} &= \sqrt{6}.
 \end{aligned}$$

$$\begin{aligned}
 7. \quad 2\sqrt{3x+1} - 8 &= 0. \\
 12x+4 &= 64. \\
 12x &= 60. \\
 x &= 5.
 \end{aligned}$$

$$\begin{aligned}
 \text{Check. } 2\sqrt{15+1} - 8 &= 0. \\
 8 - 8 &= 0.
 \end{aligned}$$

$$8. \quad 2\sqrt{5x-4} + 3 = 11.$$

$$5x - 4 = 16.$$

$$5x = 20.$$

$$x = 4.$$

$$\text{Check. } 2\sqrt{20-4} + 3 = 11.$$

$$2 \cdot 4 + 3 = 11.$$

$$11 = 11.$$

$$9. \quad \sqrt[3]{x-1} = 2.$$

$$x - 1 = 8.$$

$$x = 9.$$

$$\text{Check. } \sqrt[3]{9-1} = 2.$$

$$\sqrt[3]{8} = 2.$$

$$10. \quad \sqrt[3]{2x+1} = 3.$$

$$2x + 1 = 27.$$

$$2x = 26.$$

$$x = 13.$$

$$\text{Check. } \sqrt[3]{26+1} = 3.$$

$$3 = 3.$$

$$11. \quad \sqrt[3]{4x-6} = 4.$$

$$4x - 6 = 64.$$

$$x = \frac{35}{2}.$$

$$\text{Check. } \sqrt[3]{70-6} = 4.$$

$$4 = 4.$$

$$12. \quad \sqrt[3]{2x-1} + 5 = 0.$$

$$2x - 1 = -125.$$

$$x = -62.$$

$$\text{Check. } \sqrt[3]{-124-1} + 5 = 0.$$

$$\sqrt[3]{-125} + 5 = 0.$$

$$-5 + 5 = 0.$$

$$13. \quad 3\sqrt[3]{2x+7} = 6.$$

$$2x + 7 = 8.$$

$$x = \frac{1}{2}.$$

$$\text{Check. } 3\sqrt[3]{1+7} = 6.$$

$$6 = 6.$$

$$14. \quad 2\sqrt[3]{3x-9} + 4 = 0.$$

$$3x - 9 = -8.$$

$$x = \frac{1}{3}.$$

$$\text{Check. } 2\sqrt[3]{1-9} + 4 = 0.$$

$$-4 + 4 = 0.$$

$$15. \quad 3\sqrt{x-2} = \sqrt{2x+3}.$$

$$9x - 18 = 2x + 3.$$

$$7x = 21.$$

$$x = 3.$$

$$\text{Check. } 3\sqrt{3-2} = \sqrt{6+3}.$$

$$3 = 3.$$

$$16. \quad 2\sqrt{x+3} = \sqrt{x+18}.$$

$$4x + 12 = x + 18.$$

$$3x = 6.$$

$$x = 2.$$

$$\text{Check. } 2\sqrt{2+3} = \sqrt{2+18}.$$

$$2\sqrt{5} = 2\sqrt{5}.$$

$$17. \quad 3\sqrt{2x-6} = 2\sqrt{x+4}.$$

$$18x - 54 = 4x + 16.$$

$$x = 5.$$

$$\text{Check. } 3\sqrt{10-6} = 2\sqrt{5+4}.$$

$$6 = 6.$$

$$18. \quad \sqrt{2x-1} - \sqrt{2-x} = 0.$$

$$2x - 1 = 2 - x.$$

$$x = 1.$$

$$\text{Check. } \sqrt{2-1} - \sqrt{2-1} = 0.$$

$$1 - 1 = 0.$$

$$19. \quad \sqrt{4x-3} - \sqrt{3x+5} = 0.$$

$$4x - 3 = 3x + 5.$$

$$x = 8.$$

$$\text{Check. } \sqrt{32-3} - \sqrt{24+5} = 0.$$

$$\sqrt{29} - \sqrt{29} = 0.$$

$$20. \quad \sqrt{6x-5} - \sqrt{5x+7} = 0.$$

$$6x-5 = 5x+7.$$

$$x = 12.$$

Check.

$$\sqrt{72-5} - \sqrt{60+7} = 0.$$

$$\sqrt{67} - \sqrt{67} = 0.$$

$$21. \quad 2\sqrt{3x-3} - 3\sqrt{2+x} = 0.$$

$$12x-12 = 18+9x.$$

$$x = 10.$$

$$\text{Check. } 2\sqrt{30-3} - 3\sqrt{2+10} = 0.$$

$$6\sqrt{3} - 6\sqrt{3} = 0.$$

22. From Hint

$$x+3 = 4.$$

$$x = 1.$$

Check.

$$3 - \sqrt{1+3} = \sqrt{1}.$$

$$3 - 2 = 1.$$

$$1 = 1.$$

24.

$$7 - \sqrt{3x} = \sqrt{3x+7}.$$

$$49 - 14\sqrt{3x} + 3x = 3x+7.$$

$$\sqrt{3x} = 3.$$

$$x = 3.$$

Check.

$$7 - \sqrt{9} = \sqrt{9+7}.$$

$$7 - 3 = 4.$$

23.

$$5 - \sqrt{2x+5} = \sqrt{2x}.$$

$$25 - 10\sqrt{2x+5} + 2x+5 = 2x.$$

$$\sqrt{2x+5} = 3.$$

$$2x+5 = 9.$$

$$x = 2.$$

Check.

$$5 - \sqrt{4+5} = \sqrt{4}.$$

$$5 - 3 = 2.$$

$$25. \quad \sqrt{3x-5} + \sqrt{3x+7} = 6.$$

$$3x-5 = 3x+7-12\sqrt{3x+7}+36.$$

$$\sqrt{3x+7} = 4.$$

$$3x+7 = 16.$$

$$x = 3.$$

Check.

$$\sqrt{9-5} + \sqrt{9+7} = 6.$$

$$2 + 4 = 6.$$

26.

$$\sqrt{x} + \sqrt{2} = \sqrt{x+2}.$$

$$x + 2\sqrt{2x} + 2 = x+2.$$

$$x = 0.$$

Check.

$$0 + \sqrt{2} - \sqrt{2} = 0.$$

27.

$$\sqrt{x+1} - \sqrt{2x-3} = \sqrt{3x-2}.$$

$$x+1 - 2\sqrt{(x+1)(2x-3)} + 2x-3 = 3x-2.$$

$$\sqrt{(x+1)(2x-3)} = 0.$$

$$x+1 = 0, \text{ and } x = -1.$$

$$2x-3 = 0, \text{ and } x = \frac{3}{2}.$$

Check.

$$\sqrt{-1+1} - \sqrt{-2-3} = \sqrt{-3-2}.$$

$$-\sqrt{-5} = \sqrt{-5}.$$

$\therefore -1$ is not a root.

$$\sqrt{\frac{3}{2}+1} - \sqrt{3-3} = \sqrt{\frac{9}{2}-2}.$$

$$\sqrt{\frac{5}{2}} - 0 = \sqrt{\frac{5}{2}}.$$

$\therefore \frac{3}{2}$ is a root.

28.

$$\begin{aligned}\sqrt{x-3} + \sqrt{2x+4} &= \sqrt{3x+1}. \\ x-3 + 2\sqrt{(x-3)(2x+4)} + 2x+4 &= 3x+1. \\ \sqrt{(x-3)(2x+4)} &= 0. \\ \therefore x-3 &= 0, \text{ and } x=3. \\ 2x+4 &= 0, \text{ and } x=-2.\end{aligned}$$

Check.

$$\begin{aligned}\sqrt{3-3} + \sqrt{6+4} &= \sqrt{9+1}. \\ 0 + \sqrt{10} &= \sqrt{10}. \\ \therefore x=3 &\text{ is a root.} \\ \sqrt{-2-3} + \sqrt{-4+4} &= \sqrt{-6+1}. \\ \sqrt{-5} + 0 &= \sqrt{-5}. \\ \therefore x=-2 &\text{ is a root.}\end{aligned}$$

29.

$$\begin{aligned}\sqrt{x+3} + \sqrt{x-4} &= \sqrt{4x-3}. \\ x+3 + 2\sqrt{(x+3)(x-4)} + x-4 &= 4x-3. \\ \sqrt{(x+3)(x-4)} &= x-1. \\ x^2 - x - 12 &= x^2 - 2x + 1. \\ \therefore x &= 13.\end{aligned}$$

Check.

$$\begin{aligned}\sqrt{13+3} + \sqrt{13-4} &= \sqrt{52-3}. \\ 4+3 &= 7.\end{aligned}$$

30.

$$\begin{aligned}\sqrt{x+4} + \sqrt{x-3} &= \sqrt{4x-3}. \\ x+4 + 2\sqrt{x^2+x-12} + x-3 &= 4x-3. \\ \sqrt{x^2+x-12} &= x-2. \\ x^2+x-12 &= x^2-4x+4. \\ 5x &= 16. \\ x &= \frac{16}{5}.\end{aligned}$$

Check.

$$\begin{aligned}\sqrt{\frac{16}{5}+4} + \sqrt{\frac{16}{5}-3} &= \sqrt{\frac{64}{5}-3}. \\ 6\sqrt{\frac{1}{5}} + \sqrt{\frac{1}{5}} &= 7\sqrt{\frac{1}{5}}.\end{aligned}$$

31.

$$\begin{aligned}\frac{\sqrt{x-3}}{\sqrt{x+1}} &= \frac{\sqrt{x-4}}{\sqrt{x-2}}. \\ x^2-5x+6 &= x^2-3x-4. \\ x &= 5.\end{aligned}$$

32.

$$\begin{aligned}\frac{\sqrt{x-3}}{\sqrt{x+2}} &= \frac{\sqrt{x-5}}{\sqrt{x-1}}. \\ x^2-4x+3 &= x^2-3x-10. \\ x &= 13.\end{aligned}$$

Check.

$$\begin{aligned}\frac{\sqrt{5-3}}{\sqrt{5+1}} &= \frac{\sqrt{5-4}}{\sqrt{5-2}}. \\ \sqrt{\frac{2}{6}} &= \sqrt{\frac{1}{3}}.\end{aligned}$$

Check.

$$\begin{aligned}\frac{\sqrt{13-3}}{\sqrt{13+2}} &= \frac{\sqrt{13-5}}{\sqrt{13-1}}. \\ \frac{\sqrt{10}}{\sqrt{15}} &= \frac{\sqrt{8}}{\sqrt{12}}, \\ \text{or } \sqrt{\frac{2}{3}} &= \sqrt{\frac{2}{3}}.\end{aligned}$$

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1. $\sqrt{10} \div \sqrt{2} = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}.$
2. $\sqrt{18} \div \sqrt{3} = \frac{\sqrt{18}}{\sqrt{3}} = \sqrt{6}.$
4. $\sqrt{7} \div \sqrt{3} = \frac{\sqrt{7}\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{1}{3}\sqrt{21}.$
5. $2\sqrt{6} \div 3\sqrt{2} = \frac{2\sqrt{6}}{3\sqrt{2}} = \frac{2}{3}\sqrt{3}.$
6. $\sqrt{10} \div \sqrt{3} = \frac{\sqrt{10} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{1}{3}\sqrt{30}.$
7. $2\sqrt{7} \div \sqrt{2} = \frac{2\sqrt{7}\sqrt{2}}{\sqrt{2}\sqrt{2}} = \sqrt{14}.$
8. $\sqrt{6} \div \sqrt{18} = \frac{\sqrt{6}}{\sqrt{18}} = \sqrt{\frac{1}{3}} = \frac{1}{3}\sqrt{3}.$
9. $6 \div 2\sqrt{2} = \frac{6\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \frac{3}{2}\sqrt{2}.$
10. $8\sqrt{15} \div 4\sqrt{5} = \frac{8\sqrt{15}}{\sqrt{5}} = 2\sqrt{3}.$
11. $3\sqrt{2} \div 15\sqrt{8} = \frac{3\sqrt{2}}{15\sqrt{8}} = \frac{1}{5\sqrt{4}} = \frac{1}{10}.$
12. $\sqrt{6} - \sqrt{18} \div \sqrt{2} = \sqrt{6} - \frac{\sqrt{18}}{\sqrt{2}} = \sqrt{6} - 3.$
13. $(\sqrt{12} - \sqrt{24}) \div 3\sqrt{3} = \frac{\sqrt{12}}{3\sqrt{3}} - \frac{\sqrt{24}}{3\sqrt{3}} = \frac{2}{3} - \frac{2}{3}\sqrt{2}.$
14. $(\sqrt{6} - \sqrt{9} + 18) \div 2\sqrt{2} = \frac{\sqrt{6}}{2\sqrt{2}} + \frac{15\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \frac{1}{2}\sqrt{3} + \frac{15}{4}\sqrt{2}.$
16. $9 \div (3 - \sqrt{2}) = \frac{9(3 + \sqrt{2})}{(3 - \sqrt{2})(3 + \sqrt{2})} = \frac{27 + 9\sqrt{2}}{9 - 2} = \frac{27}{7} + \frac{9}{7}\sqrt{2}.$
17. $\frac{\sqrt{5}}{\sqrt{5} - \sqrt{2}} = \frac{\sqrt{5}(\sqrt{5} + \sqrt{2})}{(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})} = \frac{5 + \sqrt{10}}{5 - 2} = \frac{5}{3} + \frac{1}{3}\sqrt{10}.$
18. $\frac{2\sqrt{3}}{\sqrt{3} + \sqrt{2}} = \frac{2\sqrt{3}(\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})} = 6 - 2\sqrt{6}.$
19. $\frac{4\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} = \frac{(4\sqrt{5} + \sqrt{3})(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})} = \frac{23 + 5\sqrt{15}}{2}.$
20. $\frac{\sqrt{3} + \sqrt{7}}{\sqrt{7} - 2} = \frac{(\sqrt{3} + \sqrt{7})(\sqrt{7} + 2)}{(\sqrt{7} - 2)(\sqrt{7} + 2)} = \frac{7 + \sqrt{21} + 2\sqrt{3} + 2\sqrt{7}}{3}.$
21. $5 + 3\sqrt{2} = 5 + 3(1.4142) = 5 + 4.242 = 9.242.$
22. $7 - 3\sqrt{5} = 7 - 3(2.23607) = 7 - 6.7082 = .291.$
23. $\frac{7 + \sqrt{6}}{3} = \frac{7 + 2.449}{3} = 3.149.$
24. $\frac{\sqrt{2}}{\sqrt{3}} = \frac{1}{3}\sqrt{6} = \frac{1}{3}(2.449) = .816.$

$$25. \frac{\sqrt{5} + 1}{\sqrt{5}} = 1 + \frac{1}{5}\sqrt{5} = 1 + \frac{1}{5}(2.236) = 1.447.$$

$$26. \frac{2}{\sqrt{3} - \sqrt{2}} = \frac{2\sqrt{3} + 2\sqrt{2}}{3 - 2} = 2(1.7320) + 2(1.4142) = 6.292.$$

$$27. 3\sqrt{8} - \sqrt{7} = 3(2.8284) - 2.6457 = 5.839.$$

$$28. \sqrt{3 - \sqrt{2}} = \sqrt{3 - 1.414213} \\ = 1.259.$$

$$30. \frac{\sqrt{3a^2}}{\sqrt{ax^2}} = \frac{1}{x}\sqrt{3a}.$$

$$29. \frac{\sqrt{a}}{\sqrt{x}} = \frac{\sqrt{a}\sqrt{x}}{\sqrt{x}\sqrt{x}} = \frac{1}{x}\sqrt{ax}.$$

$$31. \frac{2\sqrt{ax}}{3\sqrt{bx}} = \frac{2}{3}\sqrt{\frac{a}{b}} = \frac{2}{3b}\sqrt{ab}.$$

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1.

Transposing,

Adding $(-\frac{4}{2})^2 = 4$,

Extracting the square root,

$$x^2 - 4x - 32 = 0.$$

$$x^2 - 4x = 32.$$

$$x^2 - 4x + 4 = 36.$$

$$x - 2 = \pm 6.$$

$$x = 8, -4.$$

2.

Transposing,

Completing the square,

Extracting the square root,

$$x^2 - 2x - 15 = 0.$$

$$x^2 - 2x = 15.$$

$$x^2 - 2x + 1 = 16.$$

$$x - 1 = \pm 4.$$

$$x = 5, -3.$$

3.

Extracting the square root,

$$x^2 = 18.$$

$$x = \pm 4.24.$$

4.

Transposing,

Extracting the square root,

$$x^2 - 25 = 0.$$

$$x^2 = 25.$$

$$x = \pm 5.$$

5.

Transposing,

Completing the square,

Extracting the square root,

$$x^2 - 7x - 18 = 0.$$

$$x^2 - 7x = 18.$$

$$x^2 - 7x + \frac{49}{4} = 18 + \frac{49}{4} = 1\frac{21}{4}.$$

$$x - \frac{7}{2} = \pm \frac{11}{2}.$$

$$x = 9, -2.$$

$$6. x(x + 4) - 3(x + 4) = 0.$$

Removing parentheses and combining,

$$x^2 + x - 12 = 0.$$

$$x^2 + x = 12.$$

$$x^2 + x + \frac{1}{4} = 12 + \frac{1}{4} = 12\frac{1}{4}.$$

$$x + \frac{1}{2} = \pm \frac{7}{2}.$$

$$x = 3, -4.$$

7.

$$y + 6 = y^2.$$

$$y^2 - y = 6.$$

Completing the square,

$$y^2 - y + \frac{1}{4} = 6 + \frac{1}{4}.$$

$$y - \frac{1}{2} = \pm \frac{5}{2}.$$

$$y = 3, -2.$$

$$\begin{aligned}
 8. \quad & 4x^2 - 4x - 3 = 0. \\
 & x^2 - x - \frac{3}{4} = 0. \\
 & x^2 - x = \frac{3}{4}. \\
 & x^2 - x + \frac{1}{4} = 1. \\
 & x - \frac{1}{2} = \pm 1. \\
 & x = \frac{3}{2}, -\frac{1}{2}.
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & z^2 - 5\frac{1}{2}z + 2\frac{1}{2} = 0. \\
 & z^2 - \frac{11}{2}z = -\frac{5}{2}. \\
 & z^2 - \frac{11}{2}z + (-\frac{11}{4})^2 = -\frac{5}{2} + \frac{121}{16} \\
 & \quad = \frac{81}{16}. \\
 & z - \frac{11}{4} = \pm \frac{9}{4}. \\
 & z = 5, \frac{1}{2}.
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & \frac{5}{4} - x^2 = 2x. \\
 & x^2 + 2x = \frac{5}{4}. \\
 & x^2 + 2x + 1 = \frac{9}{4}. \\
 & x + 1 = \pm \frac{3}{2}. \\
 & x = \frac{1}{2}, -\frac{5}{2}.
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & 2x^2 - 9x + 4 = 0. \\
 & x^2 - \frac{9}{2}x + 2 = 0. \\
 & x^2 - \frac{9}{2}x = -2. \\
 & x^2 - \frac{9}{2}x + (-\frac{9}{4})^2 = -2 + \frac{81}{16} \\
 & \quad = \frac{49}{16}. \\
 & x - \frac{9}{4} = \pm \frac{7}{4}. \\
 & x = 4, \frac{1}{2}.
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & 2t^2 - 3t = 9. \\
 & t^2 - \frac{3}{2}t = \frac{9}{2}. \\
 & t^2 - \frac{3}{2}t + (-\frac{3}{4})^2 = \frac{81}{4} + \frac{9}{16}. \\
 & \quad = \frac{209}{16}. \\
 & t - \frac{3}{4} = \pm \frac{9}{4}. \\
 & t = 3, -\frac{3}{2}.
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & 3x^2 - 5 = 70. \\
 & 3x^2 = 75. \\
 & x^2 = 25. \\
 & x = \pm 5.
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & x^2 + 8 = 4x^2 - 40. \\
 & 3x^2 = 48. \\
 & x^2 = 16. \\
 & x = \pm 4.
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & 5x = 6x^2 - 14. \\
 & 14 = 6x^2 - 5x. \\
 & \frac{7}{3} = x^2 - \frac{5}{6}x. \\
 & \frac{7}{3} + \frac{25}{144} = \frac{361}{144} \\
 & \quad = x^2 - \frac{5}{6}x + (-\frac{5}{12})^2. \\
 & \pm \frac{1}{12} = x - \frac{5}{12}. \\
 & x = 2, -\frac{7}{6}.
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & x^2 - \frac{7x}{12} - 1 = 0. \\
 & x^2 - \frac{7}{12}x = 1. \\
 & x^2 - \frac{7}{12}x + (-\frac{7}{24})^2 = 1 + \frac{49}{576} \\
 & \quad = \frac{625}{576}. \\
 & x - \frac{7}{24} = \pm \frac{25}{24}. \\
 & x = \frac{4}{3}, -\frac{3}{4}.
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & 6t^2 - 13t + 6 = 0. \\
 & t^2 - \frac{13}{6}t = -1. \\
 & t^2 - \frac{13}{6}t + (-\frac{13}{12})^2 = -1 + \frac{169}{144} \\
 & \quad = \frac{25}{144}. \\
 & t - \frac{13}{12} = \pm \frac{5}{12}. \\
 & t = \frac{3}{2}, \frac{2}{3}.
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & 42 + 2x^2 = -19x. \\
 & x^2 + \frac{19}{2}x = -21. \\
 & x^2 + \frac{19}{2}x + (\frac{19}{4})^2 = -21 + \frac{361}{16} \\
 & \quad = \frac{25}{16}. \\
 & x + \frac{19}{4} = \pm \frac{5}{4}. \\
 & x = -6, -\frac{7}{2}.
 \end{aligned}$$

19. Writing the double sign before both members would be equivalent to writing the two equations

$$x + 2 = \pm 5, \quad (1)$$

$$-(x + 2) = \pm 5. \quad (2)$$

But (2) may be multiplied in both members by -1 , giving

$$x + 2 = \mp 5. \quad (3)$$

Now (3) plainly is equivalent to (1), and because of the way in which it was obtained it is equivalent to (2). Hence (2) and (1) are equivalent, and no root is retained by attaching the double sign to both members.

$$21. \quad p^2 + 10p + 17 = 0.$$

$$p^2 + 10p = -17.$$

$$p^2 + 10p + 25 = 8.$$

$$p + 5 = \pm 2\sqrt{2}.$$

$$p = -5 \pm 2\sqrt{2}.$$

Extracting,

$$\sqrt{2} = 1.4142.$$

Hence

$$p = -5 \pm 2.8284.$$

$$p = -7.828,$$

$$-2.172.$$

$$22. \quad x^2 + 2x - 4 = 0.$$

$$x^2 + 2x = 4.$$

$$x^2 + 2x + 1 = 5.$$

$$x + 1 = \pm\sqrt{5}.$$

$$x = -1 \pm\sqrt{5}.$$

Extracting,

$$\sqrt{5} = 2.236.$$

Hence

$$x = -1 \pm 2.236$$

$$= 1.236,$$

$$-3.236.$$

$$23. \quad x^2 - 2x - 3\frac{1}{2} = 0.$$

$$x^2 - 2x = \frac{7}{2}.$$

$$x^2 - 2x + 1 = \frac{9}{2}.$$

$$x - 1 = \pm\frac{3}{2}\sqrt{2}.$$

$$(\text{See Ex. 21}) \quad x = 1 \pm \frac{3}{2}(1.4142)$$

$$= 1 \pm 2.1213.$$

$$x = 3.121,$$

$$-1.121.$$

$$24. \quad 3x^2 - 6x + 1 = 0.$$

$$x^2 - 2x = -\frac{1}{3}.$$

$$x^2 - 2x + 1 = \frac{2}{3}.$$

$$x - 1 = \pm\frac{1}{3}\sqrt{6}.$$

Extracting,

$$\sqrt{6} = 2.449.$$

Hence

$$x = 1 \pm .816$$

$$= 1.816, .184.$$

25.

$$9x = 5x^2 - 2.$$

$$2 = 5x^2 - 9x.$$

$$x^2 - \frac{9}{5}x = \frac{2}{5}.$$

$$x^2 - \frac{9}{5}x + (-\frac{9}{10})^2 = \frac{2}{5} + \frac{81}{100}$$

$$= \frac{1}{10} \frac{21}{10}.$$

$$x - \frac{9}{10} = \pm \frac{1}{10}.$$

$$x = 2, -\frac{1}{5}.$$

26.

$$1 - 4v^2 = 2v.$$

$$1 = 4v^2 + 2v.$$

$$v^2 + \frac{1}{2}v = \frac{1}{4}.$$

$$v^2 + \frac{1}{2}v + \frac{1}{16} = \frac{1}{4} + \frac{1}{16} = \frac{5}{16}.$$

$$(\text{See Ex. 22}) \quad v + \frac{1}{4} = \pm \frac{1}{4}\sqrt{5}$$

$$= \pm \frac{1}{4}(2.236).$$

$$v = -\frac{1}{4} \pm .559$$

$$= .309, -.809.$$

27.

$$x^2 - 6\sqrt{3}x = -9.$$

$$x^2 - 6\sqrt{3}x + (-3\sqrt{3})^2 = -9 + 27 = 18.$$

$$x - 3\sqrt{3} = \pm 3\sqrt{2}.$$

$$x = 3(\sqrt{3} \pm \sqrt{2}).$$

But

$$\sqrt{3} = 1.7321,$$

$$\sqrt{2} = 1.4142.$$

$$x = 3(3.1463),$$

$$3(.3179)$$

$$= 9.439, .954.$$

28.

$$q^2 - 3q\sqrt{2} + 4 = 0.$$

$$q^2 - 3\sqrt{2}q + (-\frac{3}{2}\sqrt{2})^2 = -4 + \frac{9}{2} = \frac{1}{2}.$$

$$q - \frac{3}{2}\sqrt{2} = \pm \frac{1}{2}\sqrt{2}.$$

$$(\text{See Ex. 21}) \quad q = (\frac{3}{2} \pm \frac{1}{2})\sqrt{2}$$

$$= 2.828, 1.414.$$

29.

$$x^2 + 5 = 6 - 4x - 3x^2.$$

$$4x^2 + 4x = 1.$$

$$x^2 + x = \frac{1}{4}.$$

$$x^2 + x + \frac{1}{4} = \frac{1}{2}.$$

$$(\text{See Ex. 21}) \quad x + \frac{1}{2} = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

$$= \pm .707.$$

$$x = -\frac{1}{2} \pm .707$$

$$= .207, -1.207.$$

30. $3 + \frac{1}{3p} = 3p.$ $x^4 - \frac{8}{9}x^2 + (-\frac{4}{9})^2 = -1 + \frac{16}{81}$
 $\frac{1}{3p} = 3p - 3.$ $= \frac{16}{81} - 1 = \frac{16 - 81}{81} = -\frac{65}{81}.$
 $9p + 1 = 9p^2.$ $x^2 - \frac{4}{9} = \pm \frac{4}{9}.$
 $1 = 9p^2 - 9p.$ $x^2 = 9, \frac{1}{9}.$
 $p^2 - p = \frac{1}{9}.$ $x = \pm 3, \pm \frac{1}{3}.$
 $p^2 - p + \frac{1}{4} = \frac{1}{9} + \frac{1}{4} = \frac{13}{36}.$ 35. $x^4 - 10x^2 + 24 = 0.$
 $p - \frac{1}{2} = \pm \frac{1}{6} \sqrt{13}$ $x^4 - 10x^2 = -24.$
 $= \pm \frac{1}{6} (3.605).$ $x^4 - 10x^2 + 25 = 1.$
 $p = .5 \pm .601$ $x^2 - 5 = \pm 1.$
 $= 1.101, -.101.$ $x^2 = 4, 6.$
 $x = \pm 2, \pm \sqrt{6}.$
32. $x^4 - 5x^2 + 4 = 0.$ 36. $4x^4 - 13x^2 + 3 = 0.$
 $x^4 - 5x^2 = -4.$ $x^4 - \frac{13}{4}x^2 = -\frac{3}{4}.$
 $x^4 - 5x^2 + (-\frac{5}{2})^2 = -4 + \frac{25}{4} = \frac{9}{4}.$ $x^4 - \frac{13}{4}x^2 + (\frac{13}{8})^2 = -\frac{3}{4} + \frac{169}{64}$
 $x^2 - \frac{5}{2} = \pm \frac{3}{2}.$ $= \frac{121}{64}.$
 $x^2 = 4, 1.$ $x^2 - \frac{13}{8} = \pm \frac{11}{8}.$
 $x = \pm 2, \pm 1.$ $x^2 = 3, \frac{1}{4}.$
 $x = \pm \sqrt{3}, \pm \frac{1}{2}.$
33. $9x^4 - 13x^2 + 4 = 0.$ 37. $16x^4 - 81x^2 + 5 = 0.$
 $x^4 - \frac{13}{9}x^2 = -\frac{4}{9}.$ $x^4 - \frac{81}{16}x^2 = -\frac{5}{16}.$
 $x^4 - \frac{13}{9}x^2 + (-\frac{13}{18})^2 = -\frac{4}{9} + \frac{169}{324}.$ $x^4 - \frac{81}{16}x^2 + (-\frac{81}{32})^2 = -\frac{5}{16} + \frac{6561}{1024}$
 $= \frac{25}{324}.$ $= \frac{6241}{1024}.$
 $x^2 - \frac{13}{9} = \pm \frac{5}{18}.$ $x^2 - \frac{81}{32} = \pm \frac{79}{32}.$
 $x^2 = 1, \frac{4}{9}.$ $x^2 = 5, \frac{1}{16}.$
 $x = \pm 1, \pm \frac{2}{3}.$ $x = \pm \sqrt{5}, \pm \frac{1}{4}.$
34. $9x^4 - 82x^2 + 9 = 0.$

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1. $12x^2 + 7x = 5.$ 3. $4x^2 + 15x = -9.$
 $x^2 + \frac{7}{12}x = \frac{5}{12}.$ $x^2 + \frac{15}{4}x = -\frac{9}{4}.$
 $x^2 + \frac{7}{12}x + (\frac{7}{24})^2 = \frac{5}{12} + \frac{49}{576}$ $x^2 + \frac{15}{4}x + (\frac{15}{8})^2 = -\frac{9}{4} + \frac{225}{64} = \frac{81}{64}.$
 $= \frac{289}{576}.$ $x + \frac{15}{8} = \pm \frac{9}{8}.$
 $x + \frac{7}{24} = \pm \frac{17}{24}.$ $x = -3, -\frac{3}{4}.$
 $x = \frac{5}{12}, -1.$ 4. $x + 1 = 20x^2.$
 $20x^2 - x = 1.$
 $x^2 - \frac{1}{20}x = \frac{1}{20}.$
 $x^2 - \frac{1}{20}x + (-\frac{1}{40})^2 = \frac{1}{20} + \frac{1}{1600}$
 $= \frac{81}{1600}.$
 $x - \frac{1}{40} = \pm \frac{9}{40}.$
 $x = \frac{1}{4}, -\frac{1}{5}.$
2. $(5x-2)(x-6) = (2x-12)(2x-4).$
 $5x^2 - 32x + 12 = 4x^2 - 32x + 48.$
 $x^2 = 36.$
 $x = \pm 6.$

$$5. 25y^2 - 20y + 12 = 0.$$

$$y^2 - \frac{4}{5}y = \frac{12}{25}.$$

$$y^2 - \frac{4}{5}y + (-\frac{2}{5})^2 = \frac{12+4}{25} = \frac{16}{25}.$$

$$y - \frac{2}{5} = \pm \frac{4}{5}.$$

$$y = \frac{6}{5}, -\frac{2}{5}.$$

$$6. 6x + x^2 + 3 = 2 - 3x^2 + 10x.$$

$$4x^2 - 4x + 1 = 0.$$

$$(2x - 1)^2 = 0.$$

$$x = \frac{1}{2}, \frac{1}{2}.$$

$$7. (3x + 6)(x - 3) = (2x + 1)(x - 2).$$

$$3x^2 - 3x - 18 = 2x^2 - 3x - 2.$$

$$x^2 = 16.$$

$$x = \pm 4.$$

$$8. \frac{x}{3} + 4 - \frac{15}{x} = 0.$$

$$x^2 + 12x - 45 = 0.$$

$$x^2 + 12x = 45.$$

$$x^2 + 12x + 36 = 81.$$

$$x + 6 = \pm 9.$$

$$x = 3, -15.$$

$$9. \frac{1}{3} + \frac{a}{9} - \frac{2}{a} = 0.$$

$$3a + a^2 - 18 = 0.$$

$$a^2 + 3a = 18.$$

$$a^2 + 3a + \frac{9}{4} = 18 + \frac{9}{4} = \frac{81}{4}.$$

$$a + \frac{3}{2} = \pm \frac{9}{2}.$$

$$a = 3, -6.$$

$$10. \frac{3x}{2} + \frac{1}{2} = \frac{1}{3x}.$$

$$9x^2 + 3x = 2.$$

$$x^2 + \frac{1}{3}x = \frac{2}{9}.$$

$$x^2 + \frac{1}{3}x + \frac{1}{36} = \frac{2}{9} + \frac{1}{36} = \frac{9}{36} = \frac{1}{4}.$$

$$x + \frac{1}{6} = \pm \frac{1}{2} = \pm \frac{3}{6}.$$

$$x = \frac{1}{3}, -\frac{2}{3}.$$

$$11. x = \frac{3}{x+2}.$$

$$x^2 + 2x = 3.$$

$$x^2 + 2x + 1 = 4.$$

$$x + 1 = \pm 2.$$

$$x = -3, 1.$$

$$12. \frac{x^2}{x-5} + \frac{5}{2} = 0.$$

$$2x^2 + 5x - 25 = 0.$$

$$x^2 + \frac{5}{2}x = \frac{25}{2}.$$

$$x^2 + \frac{5}{2}x + (\frac{5}{4})^2 = \frac{25}{2} + \frac{25}{16} = \frac{225}{16}.$$

$$x + \frac{5}{4} = \pm \frac{15}{4}.$$

$$x = \frac{5}{2}, -5.$$

$$13. \frac{2}{t-2} - \frac{3t}{2} = 0.$$

$$4 - 3t^2 + 6t = 0.$$

$$3t^2 - 6t = 4.$$

$$t^2 - 2t = \frac{4}{3}.$$

$$t^2 - 2t + 1 = \frac{7}{3}.$$

$$t - 1 = \pm \frac{1}{3}\sqrt{21}.$$

$$t = 1 \pm \frac{1}{3}\sqrt{21}.$$

$$14. \frac{x}{5-x} + \frac{5x}{12} = \frac{3}{2}.$$

$$12x + 5x(5-x) = 18(5-x).$$

Whence

$$5x^2 - 55x + 90 = 0.$$

$$x^2 - 11x + 18 = 0.$$

$$x^2 - 11x = -18.$$

$$x^2 - 11x + \frac{121}{4} = -18 + \frac{121}{4} = \frac{49}{4}.$$

$$x - \frac{11}{2} = \pm \frac{7}{2}.$$

$$x = 9, 2.$$

$$15. \frac{x}{x-2} - \frac{5}{2} = \frac{2-x}{x}.$$

$$2x^2 - 5x(x-2) = 2(2-x)(x-2).$$

Whence $x^2 - 2x - 8 = 0.$

$$x^2 - 2x = 8.$$

$$x^2 - 2x + 1 = 9.$$

$$x - 1 = \pm 3.$$

$$x = 4, -2.$$

16.
$$\frac{2}{s-2} - \frac{1}{s+2} = -\frac{15}{8}.$$

$$16(s+2) - 8(s-2) = -15(s^2-4).$$

$$15s^2 + 8s - 12 = 0.$$

$$s^2 + \frac{8}{15}s = \frac{4}{5}.$$

$$s^2 + \frac{8}{15}s + \left(\frac{4}{15}\right)^2 = \frac{4}{5} + \frac{16}{225} = \frac{196}{225}.$$

$$s + \frac{4}{15} = \pm \frac{14}{15}.$$

$$s = \frac{2}{3}, -\frac{6}{5}.$$

17.
$$\frac{3+x}{4+x} - \frac{1}{12} = \frac{5-x}{6-x}.$$

$$12(3+x)(6-x) - (4+x)(6-x) = 12(5-x)(4+x).$$

Whence
$$x^2 + 22x - 48 = 0.$$

$$x^2 + 22x = 48.$$

$$x^2 + 22x + 121 = 169.$$

$$x + 11 = \pm 13.$$

$$x = 2, -24.$$

18.
$$\frac{1+x}{2+x} - \frac{1}{6} = \frac{3-x}{5-x}.$$

$$6(1+x)(5-x) - (2+x)(5-x) = 6(3-x)(2+x).$$

Whence
$$x^2 + 15x - 16 = 0.$$

$$x^2 + 15x = 16.$$

$$x^2 + 15x + \left(\frac{15}{2}\right)^2 = 16 + \frac{225}{4} = \frac{289}{4}.$$

$$x + \frac{15}{2} = \pm \frac{17}{2}.$$

$$x = 1, -16.$$

19.
$$\frac{x+2}{x-4} + \frac{x-1}{x+5} = -1.$$

$$(x+2)(x+5) + (x-1)(x-4) = -(x-4)(x+5).$$

Whence
$$3x^2 + 3x - 6 = 0.$$

$$x^2 + x = 2.$$

$$x^2 + x + \frac{1}{4} = \frac{9}{4}.$$

$$x + \frac{1}{2} = \pm \frac{3}{2}.$$

$$x = 1, -2.$$

20.
$$\frac{x-2}{x+2} - \frac{x-1}{x+1} = 6.$$

$$(x-2)(x+1) - (x-1)(x+2) = 6(x+2)(x+1).$$

Whence
$$6x^2 + 20x + 12 = 0.$$

$$x^2 + \frac{10}{3}x = -2.$$

$$x^2 + \frac{10}{3}x + \left(\frac{5}{3}\right)^2 = -2 + \frac{25}{9} = \frac{7}{9}.$$

$$x + \frac{5}{3} = \pm \frac{1}{3}\sqrt{7}.$$

$$x = -\frac{5}{3} \pm \frac{1}{3}\sqrt{7}.$$

21. $(3x - 2)(x - 3) = (x + 1)(x + 4).$

Whence $2x^2 - 16x + 2 = 0.$

$$x^2 - 8x = -1.$$

$$x^2 - 8x + 16 = 15.$$

$$x - 4 = \pm \sqrt{15}.$$

$$x = 4 \pm \sqrt{15}.$$

22. $x^4 - 10x^2 + 16 = 0.$

$$x^4 - 10x^2 = -16.$$

$$x^4 - 10x^2 + 25 = 9.$$

$$x^2 - 5 = \pm 3.$$

$$x^2 = 8, 2.$$

$$x = \pm 2\sqrt{2}, \pm \sqrt{2}.$$

24. $6x^4 - 19x^2 + 15 = 0.$

$$x^4 - \frac{19}{6}x^2 = -\frac{5}{2}.$$

$$x^4 - \frac{19}{6}x^2 + \left(-\frac{19}{6}\right)^2 = -\frac{5}{2} + \frac{361}{36} = 1\frac{1}{4}.$$

$$x^2 - \frac{19}{6} = \pm 1\frac{1}{2}.$$

$$x^2 = \frac{5}{3}, \frac{3}{2}.$$

$$x = \pm \frac{1}{3}\sqrt{15}, \pm \frac{1}{2}\sqrt{6}.$$

23. $x^4 - 8x^2 + 15 = 0.$

$$x^4 - 8x^2 = -15.$$

$$x^4 - 8x^2 + 16 = 1.$$

$$x^2 - 4 = \pm 1.$$

$$x^2 = 3, 5.$$

$$x = \pm \sqrt{3}, \pm \sqrt{5}.$$

25. $8x^4 - 18x^2 + 9 = 0.$

$$x^4 - \frac{9}{4}x^2 = -\frac{9}{8}.$$

$$x^4 - \frac{9}{4}x^2 + \left(-\frac{9}{4}\right)^2 = \frac{81}{16} - \frac{9}{8} = \frac{9}{4}.$$

$$x^2 - \frac{9}{4} = \pm \frac{3}{2}.$$

$$x^2 = \frac{3}{2}, \frac{3}{4}.$$

$$x = \pm \frac{1}{2}\sqrt{6}, \pm \frac{1}{2}\sqrt{3}.$$

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1. Let

x = the number.

Then

$$x^2 + 2x = 8.$$

Whence

$$x = 2, -4.$$

2. Let

x = the number.

Then

$$2x^2 - x = 28.$$

Whence

$$x = 4, -\frac{7}{2}.$$

3. Let x and $x + 2$ represent the numbers.

Then

$$x(x + 2) = 675.$$

Whence

$$x = 25, -27.$$

Therefore

$$x = 25, x + 2 = 27,$$

or

$$x = -27, x + 2 = -25.$$

4. Let x , $x + 2$, and $x + 4$ represent the numbers.
 Then $3x + 6 = \frac{1}{2}(x^2 + 2x)$.
 Whence $x = 6, -2$.
 Therefore $x = 6, x + 2 = 8, x + 4 = 10$,
 or $x = -2, x + 2 = 0, x + 4 = 2$.
5. Let $x =$ the width of the field in rods.
 Then $x + 14 =$ the length in rods,
 and $x(x + 14) = 20 \cdot 160$.
 Whence $x = 50$, root $- 64$ rejected,
 and $x + 14 = 64$.
6. Let $x =$ the number.
 Then $x + \frac{1}{x} = \frac{13}{6}$.
 Whence $x = \frac{2}{3}, \frac{3}{2}$.
7. Let $x =$ the altitude of the field in rods.
 Then $x + 36 =$ the base in rods,
 and $\frac{x(x + 36)}{2} = \frac{9}{2}(160)$.
 Whence $x = 24$, root $- 60$ rejected,
 and $x + 36 = 60$.
8. Let $x =$ the side of the smaller field in rods.
 Then $x + 16 =$ the side of the larger field in rods,
 and $x^2 + (x + 16)^2 = (40)(160)$.
 Whence $x = 48$, root $- 64$ rejected,
 and $x + 16 = 64$.
9. Let $x =$ the longer leg of the triangle in feet.
 Then $x - 31 =$ the shorter leg in feet,
 and $x^2 + (x - 31)^2 = 41^2 = 1681$.
 Whence $x = 40$, root $- 9$ rejected,
 and $x - 31 = 9$.
10. Let $x =$ the longer leg of the triangle.
 Then $\frac{3}{4}x =$ the shorter leg,
 and $x^2 + (\frac{3}{4}x)^2 = 900$.
 Whence $x = 24$, root $- 24$ rejected,
 and $\frac{3}{4}x = 18$.

11. Let x = the side of the square in inches.

Then $\frac{x}{12}$ = the side in feet,

and $4x = \left(\frac{x}{12}\right)^2$.

Whence $x = 576$, root 0 rejected.

12. Let x and $x + 1$ represent the dimensions.

Then the longest line is the diagonal.

Therefore $x^2 + (x + 1)^2 = (x + 2)^2$.

Whence $x = 3$, root -1 rejected,

and $x + 1 = 4$.

13. Let x and $x + 1$ represent the edges of the bins in yards.

Then $(x + 1)^3 - x^3 = 127$.

Whence $x = 6$, root -7 rejected,

and $x + 1 = 7$.

14. Let

x = the greater rate in miles per hour.

Then

$x - 6$ = the less rate in miles per hour,

and

$$\frac{252}{x - 6} = \frac{252}{x} + 1.$$

Whence

$x = 42$, root -36 rejected,

and

$$x - 6 = 36.$$

15. Let

x = the rate going in miles per hour.

Then

$x + 3$ = the rate returning in miles per hour,

and

$$\frac{90}{x} + \frac{90}{x + 3} = 11.$$

Whence

$x = 15$, root $-\frac{18}{11}$ rejected,

and

$$x + 3 = 18.$$

16. Let

x = the width in inches.

Then

$x + 2$ = the length in inches,

$x - 2$ = the width of the printed portion,

and

x = its length.

Then

$$x(x - 2) = \frac{1}{2}x(x + 2).$$

Whence

$x = 6$, root 0 rejected,

and

$$x + 2 = 8.$$

17. Let $x =$ the original price of one orange in cents.
 Then $12x + 10 =$ the new price per dozen in cents,
 and $\frac{12x + 10}{12} = \frac{6x + 5}{6} =$ the new price of one orange in cents.
 Therefore $\frac{100}{x} - 6 = \frac{100}{\frac{6x + 5}{6}},$
 or $\frac{100}{x} - 6 = \frac{600}{6x + 5}.$
 Whence $x = 3\frac{1}{3}$, root $-2\frac{5}{6}$ rejected,
 and $12x = 40.$ *Ans.* = 40.
 Hence the original price was 40 cents per dozen.

18. Let $x =$ number of minutes by slower pump alone.
 Then $x - 32 =$ number of minutes by faster pump alone,
 and $\frac{1}{x} + \frac{1}{x - 32} = \frac{1}{30}.$
 Whence $x = 12, 80.$
 If $x = 12$, $x - 32 = -20$, which must be rejected.
 Therefore $x = 80,$
 and $x - 32 = 48.$

19. In the equation $s = 16t^2$
 we have given $s = 1600.$
 Therefore $16t^2 = 1600.$
 Whence $t = 10$, root -10 rejected.

20. We know that $s = 16t^2,$
 and since $t =$ the time in falling in seconds,
 we have $6\frac{1}{2} - t =$ the number of seconds for the sound
 to return.

But $1152 =$ the rate of the sound in feet per second.
 Therefore $s = 1152(\frac{13}{2} - t) = 1152\left(\frac{13 - 2t}{2}\right)$
 $= 576(13 - 2t).$

Equating the two expressions for s ,

$$\begin{aligned} 16t^2 &= 576(13 - 2t), \\ t^2 &= 36(13 - 2t). \\ t^2 + 72t - 468 &= 0, \\ (t + 78)(t - 6) &= 0. \end{aligned}$$

Whence $t = 6$, root -78 rejected.
 But if $t = 6,$
 then $s = 16 \cdot 6^2 = 576$ feet.

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1. $x^2 + 2bx = 3b^2.$
 $x^2 + 2bx + b^2 = 4b^2.$
 $x + b = \pm 2b.$
 $x = b, -3b.$
2. $x^2 + 4cx = 5c^2.$
 $x^2 + 4cx + 4c^2 = 9c^2.$
 $x + 2c = \pm 3c.$
 $x = c, -5c.$
3. $x^2 - 8cx - 9c^2 = 0.$
 $x^2 - 8cx + 16c^2 = 25c^2.$
 $x - 4c = \pm 5c.$
 $x = -c, 9c.$
4. $x^2 + ax - 2a^2 = 0.$
 $x^2 + ax = 2a^2.$
 $x^2 + ax + \frac{a^2}{4} = \frac{9a^2}{4}.$
 $x + \frac{a}{2} = \pm \frac{3a}{2}.$
 $x = a, -2a.$
5. $x^2 - 2ax = 1 - a^2.$
 $x^2 - 2ax + a^2 = 1.$
 $x - a = \pm 1.$
 $x = a \pm 1.$
6. $x^2 - a^2 = 0.$
 $x^2 = a^2.$
 $x = \pm a.$
7. $a^2x^2 = b^2.$
 $x^2 = \frac{b^2}{a^2}.$
 $x = \pm \frac{b}{a}.$
8. $x^2 + bx = 2b^2.$
 $x^2 + bx + \frac{b^2}{4} = \frac{9b^2}{4}.$
 $x + \frac{b}{2} = \pm \frac{3b}{2}.$
 $x = b, -2b.$
9. $x^2 - 12a = 3ax - 4x.$
 $x^2 + (4 - 3a)x = 12a.$
 $x^2 + (4 - 3a)x + \left(\frac{4 - 3a}{2}\right)^2 = 12a + \frac{16 - 24a + 9a^2}{4}$
 $= \frac{16 + 24a + 9a^2}{4}.$
 $x + \frac{4 - 3a}{2} = \pm \frac{4 + 3a}{2}.$
 $x = 3a, -4.$
10. $2x^2 - 3hx - 2h^2 = 0.$
 $x^2 - \frac{3h}{2}x = h^2.$
 $x^2 - \frac{3h}{2}x + \left(-\frac{3h}{4}\right)^2 = h^2 + \frac{9}{16}h^2 = \frac{25}{16}h^2.$
 $x - \frac{3h}{4} = \pm \frac{5h}{4}.$
 $x = 2h, -\frac{h}{2}.$

11.

$$10x^2 = 3a^2 - ax.$$

$$x^2 + \frac{a}{10}x = \frac{3a^2}{10}.$$

$$x^2 + \frac{a}{10}x + \frac{a^2}{400} = \frac{3a^2}{10} + \frac{a^2}{400} = \frac{121a^2}{400}.$$

$$x + \frac{a}{20} = \pm \frac{11a}{20}.$$

$$x = \frac{a}{2}, -\frac{3}{5}a.$$

12.

$$2x^2 - 5hx + 2h^2 = 0.$$

$$x^2 - \frac{5hx}{2} = -h^2.$$

$$x^2 - \frac{5hx}{2} + \left(-\frac{5h}{4}\right)^2 = -h^2 + \frac{25h^2}{16} = \frac{9h^2}{16}.$$

$$x - \frac{5h}{4} = \pm \frac{3h}{4}.$$

$$x = 2h, \frac{h}{2}.$$

13.

$$ax^2 - 5x - 2ax + 10 = 0.$$

$$x^2 - \frac{5+2a}{a}x = -\frac{10}{a}.$$

$$x^2 - \frac{5+2a}{a}x + \left(-\frac{5+2a}{2a}\right)^2 = -\frac{10}{a} + \frac{25+20a+4a^2}{4a^2} = \frac{25-20a+4a^2}{4a^2}.$$

$$x - \frac{5+2a}{2a} = \pm \frac{5-2a}{2a}.$$

$$x = \frac{5}{a}, 2.$$

14.

$$cx^2 + 2x = 3cx + 6.$$

$$x^2 + \frac{2-3c}{c}x = \frac{6}{c}.$$

$$x^2 + \frac{2-3c}{c}x + \left(\frac{2-3c}{2c}\right)^2 = \frac{6}{c} + \frac{4-12c+9c^2}{4c^2} = \frac{4+12c+9c^2}{4c^2}.$$

$$x + \frac{2-3c}{2c} = \pm \frac{2+3c}{2c}.$$

$$x = 3, -\frac{2}{c}.$$

15.

$$ax^2 + a = a^2x + x.$$

$$ax^2 - (a^2 + 1)x = -a.$$

$$x^2 - \frac{a^2 + 1}{a}x = -1.$$

$$x^2 - \frac{a^2 + 1}{a}x + \left(-\frac{a^2 + 1}{2a}\right)^2 = -1 + \frac{a^4 + 2a^2 + 1}{4a^2} = \frac{a^4 - 2a^2 + 1}{4a^2}.$$

$$x - \frac{a^2 + 1}{2a} = \pm \frac{a^2 - 1}{2a}.$$

$$x = a, \frac{1}{a}.$$

16.

$$cx^2 + ax = 2cx + 2a.$$

$$x^2 + \frac{a - 2c}{c}x = \frac{2a}{c}.$$

$$x^2 + \frac{a - 2c}{c}x + \left(\frac{a - 2c}{2c}\right)^2 = \frac{2a}{c} + \frac{a^2 - 4ac + 4c^2}{4c^2} = \frac{a^2 + 4ac + 4c^2}{4c^2}.$$

$$x + \frac{a - 2c}{2c} = \pm \frac{a + 2c}{2c}.$$

$$x = 2, -\frac{a}{c}.$$

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1.

$$x^2 + y = 8, \quad (1)$$

$$2x + y = 5. \quad (2)$$

From (2),

$$y = 5 - 2x. \quad (3)$$

Then in (1),

$$x^2 + 5 - 2x = 8.$$

Whence

$$x = 3, -1,$$

and in (3),

$$y = -1, 7.$$

2.

$$x + y = 5, \quad (1)$$

$$x^2 + y^2 = 13. \quad (2)$$

From (1),

$$y = 5 - x. \quad (3)$$

Then in (2),

$$x^2 + (5 - x)^2 = 13.$$

Whence

$$x = 2, 3,$$

and in (3),

$$y = 3, 2.$$

3.

$$x^2 + y^2 = 25, \quad (1)$$

$$3x - 4y = 0. \quad (2)$$

From (2),

$$y = \frac{3}{4}x. \quad (3)$$

Then in (1),
Whence
and in (3),

$$\begin{aligned}x^2 + \frac{9}{16}x^2 &= 25. \\x &= 4, -4, \\y &= 3, -3.\end{aligned}$$

4.

$$x^2 + y^2 = 25, \quad (1)$$

$$x - 3y = 5. \quad (2)$$

From (2),

$$x = 3y + 5. \quad (3)$$

Then in (1),

$$(3y + 5)^2 + y^2 = 25.$$

Whence

$$y = 0, -3,$$

and in (3),

$$x = 5, -4.$$

5.

$$xy = 12, \quad (1)$$

$$2x + y - 10 = 0. \quad (2)$$

From (2),

$$y = 10 - 2x. \quad (3)$$

Then in (1),

$$x(10 - 2x) = 12.$$

Whence

$$x = 2, 3,$$

and in (3),

$$y = 6, 4.$$

6.

$$x^2 + 3xy = 25, \quad (1)$$

$$2x + y = 10. \quad (2)$$

From (2),

$$y = 10 - 2x. \quad (3)$$

Then in (1),

$$x^2 + 3x(10 - 2x) = 25.$$

Whence

$$x = 5, 1,$$

and in (3),

$$y = 0, 8.$$

7.

$$4p + 5q = 6, \quad (1)$$

$$pq + 2 = 0. \quad (2)$$

From (1),

$$p = \frac{6 - 5q}{4}. \quad (3)$$

In (2),

$$\left(\frac{6 - 5q}{4}\right)q + 2 = 0.$$

Whence

$$q = 2, -\frac{4}{5},$$

and in (3),

$$p = -1, \frac{5}{2}.$$

8.

$$4x - 3y = 30, \quad (1)$$

$$xy + 12 = 0. \quad (2)$$

From (1),

$$y = \frac{4x - 30}{3}. \quad (3)$$

In (2),

$$x\left(\frac{4x - 30}{3}\right) + 12 = 0.$$

Whence

$$x = 6, \frac{3}{2},$$

and in (3),

$$y = -2, -8.$$

$$9. \quad 3A + 2B = 5, \quad (1)$$

$$AB + 3 = 6A. \quad (2)$$

$$\text{From (1),} \quad B = \frac{5 - 3A}{2}. \quad (3)$$

$$\text{In (2),} \quad A \left(\frac{5 - 3A}{2} \right) + 3 = 6A.$$

$$\begin{aligned} \text{Whence} \quad A &= -3, \frac{2}{3}, \\ \text{and in (3),} \quad B &= 7, \frac{3}{2}. \end{aligned}$$

$$10. \quad x^2 + xy = 5, \quad (1)$$

$$x - y = 9. \quad (2)$$

$$\text{From (2),} \quad y = x - 9. \quad (3)$$

$$\text{In (1),} \quad x^2 + x(x - 9) = 5.$$

$$\begin{aligned} \text{Whence} \quad x &= 5, -\frac{1}{2}, \\ \text{and in (3),} \quad y &= -4, -\frac{19}{2}. \end{aligned}$$

$$11. \quad x^2 + y^2 + y = 46, \quad (1)$$

$$2x + y = 10. \quad (2)$$

$$\text{From (2),} \quad y = 10 - 2x. \quad (3)$$

$$\text{In (1),} \quad x^2 + (10 - 2x)^2 + (10 - 2x) = 46.$$

$$\begin{aligned} \text{Whence} \quad x &= 2, \frac{32}{5}, \\ \text{and in (3),} \quad y &= 6, -\frac{14}{5}. \end{aligned}$$

$$12. \quad x^2 + 3xy + y^2 = 22, \quad (1)$$

$$2x = y. \quad (2)$$

$$\text{Then (1) becomes} \quad x^2 + 6x^2 + 4x^2 = 22.$$

$$\begin{aligned} \text{Whence} \quad x &= \sqrt{2}, -\sqrt{2}, \\ \text{and in (2),} \quad y &= 2\sqrt{2}, -2\sqrt{2}. \end{aligned}$$

$$13. \quad x^2 + y^2 - 4x - 2y = 20, \quad (1)$$

$$3x - y = 10. \quad (2)$$

$$\text{From (2),} \quad y = 3x - 10. \quad (3)$$

$$\text{In (1),} \quad x^2 + (3x - 10)^2 - 4x - 2(3x - 10) = 20.$$

$$\begin{aligned} \text{Whence} \quad x &= 2, 5, \\ \text{and in (3),} \quad y &= -4, 5. \end{aligned}$$

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$$1. \quad x - y = 3,$$

$$x^2 - y^2 = 45.$$

$$\text{Whence} \quad x = 9; y = 6.$$

$$2. \quad x + y = 18,$$

$$x^2 + y^2 = 170.$$

$$\text{Whence} \quad x = 7, 11;$$

$$y = 11, 7.$$

3. Let x = the length of the field in rods,
 and y = the width in rods.
 Then $x - y = 39$,
 and $xy = 1600$. (See Ex. 5, p. 276.)
 Whence $x = 64$, root — 25 rejected;
 $y = 25$.
4. Let x and y represent the sides in feet.
 Then $4x - 4y = 32$,
 and $x^2 - y^2 = 208$.
 Whence $x = 17$;
 $y = 9$.
5. Let x = the length of the rectangle in feet,
 and y = the width in feet.
 Then $2x + 2y = 92$,
 and $xy = 504$.
 Whence $x = 28, 18$;
 $y = 18, 28$.
6. Let y = the altitude of the triangle in
 inches,
 and x = the base in inches.
 Then $x - y = 5$,
 and $\frac{xy}{2} = \frac{25}{24} \cdot 144 = 150$.
 Whence $y = 15$, root — 20 rejected;
 $x = 20$.
7. As in Ex. 5, $2x + 2y = 3c$,
 $xy = \frac{c^2}{2}$.
 Whence $x = c, \frac{c}{2}$;
 $y = \frac{c}{2}, c$.
8. We are asked whether the system
 $x - y = 3$, (1)
 $x^2 + y^2 = 115$ (2)
 has a solution in positive integers.
 From (1), $y = x - 3$.
 In (2), $x^2 + (x - 3)^2 = 115$.
 Whence $x^2 - 3x - 53 = 0$.
 Solving, $x = \frac{3}{2} \pm \frac{1}{2}\sqrt{221}$.
 Since neither of these values is an integer, the answer is "No."

9. Let x = the tens' digit,
 and y = the units' digit.
 Then $(10x + y)(x + y) = 576$,
 and $10x + y + 3(x + y) = 10y + x$.
 Whence $x = 4$, root $- 4$ rejected;
 $y = 8$. Ans. = 48.

10. Let p = the principal in dollars,
 and r = the rate per cent.
 Then $\frac{pr}{100} = 48$, (1)
 and $\frac{(p + 200)(r - 1)}{100} = 50$. (2)
 From (1), $pr = 4800$. (3)
 From (2), $pr + 200r - p = 5200$. (4)
 (4) - (3), $200r - p = 400$, (5)
 and $p = 200r - 400$. (6)
 Then (3) becomes $(200r - 400)r = 4800$.
 Whence $r^2 - 2r = 24$,
 and $r = 6$, root $- 4$ rejected.
 Then in (6) $p = 800$.

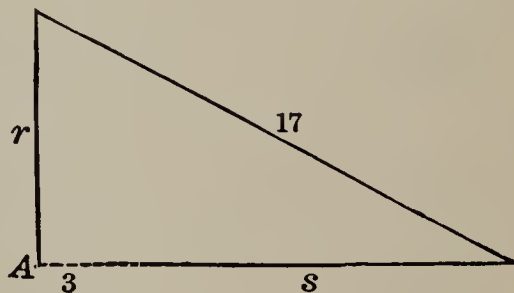
11. Let r = the rate in miles per hour of the slower,
 and s = the rate in miles per hour of the faster.

In one hour they travel r and s miles respectively.

Hence $r^2 + (s + 3)^2 = 17^2$,
 $s = \frac{3}{2}r$.

Whence

$r = 8$, root $- 1\frac{4}{3}$ rejected;
 $s = 12$.



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1. See p. 13.
2. $6 - 2 + 8 - 7 = 5$.
3. $8 + 12 \div 3 - 5 = 8 + 4 - 5 = 7$.
4. $8 \cdot 6 \div 4 = 12$.
5. $24 \div 2 \div 3 = 4$.
6. $36 \div 4 \cdot 3 - 8 + 2 = 27 - 8 + 2 = 21$.
7. $(19 - 3 \cdot 5)(8 - 5) \div (14 \div 7) = 4 \cdot 3 \div 2 = 6$.
8. $50 - 4(20 - 4 \cdot 2) \div 3 + 2 \cdot 7 = 50 - 4 \cdot 12 \div 3 + 14 = 50 - 16 + 14 = 48$.
9. $12 - 8 \div 2 + 10 \cdot 3 - 6 + 8 \cdot 2 = 12 - 4 + 30 - 6 + 16 = 48$.
10. $(28 \div 14 \cdot 24 \div 8 - 3 + 10) \times (30 \div 3 \div 5 - 2) = (6 - 3 + 10)(0) = 0$.
11. $(16 \div 32 \times 48 \div 8 - 8 + 3) + (42 \div 6 \cdot 7 - 42 - 6) \cdot 6$
 $= (3 - 8 + 3) + (49 - 42 - 6)6 = -2 + 6 = 4$.

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1. See p. 23.

2. See p. 22

3. See p. 51.

4. See p. 33.

5. See pp. 33 and 35.

6. See p. 51.

7. $12 - 8 + 4 - 3 + 6 = 11$.8. $4a + 3a - 7a + 6a - 2a = 4a$.

$$\begin{array}{r} 9. \quad 5a - 3c + 6 \\ \quad 2a - 6c + 11 \\ \quad 4a - \quad c - 9 \\ \hline 11a - 10c + 8 \end{array}$$

$$\begin{array}{r} 10. \quad 3s - 4t + 6 \\ \quad 6s - 7t - 8 \\ \quad s + \quad t + 10 \\ \hline 10s - 10t + 8 \end{array}$$

$$\begin{array}{r} 11. \quad a^2 - 3a + 1 \\ \quad - 2a^2 - 7a + 6 \\ \quad 3a^2 + 5a - 4 \\ \hline 2a^2 - 5a + 3 \end{array}$$

12. $6a - 4a = 2a$.

$$\begin{array}{r} 13. \quad 8a^3 - 15a^3 \\ \quad = -7a^3. \end{array}$$

$$\begin{array}{r} 14. \quad 4x + 3 \\ \quad 8x + 6 \\ \hline -4x - 3 \end{array}$$

$$\begin{array}{r} 15. \quad 7x^2 - 10 \\ \quad 5x^2 + 20 \\ \hline 2x^2 - 30 \end{array}$$

$$\begin{array}{r} 16. \quad 5x - 6 \\ \quad 2x + 8 \\ \hline 3x - 14 \end{array}$$

$$\begin{array}{r} 17. \quad x^2 - 5x + 6 \\ \quad 2x^2 + 8x - 10 \\ \hline -x^2 - 13x + 16 \end{array}$$

$$\begin{array}{r} 18. \quad a^2 - 4ac - 3c^2 \\ \quad 4a^2 + 10ac - c^2 \\ \hline -3a^2 - 14ac - 2c^2 \end{array}$$

$$\begin{array}{r} 19. \quad 3a - 2b - 6c \\ \quad 4a + 6b - 7c - 2 \\ \hline -a - 8b + c + 2 \end{array}$$

$$\begin{array}{r} 20. \quad 3a^2 - 2c^2 - 6ac \\ \quad 5a^2 + 4c^2 - 3ac \\ \hline -2a^2 - 6c^2 - 3ac \end{array}$$

$$\begin{array}{r} 21. \quad x - 3y^2 + z - 4ac + 7ax \\ \quad 4x - y^2 + 9ac - 5ax + 8 \\ \hline -3x - 2y^2 + z - 13ac + 12ax - 8 \end{array}$$

$$\begin{array}{r} 22. \quad a^3 - c + 3x - a^2m - 8ac \\ \quad 4a^3 - 8x + 4a^2m - 10ac + m \\ \hline -3a^3 - c + 11x - 5a^2m + 2ac - m \end{array}$$

$$\begin{array}{r} 23. \quad 6x^2 - 11x + 8 \\ \quad 3x^2 - 5x + 2 \\ \hline 3x^2 - 6x + 6 \end{array}$$

$$\begin{array}{r} 24. \quad 10x^2 + 8cx - 9c^2 \\ \quad 4x^2 - 3cx + c^2 \\ \hline 6x^2 + 11cx - 10c^2 \end{array}$$

$$\begin{array}{r} 25. \quad 7a^2 - 10ab + 6b^2 \\ \quad 4a^2 - 2ab + b^2 \\ \hline 3a^2 - 8ab + 5b^2 \end{array}$$

$$\begin{array}{r} 26. \quad 9x^2 - 10cx + 4 + c^2 \\ \quad 8x^2 - 6cx + c^2 \\ \hline x^2 - 4cx + 4 \end{array}$$

$$\begin{array}{r} 27. \quad t^2 - 4t - 9 \\ \quad 3t^2 - 8t + 1 \\ \quad 4t^2 - 12t - 8 \\ \hline -4t^2 + 7t - 1 \\ \hline 8t^2 - 19t - 7 \text{ (Ans.)} \end{array}$$

$$\begin{array}{r} 28. \quad ax - ac - 4c^2 \\ \quad - 3ac + 4c^2 \\ \hline ax - 4ac \\ \quad - 5ax + 4ac - c^2 + a^2 \\ \hline 6ax - 8ac + c^2 - a^2 \text{ (Ans.)} \end{array}$$

$$\begin{array}{r} 4c^2 - 8ax + a^2 \\ \quad - 5c^2 + 3ax + 4ac \\ \hline -c^2 - 5ax + a^2 + 4ac \end{array}$$

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1. See index.

2. See p. 9.

3. See pp. 72-73.

4. See p. 75.

$$5. \quad b^4 + 4ab^3 - 6a^2b^2 \\ - 4a^3b + a^4.$$

$$6. \quad -5 + t - 2t^2 + t^3 - 3t^4.$$

7. Yes. It makes for orderly arrangement and accuracy.

8. See p. 72.

$$9. \quad x^a \cdot x^b = x^{a+b}.$$

10. See p. 76.

$$11. \quad (6x^2 - 3x + 4)3x \\ = 18x^3 - 9x^2 + 12x.$$

$$12. \quad \begin{array}{r} 3x - 5 \\ 4x + 3 \\ \hline 12x^2 - 20x \\ + 9x - 15 \\ \hline 12x^2 - 11x - 15 \end{array}$$

$$13. \quad \begin{array}{r} 2s - 3t \\ 4s + 5t \\ \hline 8s^2 - 12st \\ + 10st - 15t^2 \\ \hline 8s^2 - 2st - 15t^2 \end{array}$$

$$14. \quad \begin{array}{r} x^2 - x + 2 \\ 3x - 4 \\ \hline 3x^3 - 3x^2 + 6x \\ - 4x^2 + 4x - 8 \\ \hline 3x^3 - 7x^2 + 10x - 8 \end{array}$$

$$15. \quad 2x^2 - 5x + 3$$

$$\begin{array}{r} x^2 - 5x + 6 \\ \hline 2x^4 - 5x^3 + 3x^2 \\ - 10x^3 + 25x^2 - 15x \\ + 12x^2 - 30x + 18 \\ \hline 2x^4 - 15x^3 + 40x^2 - 45x + 18 \end{array}$$

$$16. \quad a^4 + 2a^2 - 4$$

$$\begin{array}{r} a^2 - 2a - 3 \\ \hline a^6 + 2a^4 - 4a^2 \\ - 2a^5 - 4a^3 + 8a \\ - 3a^4 - 6a^2 + 12 \\ \hline a^6 - 2a^5 - a^4 - 4a^3 - 10a^2 + 8a + 12 \end{array}$$

$$17. \quad t^3 - 2t + 6$$

$$\begin{array}{r} t^4 - 3t^3 - t^2 \\ \hline t^7 - 2t^5 + 6t^4 \\ - 3t^6 + 6t^4 - 18t^3 \\ - t^5 + 2t^3 - 6t^2 \\ \hline t^7 - 3t^6 - 3t^5 + 12t^4 - 16t^3 - 6t^2 \end{array}$$

$$18. \quad 2s^2 - 3st + t^2$$

$$\begin{array}{r} s^2 + 5st - 4t^2 \\ \hline 2s^4 - 3s^3t + s^2t^2 \\ + 10s^3t - 15s^2t^2 + 5st^3 \\ - 8s^2t^2 + 12st^3 - 4t^4 \\ \hline 2s^4 + 7s^3t - 22s^2t^2 + 17st^3 - 4t^4 \end{array}$$

$$19. \quad a^2 - ac + c^2$$

$$\begin{array}{r} a^2 + ac + c^2 \\ \hline a^4 - a^3c + a^2c^2 \\ + a^3c - a^2c^2 + ac^3 \\ + a^2c^2 - ac^3 + c^4 \\ \hline a^4 + a^2c^2 + c^4 \end{array}$$

$$20. \quad a^2 + 2ab + b^2$$

$$\begin{array}{r} a^2 - ab + b \\ \hline a^4 + 2a^3b + a^2b^2 \\ - a^3b - 2a^2b^2 - ab^3 \\ + a^2b + 2ab^2 + b^3 \\ \hline a^4 + a^3b - a^2b^2 - ab^3 + a^2b + 2ab^2 + b^3 \end{array}$$

$$\begin{array}{r}
 21. \quad 4h^2 + 9hk + 6k^2 \\
 \underline{4h^2 - 9hk + 6k^2} \\
 16h^4 + 36h^3k + 24h^2k^2 \\
 \quad - 36h^3k - 81h^2k^2 - 54hk^3 \\
 \quad \quad + 24h^2k^2 + 54hk^3 + 36k^4 \\
 \hline
 16h^4 \quad \quad - 33h^2k^2 \quad \quad + 36k^4
 \end{array}$$

$$\begin{array}{r}
 22. \quad 2a^2 + 4ac^3 - 3c^6 \\
 \underline{2a^2 - 4ac^3 - 3c^6} \\
 4a^4 + 8a^3c^3 - 6a^2c^6 \\
 \quad - 8a^3c^3 - 16a^2c^6 + 12ac^9 \\
 \quad \quad - 6a^2c^6 - 12ac^9 + 9c^{12} \\
 \hline
 4a^4 \quad \quad - 28a^2c^6 \quad \quad + 9c^{12}
 \end{array}$$

$$\begin{array}{r}
 23. \quad t^2 + t + 1 \\
 \underline{t^2 - t + 1} \\
 t^4 + t^3 + t^2 \\
 \quad - t^3 - t^2 - t \\
 \quad \quad + t^2 + t + 1 \\
 \hline
 t^4 \quad \quad + t^2 \quad \quad + 1 \\
 t^4 \quad \quad - t^2 \quad \quad + 1 \\
 \hline
 t^8 \quad \quad + t^6 \quad \quad + t^4 \\
 \quad \quad - t^6 \quad \quad - t^4 - t^2 \\
 \quad \quad \quad + t^4 + t^2 + 1 \\
 \hline
 t^8 \quad \quad \quad + t^4 \quad \quad + 1
 \end{array}$$

$$\begin{array}{r}
 24. \quad x^2 - 4x + 3xy + 12y + 9y^2 + 16 \\
 \underline{x - 3y + 4} \\
 x^3 - 4x^2 + 3x^2y + 12xy + 9xy^2 + 16x \\
 \quad - 3x^2y + 12xy - 9xy^2 \quad \quad - 36y^2 - 27y^3 - 48y \\
 \quad + 4x^2 \quad \quad + 12xy \quad \quad - 16x + 36y^2 \quad \quad + 48y + 64 \\
 \hline
 x^3 \quad \quad + 36xy \quad \quad - 27y^3 \quad \quad + 64
 \end{array}$$

$$25. \quad 3x^2 + 2x + 5 = 3 \cdot 5^2 + 2 \cdot 5 + 5 = 75 + 10 + 5 = 90.$$

$$26. \quad 3t^2 - 4t + 8 = 3(-3)^2 - 4(-3) + 8 = +27 + 12 + 8 = 47.$$

$$27. \quad 9 - 8t + 5t^2 - 3t^3 = 9 - 16 + 20 - 24 = -11.$$

$$\begin{aligned}
 28. \quad 2a^3 - 3a^2c + 4ac^2 - 3c^2 &= 2(3)^3 - 3 \cdot 3^2(-2) + 4(3)(-2)^2 - 3(-2)^2 \\
 &= 54 + 54 + 48 - 12 \\
 &= 144.
 \end{aligned}$$

$$29. \quad 15(x - a) - 6(x + a) = 3(5a - 3x).$$

$$15(2a - a) - 6(2a + a) = 3(5a - 6a).$$

$$15a - 18a = -3a.$$

$$-3a = -3a. \quad \text{Yes.}$$

30.

$$\begin{aligned}
 ax(a+3) + a(10-a^2) &= x+3. \\
 a(a-3)(a+3) + a(10-a^2) &= a-3+3. \\
 a^3-9a+10a-a^3 &= a. \\
 a &= a. \text{ Yes.}
 \end{aligned}$$

$$\begin{aligned}
 31. \quad \frac{3}{x-1} - \frac{2x}{x+1} &= \frac{3+5x-2x^2}{x^2-1}. & \frac{3}{-2-1} - \frac{2 \cdot -2}{-2+1} \\
 \frac{3}{0-1} - \frac{2 \cdot 0}{0+1} &= \frac{3+5 \cdot 0-2 \cdot 0}{0-1}. & = \frac{3+5(-2)-2(-2)^2}{(-2)^2-1} \\
 -3 &= -3. \text{ Yes.} & -1-4 = \frac{3-10-8}{4-1} \\
 & & -5 = -5. \text{ Yes.}
 \end{aligned}$$

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1. See pp. 87-88.

2. See pp. 71 and 88.

3. See p. 91.

4. See p. 93.

$$\begin{array}{r}
 5. \quad 2x^2 - 5x + 3 \overline{) 2x - 3} \\
 \underline{2x^2 - 3x} \\
 -2x \\
 \underline{-2x + 3} \\
 0
 \end{array}$$

$$\begin{array}{r}
 6. \quad 6x^2 - 13x + 6 \overline{) -2x + 3} \\
 \underline{6x^2 - 9x} \\
 -4x \\
 \underline{-4x + 6} \\
 0
 \end{array}$$

$$\begin{array}{r}
 7. \quad 6a^2 + 23at - 55t^2 \overline{) 3a - 5t} \\
 \underline{6a^2 - 10at} \\
 33at \\
 \underline{33at - 55t^2} \\
 0
 \end{array}$$

$$\begin{array}{r}
 8. \quad 6a^3 + 6a^2 - 26a - 28 \overline{) 2a + 4} \\
 \underline{6a^3 + 12a^2} \\
 -6a^2 \\
 \underline{-6a^2 - 12a} \\
 -14a \\
 \underline{-14a - 28} \\
 0
 \end{array}$$

$$\begin{array}{r}
 9. \quad 6x^6 - 17x^5 - 5x^4 + 25x^3 \overline{) -2x^3 + 5x^2} \\
 \underline{6x^6 - 15x^5} \\
 -2x^5 \\
 \underline{-2x^5 + 5x^4} \\
 -10x^4 \\
 \underline{-10x^4 + 25x^3} \\
 0
 \end{array}$$

$$\begin{array}{r}
 10. \quad s^6 - 7s^3 \\
 \underline{s^6 + 2s^5 + 4s^4} \\
 -2s^5 - 4s^4 - 7s^3 \\
 \underline{-2s^5 - 4s^4 - 8s^3} \\
 s^3 \\
 \underline{s^3 + 2s^2 + 4s} \\
 -2s^2 - 4s \\
 \underline{-2s^2 - 4s - 8} \\
 -8
 \end{array}$$

$$\begin{array}{r}
 11. \quad x^3 \qquad \qquad - 5a^2x + 2a^3 \quad \overline{) x^2 + 2ax - a^2} \\
 \underline{x^3 + 2ax^2 - a^2x} \qquad \qquad \quad \underline{x - 2a} \\
 \qquad - 2ax^2 - 4a^2x \\
 \qquad \underline{- 2ax^2 - 4a^2x + 2a^3}
 \end{array}$$

$$\begin{array}{r}
 12. \quad 2t^4 - 7t^3 - 12t^2 + 11t - 2 \quad \overline{) - 2t^2 - 3t + 1} \\
 \underline{2t^4 + 3t^3 - t^2} \qquad \qquad \quad \underline{- t^2 + 5t - 2} \\
 \qquad - 10t^3 - 11t^2 \\
 \qquad \underline{- 10t^3 - 15t^2 + 5t} \\
 \qquad \qquad 4t^2 + 6t \\
 \qquad \qquad \underline{4t^2 + 6t - 2}
 \end{array}$$

$$\begin{array}{r}
 13. \quad 32x^4 - 104x^3 + 92x^2 - 2x - 60 \quad \overline{) - 4x^2 + 6x + 5} \\
 \underline{32x^4 - 48x^3 - 40x^2} \qquad \qquad \quad \underline{- 8x^2 + 14x - 12} \\
 \qquad - 56x^3 + 132x^2 \\
 \qquad \underline{- 56x^3 + 84x^2 + 70x} \\
 \qquad \qquad 48x^2 - 72x \\
 \qquad \qquad \underline{48x^2 - 72x - 60}
 \end{array}$$

$$\begin{array}{r}
 14. \quad a^2 - 2ab \qquad \qquad + b^2 \qquad \qquad - 9x^2 \quad \overline{) - a + b + 3x} \\
 \underline{a^2 - ab - 3ax} \qquad \qquad \quad \underline{- a + b - 3x} \\
 \qquad - ab + 3ax + b^2 \\
 \qquad \underline{- ab \qquad \qquad + b^2 + 3bx} \\
 \qquad \qquad 3ax \qquad - 3bx \\
 \qquad \qquad \underline{3ax \qquad - 3bx - 9x^2}
 \end{array}$$

$$\begin{array}{r}
 15. \quad - 4c^2 + 4bc \qquad - b^2 \qquad + 1 \quad \overline{) 2c - b - 1} \\
 \underline{- 4c^2 + 2bc + 2c} \qquad \qquad \quad \underline{- 2c + b - 1} \\
 \qquad 2bc - 2c - b^2 \\
 \qquad \underline{2bc \qquad - b^2 - b} \\
 \qquad \qquad - 2c \qquad + b \\
 \qquad \qquad \underline{- 2c \qquad + b + 1}
 \end{array}$$

$$\begin{array}{r}
 16. \quad - 3a^9 \qquad \qquad + 3ab^4 - 18ab^2 + 27a \quad \overline{) a^4 - b^2 + 3} \\
 \underline{- 3a^9 + 3a^5b^2 - 9a^5} \qquad \qquad \quad \underline{- 3a^5 - 3ab^2 + 9a} \\
 \qquad - 3a^5b^2 + 9a^5 \\
 \qquad \underline{- 3a^5b^2 \qquad + 3ab^4 - 9ab^2} \\
 \qquad \qquad 9a^5 \qquad - 9ab^2 \\
 \qquad \qquad \underline{9a^5 \qquad - 9ab^2 + 27a}
 \end{array}$$

$$\begin{array}{r}
 20. \ x^3 \\
 \hline
 x^3 + x^2y + x^2z \\
 \hline
 - x^2y - x^2z \\
 \hline
 - x^2y \\
 \hline
 - x^2z \\
 \hline
 - x^2z - xz^2 - xy^2 \\
 \hline
 xz^2 - xyz + xy^2 \\
 \hline
 xz^2 \\
 \hline
 - xyz + xy^2 \\
 \hline
 - xyz \\
 \hline
 \begin{array}{r}
 xy^2 + y^2z \\
 xy^2 + y^2z
 \end{array} \\
 \hline
 \begin{array}{r}
 + yz^2 \\
 - yz^2 + y^3 \\
 - y^2z - yz^2
 \end{array} \\
 \hline
 \begin{array}{r}
 + yz^2 \\
 - yz^2 + y^3 \\
 - y^2z - yz^2
 \end{array} \\
 \hline
 \begin{array}{r}
 + y^3 \\
 - y^3 + z^3 \\
 x^2 - xy - xz + z^2 - yz + y^2
 \end{array} \\
 \hline
 \begin{array}{r}
 + y^3 + z^3 \\
 x + y + z
 \end{array}
 \end{array}$$

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1. $a + (a - b) - (a - 3b) = a + a - b - a + 3b = a + 2b.$
2. $a - (a - c) + (2c - 3a) = a - a + c + 2c - 3a = 3c - 3a.$
3. $3b - [(a - b) - (c - a)] = 3b - (a - b) + (c - a) = 3b - a + b + c - a = -2a + 4b + c.$
4. $2c - 2(a - c) + 3(c - a) = 2c - 2a + 2c + 3c - 3a = 7c - 5a.$
5. $x - y + 3(x - y) - 4(2x - y) = x - y + 3x - 3y - 8x + 4y = -4x.$
6. $4x - a + [-(3c - x) - (2a - 3x)] = 4x - a - (3c - x) - (2a - 3x) = 4x - a - 3c + x - 2a + 3x = 8x - 3a - 3c.$
7. $a - [-(a - 3) + (3c - 2a) - 5a] + 6c = a + (a - 3) - (3c - 2a) + 5a + 6c = a + a - 3 - 3c + 2a + 5a + 6c = 9a + 3c - 3.$
8. $x - 3 - (a - 2x) - [2(a - x - 5) - 3(6 - 2a)] = x - 3 - a + 2x - [2a - 2x - 10 - 18 + 6a] = x - 3 - a + 2x - 8a + 2x + 28 = 5x - 9a + 25.$

9. $(2t^2 - 3t) - 2t(3 + t) = 2t^2 - 3t - 6t - 2t^2 = -9t.$
10. $(a + b)a - (a - b)b + 3ab = a^2 + ab - ab + b^2 + 3ab = a^2 + 3ab + b^2.$
11. $4x^2 - 3a^2 - (a - 2x)(3x - a) = 4x^2 - 3a^2 - (-a^2 + 5ax - 6x^2)$
 $= 4x^2 - 3a^2 + a^2 - 5ax + 6x^2$
 $= 10x^2 - 2a^2 - 5ax.$
12. $(x - 4)(x - 3) - (x - 3)(x + 2) = x^2 - 7x + 12 - (x^2 - x - 6)$
 $= x^2 - 7x + 12 - x^2 + x + 6$
 $= 18 - 6x.$
13. $3a(a - b) - (a + b)(a - b) = 3a^2 - 3ab - (a^2 - b^2)$
 $= 3a^2 - 3ab - a^2 + b^2$
 $= 2a^2 - 3ab + b^2.$
14. $(x - 3)(x - 4) - (x - 5)(x + 3) = x^2 - 7x + 12 - (x^2 - 2x - 15)$
 $= x^2 - 7x + 12 - x^2 + 2x + 15$
 $= 27 - 5x.$
15. $6x - [-(a - c) + (3c - 4a)] = 6x + (a - c) - (3c - 4a)$
 $= 6x + a - c - 3c + 4a$
 $= 6x + 5a - 4c.$
16. $7c - [(3c - 4) - 6 - (4x - 3a - c)] = 7c - (3c - 4) + 6 + (4x - 3a - c)$
 $= 7c - 3c + 4 + 6 + 4x - 3a - c$
 $= 3c + 4x - 3a + 10.$
17. $4x - 2(x - 3) - 3[x - 3(4 - 2x) + 8]$
 $= 4x - 2x + 6 - 3x + 9(4 - 2x) - 24$
 $= 4x - 2x + 6 - 3x + 36 - 18x - 24$
 $= 18 - 19x.$
18. $3x - 2[1 - 3(2x - 3 - a) - 5\{a - (3x - 2a) - 4\}]$
 $= 3x - 2 + 6(2x - 3 - a) + 10\{a - (3x - 2a) - 4\}$
 $= 3x - 2 + 12x - 18 - 6a + 10a - 30x + 20a - 40$
 $= -15x + 24a - 60.$
19. (a) $x + (-y - 3).$
 (b) $x - (y + 3).$
20. (a) $2x + (y - 8).$
 (b) $2x - (8 - y).$
21. (a) $7x + (10 - 4y).$
 (b) $7x - (4y - 10).$
22. (a) $x + (y + z).$
 (b) $x - (-y - z).$
23. (a) $x + (4z - 3y).$
 (b) $x - (3y - 4z).$
24. (a) $3x + (4y - 7z).$
 (b) $3x - (7z - 4y).$
25. (a) $2x + (5y - z).$
 (b) $2x - (z - 5y).$
26. (a) $3x + (6z - 8y).$
 (b) $3x - (8y - 6z).$
27. (a) $x + (-y - z).$
 (b) $x - (y + z).$

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1. $5x + 1 = 2x + 7.$
 $5x - 2x = 7 - 1.$
 $3x = 6.$
 $x = 2.$
2. $1 + x + 5x + 17 = 0.$
 $6x = -18.$
 $x = -3.$
3. $15x = 3(4x - 5).$
 $15x = 12x - 15.$
 $15x - 12x = -15.$
 $3x = -15.$
 $x = -5.$
4. $2(x + 2) - (x + 5) = 0.$
 $2x + 4 - x - 5 = 0.$
 $x = 1.$
5. $3(2x - 7) - 2(5 - 2x) + 1 = 0.$
 $6x - 21 - 10 + 4x + 1 = 0.$
 $10x = 30.$
 $x = 3.$
6. $3(x + 2) - 5(2x - 3) = 0.$
 $3x + 6 - 10x + 15 = 0.$
 $-7x = -21.$
 $x = 3.$
7. $4(4x - 1) + 3 - 2(3 + x) = 0.$
 $16x - 4 + 3 - 6 - 2x = 0.$
 $14x = 7.$
 $x = \frac{1}{2}.$
8. $4(x - 2) + 3(2 - x) - 3x = 6(x + 1).$
 $4x - 8 + 6 - 3x - 3x = 6x + 6.$
 $-2x - 6x = 6 + 2.$
 $-8x = 8.$
 $x = -1.$
9. $(x + 1)(x - 2) = x^2 + 3.$
 $x^2 - x - 2 = x^2 + 3.$
 $-x = 5.$
 $x = -5.$
10. $(4x - 3)(2x - 5) - (4x - 7)(2x - 1) = 0.$
 $8x^2 - 26x + 15 - 8x^2 + 18x - 7 = 0.$
 $-8x = -8.$
 $x = 1.$
11. $(x + 2)^2 + 48 = (x - 4)^2.$
 $x^2 + 4x + 4 + 48 = x^2 - 8x + 16.$
 $4x + 8x = 16 - 52.$
 $12x = -36.$
 $x = -3.$
12. $(x + 4)^2 - (2 - x)^2 = 84.$
 $x^2 + 8x + 16 - 4 + 4x - x^2 = 84.$
 $12x = 72.$
 $x = 6.$

$$13. (x + 3)(6x + 5) - (2x + 4)(3x - 8) = 38.$$

$$6x^2 + 23x + 15 - 6x^2 + 4x + 32 = 38.$$

$$27x = 38 - 47.$$

$$27x = -9.$$

$$x = -\frac{1}{3}.$$

$$14. (y - 4)(6 - y) + (y + 2)(y - 4) = 0.$$

$$-y^2 + 10y - 24 + y^2 - 2y - 8 = 0.$$

$$8y = 32.$$

$$y = 4.$$

$$15. (x + 4)(x + 3) = (x + 2)(x + 1) + 42.$$

$$x^2 + 7x + 12 = x^2 + 3x + 2 + 42.$$

$$4x = 32.$$

$$x = 8.$$

$$16. 5 - (7 - 3) = 1.$$

$$17. \text{ See p. 57.}$$

$$18. \text{ See p. 55.}$$

$$19. x - 2a = 4a - x.$$

$$2x = 6a.$$

$$x = 3a.$$

$$20. 8s - x = x - 4r.$$

$$-2x = -8s - 4r.$$

$$x = 4s + 2r.$$

$$21. ax - 2ab = 4ab - ax.$$

$$2ax = 6ab.$$

$$x = 3b.$$

$$22. m(x - 5a) - 3m(a - x) = 0.$$

$$mx - 5am - 3am + 3mx = 0.$$

$$4mx = 8am.$$

$$x = 2a.$$

$$23. x(b + a) = ab + a^2.$$

$$x(b + a) = a(b + a).$$

$$x = a.$$

$$24. x(a - c) = a^2 - c^2.$$

$$x = a + c.$$

$$25. ax - 2cx = a^2 - 4c^2.$$

$$(a - 2c)x = (a - 2c)(a + 2c).$$

$$x = a + 2c.$$

$$26. 2ax - 4a^2 + 4a = 1 + x.$$

$$2ax - x = 4a^2 - 4a + 1.$$

$$(2a - 1)x = (2a - 1)^2.$$

$$x = 2a - 1.$$

$$27. ax + 1 - a^3 = x.$$

$$ax - x = a^3 - 1.$$

$$(a - 1)x = (a - 1)(a^2 + a + 1).$$

$$x = a^2 + a + 1.$$

$$28. (x + a)(x + b) = x^2 + 2ax + 3ab.$$

$$x^2 + (a + b)x + ab = x^2 + 2ax + 3ab.$$

$$(a + b)x = 2ax + 2ab.$$

$$x = 2a.$$

$$29. (a - c)(x - m) = (m - c)(x - a).$$

$$ax - cx - am + cm = mx - cx - am + ac.$$

$$ax - mx = ac - cm.$$

$$(a - m)x = c(a - m).$$

$$x = c.$$

$$\begin{aligned}
 30. \quad & a^2x - a^3 + 3ax = 3 - 10a + x. \\
 & (a^2 + 3a - 1)x = a^3 - 10a + 3. \\
 & x = a - 3.
 \end{aligned}$$

$$\begin{aligned}
 31. \quad & c(1 + x) + m(x + 1) - x(m + c + 1) = 0. \\
 & c + cx + mx + m - mx - cx - x = 0. \\
 & -x = -c - m. \\
 & x = c + m.
 \end{aligned}$$

$$32. \quad x - 4 = 50 - 5x; \quad 2x + 47 = 17.$$

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$$\begin{aligned}
 1. \text{ Let } & x = \text{the less number.} \\
 \text{Then } & 5x = \text{the greater number.} \\
 & x + 5x = 102. \\
 & 6x = 102. \\
 & x = 17. \\
 & 5x = 85.
 \end{aligned}$$

$$\begin{aligned}
 2. \text{ Let } & x = \text{one of the numbers.} \\
 \text{Then } & 54 - x = \text{the other number.} \\
 & 10x = 2(54 - x). \\
 & 10x = 108 - 2x. \\
 & 12x = 108. \\
 & x = 9. \\
 & 54 - x = 45.
 \end{aligned}$$

$$\begin{aligned}
 3. \text{ Let } & x = \text{the least number.} \\
 \text{Then } & x + 2 = \text{the next consecutive even number,} \\
 \text{and } & x + 4 = \text{the following consecutive even number.} \\
 & x + x + 2 + x + 4 = 1044. \\
 & 3x = 1038. \\
 & x = 346. \\
 & x + 2 = 348. \\
 & x + 4 = 350.
 \end{aligned}$$

$$\begin{aligned}
 4. \text{ Let } & x = \text{the least number.} \\
 \text{Then } & x + 1, x + 2, x + 3 = \text{the three following consecutive numbers.} \\
 & x(x + 2) + 201 = (x + 1)(x + 3). \\
 & x^2 + 2x + 201 = x^2 + 4x + 3. \\
 & 2x = 198. \\
 & x = 99, x + 1 = 100, x + 2 = 101, x + 3 = 102.
 \end{aligned}$$

5. Let x = the number of years in age of younger pupil.
 Then $x + 4$ = the number of years in age of older pupil.

$$(x + 4) - 8 = 2(x - 8).$$

$$x - 4 = 2x - 16.$$

$$x = 12.$$

$$x + 4 = 16.$$

6. Let x = the number of years required.

$$48 + x = 2(18 + x).$$

$$48 + x = 36 + 2x.$$

$$x = 12.$$

7. Let x = the number of years in age of younger man.
 Then $3x$ = the number of years in age of older man.

$$3x - 15 = 6(x - 15).$$

$$3x - 15 = 6x - 90.$$

$$3x = 75.$$

$$x = 25.$$

$$3x = 75.$$

8. Let x = the number of years in the age of B.
 Then $2x$ = the number of years in the age of A,
 and $x - 8$ = the number of years in the age of D,
 and $3(x - 8)$ = the number of years in the age of C.
 $x + 10 + 2x + 10 + 3(x - 8) + 10 + x - 8 + 10 = 113.$
 $7x = 105.$
 $x = 15$, or B's age in years.
 $2x = 30$, or A's age in years.
 $x - 8 = 7$, or D's age in years.
 $3(x - 8) = 21$, or C's age in years.

9. Let x = the number of feet in the side of the square.

$$(x + 12)^2 - x^2 = 816.$$

$$x^2 + 24x + 144 - x^2 = 816.$$

$$24x = 672.$$

$$x = 28.$$

10. Let x = the number of inches in the width of the picture.

Then $x + 4$ = the number of inches in the length of the picture.

$$(x + 4)(x + 8) = x(x + 4) + 192.$$

$$x^2 + 12x + 32 = x^2 + 4x + 192.$$

$$8x = 160.$$

$$x = 20.$$

$$x + 4 = 24.$$

11. Let x = the number of inches in the width
of the supposed rectangle.

Then $2x + 5$ = the number of inches in the length
of the supposed rectangle.

Then $2x + 2(2x + 5)$ should equal 112 inches.

Equating, $2x + 2(2x + 5) = 112.$

$$6x = 102.$$

$$x = 17.$$

$2x + 5 = 39.$ Such a rectangle is possible.

12. Let x = the number of dollars.

Then $x + 6$ = the number of quarters,

and $2(x + 6)$ = the number of dimes.

$$100x + 25(x + 6) + 20(x + 6) = 1575.$$

$$145x = 1305.$$

$$x = 9.$$

$$x + 6 = 15.$$

$$2(x + 6) = 30.$$

13. Let x = the number of dimes.

Then $x + 7$ = the number of quarters,

and $5x - 3$ = the number of nickels.

$$x + x + 7 + 5x - 3 = 109.$$

$$7x = 105.$$

$$x = 15, x + 7 = 22, 5x - 3 = 72.$$

Amount in dimes = \$1.50.

Amount in quarters = \$5.50.

Amount in nickels = \$3.60.

Total amount = \$10.60.

14. Let x = the tens' digit.

Then $14 - x$ = the units' digit,

and $10x + (14 - x)$ = the number.

$$10x + (14 - x) = 10(14 - x) + x + 36.$$

$$10x + 14 - x = 140 - 10x + x + 36.$$

$$18x = 162.$$

$$x = 9.$$

$14 - x = 5.$ The number = 95.

15. Let x = the hundreds' digit.

Then $x + 2$ = the tens' digit,

and $x + 4$ = the units' digit.

$$x + x + 2 + x + 4 = 15.$$

$$3x = 9.$$

$$x = 3.$$

$$x + 2 = 5.$$

$$x + 4 = 7. \text{ The number is 357.}$$

16. Let r = the number of inches in the radius of smaller circle.

Then $r + 14$ = the number of inches in the radius of larger circle.

$$\pi r^2 + 1232 = \pi (r + 14)^2.$$

$$\pi r^2 + 1232 = \pi r^2 + 28\pi r + 196\pi.$$

$$28\pi r = 1232 - 196\pi.$$

$$88r = 616.$$

$$r = 7.$$

17. Let x = the number of inches increase in the circumference.

$$2\pi \cdot 8 + x = 2\pi \cdot (8 + 2).$$

$$x = 20\pi - 16\pi = 4\pi = \frac{88}{7}.$$

Let y = the number of square inches increase in area.

$$\pi \cdot 8^2 + y = \pi (8 + 2)^2.$$

$$y = 100\pi - 64\pi = 36\pi = \frac{792}{7}.$$

18. The first equation becomes

$$2\pi R + x = 2\pi (R + 2).$$

Whence $x = 4\pi.$

The second equation becomes

$$\pi R^2 + y = \pi (R + 2)^2.$$

Whence $y = \pi (4R + 4) = 2\pi (2R + 2).$

The solution leads one to observe that whatever number is added to the length of the radius increases the circumference by 2π times that number, and that it increases the area by π times the sum of twice the radius and the number, all multiplied by that number.

19. Let R = the number of feet in the earth's radius,
and let x = the number of feet distant the hoop will be.

$$2\pi R + 1 = 2\pi (R + x).$$

$$2\pi x = 1.$$

$$x = \frac{1}{2\pi} = \frac{7}{44} \text{ feet, or 1.9 inches.}$$

20. Here $\frac{3}{4}$ = the number of feet in the radius of the sphere.

Let x = the number of feet distant the hoop will be.

$$2\pi\left(\frac{3}{4}\right) + 1 = 2\pi\left(\frac{3}{4} + x\right).$$

$$x = \frac{1}{2\pi} = \frac{7}{44} \text{ feet, or 1.9 inches.}$$

21. Let x = the number of feet in the height of the pole.

Then $x + 5$ = the number of feet in the length of the rope.

$$(x + 5)^2 = x^2 + 25^2.$$

$$10x = 600.$$

$$x = 60.$$

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1. $2(a + b) + x(a + b) = (a + b)(2 + x)$.
2. $3(b + 5) + a(b + 5) = (b + 5)(3 + a)$.
3. $a(c - d) - b(c - d) = (c - d)(a - b)$.
4. $2(x - y) - x(x - y) = (x - y)(2 - x)$.
5. $h(m + 3n) - 2k(m + 3n) = (m + 3n)(h - 2k)$.
6. $x(r - s) + y(s - r) = (r - s)(x - y)$.
7. $5r(3m - 2n) - 2s(2n - 3m) = (3m - 2n)(5r + 2s)$.
8. $ac + 2cx + 3ay + 6xy = c(a + 2x) + 3y(a + 2x) = (a + 2x)(c + 3y)$.
9. $cd - 3cf + 2d - 6f = c(d - 3f) + 2(d - 3f) = (d - 3f)(c + 2)$.
10. $r^2s + 2rs - 3r^2t - 6rt = r[(rs + 2s) - (3rt + 6t)]$
 $= r[s(r + 2) - 3t(r + 2)] = r(r + 2)(s - 3t)$.
11. $x^3 - 3xy^2 - 3x^2y + 9y^3 = x(x^2 - 3y^2) - 3y(x^2 - 3y^2)$
 $= (x^2 - 3y^2)(x - 3y)$.
12. $ax + ay + bx + by + cx + cy = a(x + y) + b(x + y) + c(x + y)$
 $= (x + y)(a + b + c)$.
13. $mr - 2r + ms - 2s + mt - 2t = r(m - 2) + s(m - 2) + t(m - 2)$
 $= (m - 2)(r + s + t)$.
14. See p. 118.

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1. $a^2 - x^2 = (a - x)(a + x)$.
2. $x^2 - 9 = (x - 3)(x + 3)$.
3. $16 - x^2 = (4 - x)(4 + x)$.
4. $a^2 - 16b^2 = (a - 4b)(a + 4b)$.
5. $16x^2 - 25y^2 = (4x - 5y)(4x + 5y)$.
6. $25a^2 - 36b^2c^2 = (5a - 6bc)(5a + 6bc)$.
7. $a^4 - 25b^4 = (a^2 - 5b^2)(a^2 + 5b^2)$.

8. $x^4y^4 - 9 = (x^2y^2 - 3)(x^2y^2 + 3)$.
9. $a^4 - 16 = (a^2 + 4)(a^2 - 4) = (a^2 + 4)(a - 2)(a + 2)$.
10. $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2) = (a^2 + b^2)(a + b)(a - b)$.
11. $x^4 - 1 = (x^2 + 1)(x^2 - 1) = (x^2 + 1)(x + 1)(x - 1)$.
12. $x^8 - 1 = (x^4 + 1)(x^4 - 1)$
 $= (x^4 + 1)(x^2 + 1)(x^2 - 1)$
 $= (x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$.
13. $16 - a^4 = (4 + a^2)(4 - a^2) = (4 + a^2)(2 + a)(2 - a)$.
14. $625 - x^4 = (25 + x^2)(25 - x^2) = (25 + x^2)(5 + x)(5 - x)$.
15. $81 - c^4 = (9 + c^2)(9 - c^2) = (9 + c^2)(3 + c)(3 - c)$.
16. See p. 124.
17. $a^4 - x^8 = (a^2 + x^4)(a + x^2)(a - x^2)$.
18. $(a - 2)^2 - c^2 = (a - 2 + c)(a - 2 - c)$.
19. $16(x - y)^2 - z^4 = (4x - 4y + z^2)(4x - 4y - z^2)$.
20. $9(x - 3y)^2 - 16z^4 = (3x - 9y + 4z^2)(3x - 9y - 4z^2)$.
21. $a^2 - (b + c)^2 = (a + b + c)(a - b - c)$.
22. $4r^2s^2 - (r - s)^4 = (2rs + r^2 - 2rs + s^2)(2rs - r^2 + 2rs - s^2)$
 $= (r^2 + s^2)(4rs - r^2 - s^2)$.
23. $9m^2 - 4(n + 3)^4 = (3m + 2n^2 + 12n + 18)(3m - 2n^2 - 12n - 18)$.
24. $(a - c)^2 - (d + e)^2 = (a - c + d + e)(a - c - d - e)$.
25. $a(r + 2s)^2 - a(x - y)^2 = a(r + 2s + x - y)(r + 2s - x + y)$.
26. $4a(a - x)^2 - 9a(c - 2d)^4$
 $= a(2a - 2x + 3c^2 - 12cd + 12d^2)(2a - 2x - 3c^2 + 12cd - 12d^2)$.
27. $m^2 + 2mn + n^2 - (x^2 - 2xy + y^2) = (m + n)^2 - (x - y)^2$
 $= (m + n + x - y)(m + n - x + y)$.
28. $x^2 - 2xy + y^2 - a^2 - 2ab - b^2 = (x - y)^2 - (a + b)^2$
 $= (x - y + a + b)(x - y - a - b)$.
29. $a^2 - 22a + 121 - b^2 + 20bc - 100c^2 = (a - 11)^2 - (b - 10c)^2$
 $= (a - 11 + b - 10c)(a - 11 - b + 10c)$.
30. $4c^2 - a^2 - 2ab - b^2 = 4c^2 - (a + b)^2 = (2c + a + b)(2c - a - b)$.
31. $49x^4 - 49y^2 - 14y - 1 = 49x^4 - (7y + 1)^2$
 $= (7x^2 + 7y + 1)(7x^2 - 7y - 1)$.
32. $a^2 - b^2 - (a - b) = (a + b)(a - b) - (a - b) = (a - b)(a + b - 1)$.
33. $r^2 - rs - (r^2 - s^2) = r(r - s) - (r + s)(r - s)$
 $= (r - s)(r - r - s)$
 $= (r - s)(-s)$
 $= s(s - r)$.
34. $m + n - m^2 + n^2 = (m + n) - (m^2 - n^2)$
 $= (m + n)(1 - m + n)$.
35. $r^2 - r - 3s - 9s^2 = (r^2 - 9s^2) - (r + 3s)$
 $= (r + 3s)(r - 3s - 1)$.

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1. $x^2 + 5x + 6 = (x + 2)(x + 3)$.
2. $x^2 + 7x + 12 = (x + 3)(x + 4)$.
3. $d^2 + 11d + 18 = (d + 9)(d + 2)$.
4. $c^2 - 2c - 35 = (c - 7)(c + 5)$.
5. $r^2 - r - 90 = (r - 10)(r + 9)$.
6. $9 - 10x + x^2 = (9 - x)(1 - x)$.
7. $r^2 - 4rs + 3s^2 = (r - 3s)(r - s)$.
8. $m^2 - 7mn + 10n^2 = (m - 5n)(m - 2n)$.
9. $1 + 2n - 24n^2 = (1 + 6n)(1 - 4n)$.
10. $(a + b)^2 - 2(a + b) - 8 = (a + b - 4)(a + b + 2)$.
11. $(x - y)^2 + 4(x - y) + 3 = (x - y + 3)(x - y + 1)$.
12. $a^{2n} + 12a^n + 35 = (a^n + 7)(a^n + 5)$.
13. $c^{2a} - 7c^a - 18 = (c^a - 9)(c^a + 2)$.
14. See p. 128.
15. $2x^2 + 5x + 2 = (x + 2)(2x + 1)$.
16. $4a^2 + 7a + 3 = (4a + 3)(a + 1)$.
17. $3x^2 + 13x + 12 = (3x + 4)(x + 3)$.
18. $3x^2 + 17x + 10 = (3x + 2)(x + 5)$.
19. $4x^2 + 8x + 3 = (2x + 3)(2x + 1)$.
20. $2b^2 - 5b + 2 = (2b - 1)(b - 2)$.
21. $6c^2 + 7c + 2 = (3c + 2)(2c + 1)$.
22. $3x^2 - 11x + 8 = (3x - 8)(x - 1)$.
23. $10x^2 - 29x + 10 = (5x - 2)(2x - 5)$.
24. $12x^2 - 11xy + 2y^2 = (3x - 2y)(4x - y)$.
25. $2a^2 + 3a - 2 = (2a - 1)(a + 2)$.
26. $2a^2 - a - 15 = (2a + 5)(a - 3)$.
27. $9a^2 + 3a - 2 = (3a + 2)(3a - 1)$.
28. $12r^2 + 10r - 12 = 2(3r - 2)(2r + 3)$.
29. $6a^4 - 11a^3b - 7a^2b^2 = a^2(3a - 7b)(2a + b)$.
30. $6x^3 + 10x^2y - 4xy^2 = 2x(3x - y)(x + 2y)$.
31. $2x^2 + 5xy - 12y^2 = (2x - 3y)(x + 4y)$.
32. $6a^{2n} - 7a^n + 2 = (3a^n - 2)(2a^n - 1)$.
33. $3a^{2n} - 10a^n - 8 = (3a^n + 2)(a^n - 4)$.
34. $10a^{2n} - a^n - 3 = (5a^n - 3)(2a^n + 1)$.

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1. $x^4 + x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 - x^2y^2 = (x^2 + y^2)^2 - (xy)^2$
 $= (x^2 + y^2 + xy)(x^2 + y^2 - xy)$.
2. $c^4 + c^2d^2 + d^4 = c^4 + 2c^2d^2 + d^4 - c^2d^2 = (c^2 + d^2 + cd)(c^2 + d^2 - cd)$.
3. $a^4 + a^2b^4 + b^8 = a^4 + 2a^2b^4 + b^8 - a^2b^4$
 $= (a^2 + b^4 + ab^2)(a^2 + b^4 - ab^2)$.

4. $a^4 + 3a^2b^2 + 4b^4 = a^4 + 4a^2b^2 + 4b^4 - a^2b^2$
 $= (a^2 + 2b^2 + ab)(a^2 + 2b^2 - ab).$
5. $m^4 + m^2 + 1 = m^4 + 2m^2 + 1 - m^2 = (m^2 + 1 - m)(m^2 + 1 + m).$
6. $x^4 + 5x^2 + 9 = x^4 + 6x^2 + 9 - x^2 = (x^2 + 3 + x)(x^2 + 3 - x).$
7. $c^4 + 4c^2 + 16 = c^4 + 8c^2 + 16 - 4c^2 = (c^2 + 4 + 2c)(c^2 + 4 - 2c).$
8. $25a^4 - 19a^2 + 1 = 25a^4 - 10a^2 + 1 - 9a^2$
 $= (5a^2 - 1 + 3a)(5a^2 - 1 - 3a).$
9. $25x^4 - 11x^2 + 1 = 25x^4 - 10x^2 + 1 - x^2$
 $= (5x^2 - 1 + x)(5x^2 - 1 - x).$
10. $4r^4 - 44r^2s^2 + 49s^4 = 4r^4 - 28r^2s^2 + 49s^4 - 16r^2s^2$
 $= (2r^2 - 7s^2 + 4rs)(2r^2 - 7s^2 - 4rs).$
11. $25x^4 - 19x^2 + 9 = 25x^4 + 30x^2 + 9 - 49x^2$
 $= (5x^2 + 3 + 7x)(5x^2 + 3 - 7x).$
12. $9ax^8 - 28ax^4y^4 + 4ay^8 = a(9x^8 - 12x^4y^4 + 4y^8 - 16x^4y^4)$
 $= a(3x^4 - 2y^4 + 4x^2y^2)(3x^4 - 2y^4 - 4x^2y^2).$
13. $4a^8x + 3a^4b^4x + 9b^8x = x(4a^8 + 12a^4b^4 + 9b^8 - 9a^4b^4)$
 $= x(2a^4 + 3b^4 + 3a^2b^2)(2a^4 + 3b^4 - 3a^2b^2).$
14. $4a^4 + 1 = 4a^4 + 4a^2 + 1 - 4a^2 = (2a^2 + 1 - 2a)(2a^2 + 1 + 2a).$
15. $c^4 + 4d^4 = c^4 + 4c^2d^2 + 4d^4 - 4c^2d^2$
 $= (c^2 + 2d^2 + 2cd)(c^2 + 2d^2 - 2cd).$
16. $64a^4x^2 + x^2 = x^2(64a^4 + 16a^2 + 1 - 16a^2)$
 $= x^2(8a^2 + 1 + 4a)(8a^2 + 1 - 4a).$
17. $4a^{4n} + b^{4n} = 4a^{4n} + 4a^{2n}b^{2n} + b^{4n} - 4a^{2n}b^{2n}$
 $= (2a^{2n} + b^{2n} + 2a^nb^n)(2a^{2n} + b^{2n} - 2a^nb^n).$
18. $x^{4m} + 4y^{4n} = x^{4m} + 4x^{2m}y^{2n} + 4y^{4n} - 4x^{2m}y^{2n}$
 $= (x^{2m} + 2y^{2n} + 2x^my^n)(x^{2m} + 2y^{2n} - 2x^my^n).$

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1. $a^3 + d^3 = (a + d)(a^2 - ad + d^2).$ 3. $d^3 + 8 = (d + 2)(d^2 - 2d + 4).$
2. $a^3 + 2^3 = (a + 2)(a^2 - 2a + 4).$ 4. $x^3 + 125 = (x + 5)(x^2 - 5x + 25).$
5. $x^3 - y^3 = (x - y)(x^2 + xy + y^2).$
6. $x^3 - 2^3 = (x - 2)(x^2 + 2x + 4).$
7. $m^3 - 64 = (m - 4)(m^2 + 4m + 16).$
8. $m^3 - (2n)^3 = (m - 2n)(m^2 + 2mn + 4n^2).$
9. $8x^3 - y^3 = (2x - y)(4x^2 + 2xy + y^2).$
10. $125 - x^3 = (5 - x)(25 + 5x + x^2).$ 12. $c^3 + d^6 = (c + d^2)(c^2 - cd^2 + d^4).$
11. $a^3 + (b^2)^3 = (a + b^2)(a^2 - ab^2 + b^4).$ 13. $m^3 - n^9 = (m - n^3)(m^2 + mn^3 + n^6).$
14. $(2a)^3 - (3b)^3 = (2a - 3b)(4a^2 + 6ab + 9b^2).$
15. $125x^3 + 8y^3 = (5x + 2y)(25x^2 - 10xy + 4y^2).$
16. $(a + b)^3 + c^3 = (a + b + c)(a^2 + 2ab + b^2 - ac - bc + c^2).$

17. $m^3 - n^3 - m + n = (m^3 - n^3) - (m - n) = (m - n)(m^2 + mn + n^2 - 1)$.
 18. $x^3 - 8y^3 + x - 2y = (x - 2y)(x^2 + 2xy + 4y^2 + 1)$.
 19. $r^3 - 8s^3 + r - 2s = (r - 2s)(r^2 + 2rs + 4s^2 + 1)$.
 20. $a^3n + b^3n = (a^n + b^n)(a^{2n} - a^nb^n + b^{2n})$.
 21. $a^3m - b^3n = (a^m - b^n)(a^{2m} + a^mb^n + b^{2n})$.
 22. See p. 133.

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$$\begin{array}{r}
 1. \quad x^2 + bx \qquad \qquad \qquad + c \overline{) x - n} \\
 \underline{x^2 \qquad - nx} \qquad \qquad \qquad \underline{x + b + n} \\
 bx + nx \\
 \underline{bx \qquad \qquad - bn} \\
 nx \qquad \qquad + bn \\
 \underline{nx - n^2} \\
 n^2 + bn + c = \text{Remainder.}
 \end{array}$$

$$\begin{array}{r}
 2. \quad x^2 + bx \qquad \qquad \qquad + c \overline{) x - a} \\
 \underline{x^2 \qquad - ax} \qquad \qquad \qquad \underline{x + b + a} \\
 bx + ax \qquad \qquad \qquad + c \\
 \underline{bx \qquad \qquad - ab} \\
 ax \qquad \qquad + ab + c \\
 \underline{ax - a^2} \\
 a^2 + ab + c = \text{Remainder.}
 \end{array}$$

$$\begin{array}{r}
 3. \quad x^3 + ax^2 + \qquad \qquad \qquad bx + c \overline{) x - n} \\
 \underline{x^3 - nx^2} \qquad \qquad \qquad \underline{x^2 + (a+n)x + b + an + n^2} \\
 (a+n)x^2 + \qquad \qquad \qquad bx + c \\
 \underline{(a+n)x^2 - (an + n^2)x} \\
 (b + an + n^2)x + c \\
 \underline{(b + an + n^2)x \qquad - bn - an^2 - n^3} \\
 c + bn + an^2 + n^3 = \text{Remainder.}
 \end{array}$$

$$\begin{array}{r}
 4. \quad (a) \quad x^3 + x^2 - 5x + 3 \overline{) x - 2} \\
 \underline{x^3 - 2x^2} \qquad \qquad \qquad \underline{x^2 + 3x + 1} \\
 3x^2 - 5x \\
 \underline{3x^2 - 6x} \\
 x + 3 \\
 \underline{x - 2} \\
 5 = \text{Remainder.}
 \end{array}$$

(b) Substituting 2 for x , $x^3 + x^2 - 5x + 3 = 8 + 4 - 10 + 3 = 5$.

$$\begin{array}{r}
 5. (a) \quad x^3 \quad - \quad x + 5 \quad | \quad x - 3 \\
 \underline{x^3 - 3x^2} \quad \quad \quad \underline{x^2 + 3x + 8} \\
 3x^2 - x \\
 \underline{3x^2 - 9x} \\
 8x + 5 \\
 \underline{8x - 24} \\
 29 = \text{Remainder.}
 \end{array}$$

(b) Substituting 3 for x , $x^3 - x + 5 = 27 - 3 + 5 = 29$, Remainder.

6. Substituting 3 for x ,

$$x^3 + x^2 - 5x + 8 = 27 + 9 - 15 + 8 = 29, \text{ Remainder.}$$

7. Substituting -4 for x ,

$$x^3 - 3x - 15 = -64 + 12 - 15 = -67, \text{ Remainder.}$$

8. Substituting 5 for x ,

$$x^3 - 2x^2 - 100 = 125 - 50 - 100 = -25, \text{ Remainder.}$$

9. Substituting 3 for x ,

$$x^3 - 2x^2 - 2x - 3 = 27 - 18 - 6 - 3 = 0, \text{ Remainder.}$$

10. Substituting 2 for x ,

$$x^4 - 2x^3 + x - 2 = 16 - 16 + 2 - 2 = 0, \text{ Remainder.}$$

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1. $x^3 + x - 2 = (x - 1)(x^2 + x + 2)$.

Substituting 1 for x , $x^3 + x - 2 = 1 + 1 - 2 = 0$.

Hence $x - 1$ is a factor. Dividing by $x - 1$, the other factor is $x^2 + x + 2$.

2. $x^3 + 2x + 3 = (x + 1)(x^2 - x + 3)$.

3. $a^3 + a^2 - 36 = (a - 3)(a^2 + 4a + 12)$.

4. $x^3 + x - 10 = (x - 2)(x^2 + 2x + 5)$.

5. $d^3 + d^2 - 12 = (d - 2)(d^2 + 3d + 6)$.

6. $x^3 - x^2 + 4x - 4 = (x - 1)(x^2 + 4)$.

7. $x^3 - 2x^2 - 5x + 6 = (x - 1)(x + 2)(x - 3)$, for 1, -2 , and 3 when substituted for x each gives the remainder 0.

8. $x^3 - x^2 - 4 = (x - 2)(x^2 + x + 2)$.

9. $x^4 - 7x^2 - 6x = x(x + 1)(x + 2)(x - 3)$.

10. $y^3 - y^2 - 9y + 9 = (y - 1)(y - 3)(y + 3)$.

11. $2x^3 - 2x^2 - x - 6 = (x - 2)(2x^2 + 2x + 3)$.

12. $x^3 - 7x^2 + 4x + 12 = (x + 1)(x - 2)(x - 6)$.

13. $a^4 - 6a^3 + 11a^2 - 6a = a(a - 1)(a - 2)(a - 3)$.

14. See solution to Example, p. 302.

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1. $x^5 + z^5 = (x + z)(x^4 - x^3z + x^2z^2 - xz^3 + z^4)$.
2. $x^5 + 1 = (x + 1)(x^4 - x^3 + x^2 - x + 1)$.
3. $a^5 + 2^5 = (a + 2)(a^4 - 2a^3 + 4a^2 - 8a + 16)$.
4. $x^5 + 32 = (x + 2)(x^4 - 2x^3 + 4x^2 - 8x + 16)$.
5. $(a^2)^5 + (b^2)^5 = (a^2 + b^2)(a^8 - a^6b^2 + a^4b^4 - a^2b^6 + b^8)$.
6. $x^5 - z^5 = (x - z)(x^4 + x^3z + x^2z^2 + xz^3 + z^4)$.
7. $a^5 - 2^5 = (a - 2)(a^4 + 2a^3 + 4a^2 + 8a + 16)$.
8. $a^5 - 32x^5 = (a - 2x)(a^4 + 2a^3x + 4a^2x^2 + 8ax^3 + 16x^4)$.
9. $(2x)^5 - 243y^5 = (2x - 3y)(16x^4 + 24x^3y + 36x^2y^2 + 54xy^3 + 81y^4)$.
10. $a^7 - x^7 = (a - x)(a^6 + a^5x + a^4x^2 + a^3x^3 + a^2x^4 + ax^5 + x^6)$.
11. $1 - r^7 = (1 - r)(1 + r + r^2 + r^3 + r^4 + r^5 + r^6)$.
12. $x^7 - 128 = (x - 2)(x^6 + 2x^5 + 4x^4 + 8x^3 + 16x^2 + 32x + 64)$.
13. $x^{10} + y^{15} = (x^2 + y^3)(x^8 - x^6y^3 + x^4y^6 - x^2y^9 + y^{12})$.
14. $a^{10} + 32x^{15} = (a^2 + 2x^3)(a^8 - 2a^6x^3 + 4a^4x^6 - 8a^2x^9 + 16x^{12})$.
15. $x^7 + a^7 = (x + a)(x^6 - x^5a + x^4a^2 - x^3a^3 + x^2a^4 - xa^5 + a^6)$.
16. $1 + r^7 = (1 + r)(1 - r + r^2 - r^3 + r^4 - r^5 + r^6)$.
17. $128x^7 + 1 = (2x + 1)(64x^6 - 32x^5 + 16x^4 - 8x^3 + 4x^2 - 2x + 1)$.
18. $1 - r^n = (1 - r)(1 + r + r^2 + r^3 + r^4 + \dots + r^{n-1})$.
19. $1 + r^n = (1 + r)(1 - r + r^2 - r^3 + r^4 - r^5 \dots + r^{n-1})$ if n is odd.
20. $x^6 - y^6 = (x^3 + y^3)(x^3 - y^3) = (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$.
21. $x^8 - y^8 = (x^4 + y^4)(x^4 - y^4) = (x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$.
22. $a^{10} - b^{10} = (a^5 + b^5)(a^5 - b^5)$
 $= (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)(a - b)$
 $(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$.
23. $a^{12} - b^{12} = (a^6 + b^6)(a^6 - b^6)$
 $= (a^2 + b^2)(a^4 - a^2b^2 + b^4)(a + b)(a^2 - ab + b^2)(a - b)(a^2 + ab + b^2)$.

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1. $6x^4 + 2x^3 + 2x^2 = 2x^2(3x^2 + x + 1)$.
2. $5a^3 + 2a^2 - 15a - 6 = a^2(5a + 2) - 3(5a + 2) = (a^2 - 3)(5a + 2)$.
3. $2c^3d - 8cd^3 = 2cd(c^2 - 4d^2) = 2cd(c + 2d)(c - 2d)$.
4. $3m^3 - 3m^2 - 18m = 3m(m^2 - m - 6) = 3m(m - 3)(m + 2)$.
5. $2x^2 + 3ax + a^2 = (2x + a)(x + a)$.
6. $x^4 - 7x^2y^2 + 9y^4 = x^4 - 6x^2y^2 + 9y^4 - x^2y^2$
 $= (x^2 - 3y^2 + xy)(x^2 - 3y^2 - xy)$.
7. $a^4c - ac^4 = ac(a^3 - c^3) = ac(a - c)(a^2 + ac + c^2)$.
8. $2x^7y - 2xy^7 = 2xy(x^6 - y^6)$
 $= 2xy(x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$.
9. $x^5 - 2x^4 - 9x^2 = x^2(x^3 - 2x^2 - 9) = x^2(x - 3)(x^2 + x + 3)$.
10. $18r^3 - 24r^2s + 8rs^2 = 2r(9r^2 - 12rs + 4s^2) = 2r(3r - 2s)^2$.

11. $45x^3y - 20xy^3 = 5xy(9x^2 - 4y^2) = 5xy(3x + 2y)(3x - 2y).$
12. $2h^3k + 4h^2k^2 - 30hk^3 = 2hk(h^2 + 2hk - 15k^2) = 2hk(h + 5k)(h - 3k).$
13. $x^4 + 7x^2 + 16 = x^4 + 8x^2 + 16 - x^2 = (x^2 + 4 - x)(x^2 + 4 + x).$
14. $a^4d - 8ad^4 = ad(a^3 - 8d^3) = ad(a - 2d)(a^2 + 2ad + 4d^2).$
15. $2a^7 + 64a^2 = 2a^2(a^5 + 32) = 2a^2(a + 2)(a^4 - 2a^3 + 4a^2 - 8a + 16).$
16. $m^8n + mn^8 = mn(m^7 + n^7)$
 $= mn(m + n)(m^6 - m^5n + m^4n^2 - m^3n^3 + m^2n^4 - mn^5 + n^6).$
17. $x^3 + 4x - 5 = (x - 1)(x^2 + x + 5).$
18. $ax^2 - 4a + 3x^2 - 12 = (a + 3)(x^2 - 4) = (a + 3)(x + 2)(x - 2).$
19. $a^2c^2 - c^2 - a^2 + 1 = (a^2 - 1)(c^2 - 1) = (a + 1)(a - 1)(c + 1)(c - 1).$
20. $x^4 - 13x^2 + 36 = (x^2 - 9)(x^2 - 4)$
 $= (x + 3)(x - 3)(x + 2)(x - 2).$
21. $a^5 - a^2b^3 - a^3b^2 + b^5 = a^2(a^3 - b^3) - b^2(a^3 - b^3)$
 $= (a^2 - b^2)(a^3 - b^3)$
 $= (a + b)(a - b)(a - b)(a^2 + ab + b^2).$
22. $x^{12} - x^2 = x^2(x^{10} - 1)$
 $= x^2(x^5 + 1)(x^5 - 1)$
 $= x^2(x + 1)(x^4 - x^3 + x^2 - x + 1)(x - 1)(x^4 + x^3 + x^2 + x + 1).$
23. $x^3 - 3x^2 - 4x + 12 = x^2(x - 3) - 4(x - 3)$
 $= (x^2 - 4)(x - 3)$
 $= (x + 2)(x - 2)(x - 3).$
24. $(a + x)^2 + 10(a + x) + 25 = (a + x + 5)^2.$
25. $a^8 + 27a^2 = a^2(a^6 + 27) = a^2(a^2 + 3)(a^4 - 3a^2 + 9).$
26. $(x + y)^2 - 6(x + y)z + 9z^2 = (x + y - 3z)^2.$
27. $(a - x)^2 - 16(m - n)^2 = (a - x + 4m - 4n)(a - x - 4m + 4n).$
28. $a^4 - 11a^2 + 1 = a^4 - 2a^2 + 1 - 9a^2$
 $= (a^2 - 1)^2 - 9a^2$
 $= (a^2 - 1 - 3a)(a^2 - 1 + 3a).$
29. $a^4 + ab^{15} = a(a^3 + b^{15}) = a(a + b^5)(a^2 - ab^5 + b^{10}).$
30. $4(a - b)^2 - 12(a - b)c + 9c^2 = (2a - 2b - 3c)^2.$
31. $a^2 - 10ab + 25b^2 - c^2 = (a - 5b)^2 - c^2 = (a - 5b + c)(a - 5b - c).$
32. $x^2 - 2x(a - b) - 35(a - b)^2 = (x - 7a + 7b)(x + 5a - 5b).$
33. $x^3 - 8x^2 + 17x - 10 = (x - 1)(x - 2)(x - 5).$
34. $x^4 - x^2y^2 + x^3 - x^2y = x^2(x^2 - y^2 + x - y) = x^2(x - y)(x + y + 1).$
35. $a^2 - b^2 + (a - b)^2 = (a - b)(a + b + a - b) = 2a(a - b).$
36. $c^2 - 2cd + d^2 - 2(c - d) - 35 = (c - d)^2 - 2(c - d) - 35$
 $= (c - d - 7)(c - d + 5).$
37. $2(a + b)^2 + 3c(a + b) - 2c^2 = (2a + 2b - c)(a + b + 2c).$
38. $a^3 + b^3 + 3a^2b + 3ab^2 = (a + b)^3.$
39. $32x^8 - x^3y^{10} = x^3(32x^5 - y^{10})$
 $= x^3(2x - y^2)(16x^4 + 8x^3y^2 + 4x^2y^4 + 2xy^6 + y^8).$

40. $c^2 + 4d^2 - x^2 - 4cd = (c - 2d)^2 - x^2 = (c - 2d + x)(c - 2d - x).$
41. $(a - 2x)c^2 + (2x - a)d^2 = (a - 2x)c^2 - (a - 2x)d^2$
 $= (a - 2x)(c + d)(c - d).$
42. $6a^4 + 3a^3 - 3a^2 = 3a^2(2a^2 + a - 1) = 3a^2(2a - 1)(a + 1).$
43. $16a^4 + 7a^2 + 1 = 16a^4 + 8a^2 + 1 - a^2$
 $= (4a^2 + 1 + a)(4a^2 + 1 - a).$
44. $x^6y^6 - 64 = (x^3y^3 + 8)(x^3y^3 - 8)$
 $= (xy + 2)(x^2y^2 - 2xy + 4)(xy - 2)(x^2y^2 + 2xy + 4).$
45. $a^7 - a^2b^5 + a^2b - a^3 = a^2(a^5 - b^5 + b - a)$
 $= a^2[(a^5 - b^5) - (a - b)]$
 $= a^2(a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4 - 1).$
46. $a^4 + 8ab^3 + a^2 + 2ab = a[(a^3 + 8b^3) + (a + 2b)]$
 $= a(a + 2b)(a^2 - 2ab + 4b^2 + 1).$
47. $9x^2 - 4y^2 - 3x - 2y = (9x^2 - 4y^2) - (3x + 2y)$
 $= (3x + 2y)(3x - 2y - 1).$
48. $x^2 - 5(2x - 5) = x^2 - 10x + 25 = (x - 5)^2.$
49. $3a^2 - 14a(b - c) + 8(b - c)^2 = (3a - 2b + 2c)(a - 4b + 4c).$
50. $a^4x^4 + 4 = a^4x^4 + 4a^2x^2 + 4 - 4a^2x^2$
 $= (a^2x^2 + 2 + 2ax)(a^2x^2 + 2 - 2ax).$
51. $x^8y^6 + x^2z^6 = x^2(x^6y^6 + z^6) = x^2(x^2y^2 + z^2)(x^4y^4 - x^2y^2z^2 + z^4).$
52. $x^3 - 10x - 3 = (x + 3)(x^2 - 3x - 1).$
53. $a^8 - 5a^4 + 4 = (a^4 - 1)(a^4 - 4)$
 $= (a^2 + 1)(a + 1)(a - 1)(a^2 + 2)(a^2 - 2).$
54. $x - 1 + x^5 - x^3 = x^5 - x^3 + x - 1 = (x - 1)(x^4 + x^3 + 1).$
55. $x^3 + x - y - y^3 = (x^3 - y^3) + (x - y) = (x - y)(x^2 + xy + y^2 + 1).$
56. $x^2 + 2xy + y^2 - 25a^2 - 10a - 1 = (x + y)^2 - (5a + 1)^2$
 $= (x + y + 5a + 1)(x + y - 5a - 1).$
57. $a^4 - b^4 - 2ab(a^2 - b^2) = (a^2 - b^2)(a^2 + b^2 - 2ab) = (a + b)(a - b)^3.$
58. $2x^3 + 3x^2 + x = x(2x^2 + 3x + 1) = x(2x + 1)(x + 1).$
59. $m^7 - 8m - 7m^4 = m(m^6 - 7m^3 - 8)$
 $= m(m^3 - 8)(m^3 + 1)$
 $= m(m - 2)(m^2 + 2m + 4)(m + 1)(m^2 - m + 1).$
60. $2x^4 - 10x^2 + 4x = 2x(x^3 - 5x + 2) = 2x(x - 2)(x^2 + 2x - 1).$
61. $a^3 + 1 + 3a^2 + 3a = (a^3 + 1) + 3a(a + 1)$
 $= (a + 1)(a^2 - a + 1 + 3a)$
 $= (a + 1)(a^2 + 2a + 1)$
 $= (a + 1)^3.$
62. $12x^2 - 8x + x^4 - 6x^3 = x(x^3 - 6x^2 + 12x - 8)$
 $= x(x - 2)(x^2 + 2x + 4 - 6x)$
 $= x(x - 2)^3.$
63. $a^nx^2 + 2a^nx + a^n = a^n(x + 1)^2.$

$$64. mr - ms - nr + ns = m(r - s) - n(r - s) = (r - s)(m - n).$$

$$65. h^{2m} - 2h^mk^n + k^{2n} = (h^m - k^n)^2.$$

$$66. a^{2m} - b^{2n} = (a^m + b^n)(a^m - b^n).$$

$$67. x^{2n} + (r + s)x^n + rs = (x^n + r)(x^n + s).$$

$$68. hr x^{2n} + hr x^n + hs x^n + hs = hr x^n (x^n + 1) + hs (x^n + 1) \\ = (hr x^n + hs)(x^n + 1) \\ = h(rx^n + s)(x^n + 1).$$

$$69. a^{4m} + a^{2m}b^{2n} + b^{4n} = a^{4m} + 2a^{2m}b^{2n} + b^{4n} - a^{2m}b^{2n} \\ = (a^{2m} + b^{2n} + a^mb^n)(a^{2m} + b^{2n} - a^mb^n).$$

$$70. a^{3m} + b^{3n} = (a^m + b^n)(a^{2m} - a^mb^n + b^{2n}).$$

$$71. a^{3m} - b^{3n} = (a^m - b^n)(a^{2m} + a^mb^n + b^{2n}).$$

$$72. a^{5m} + b^{5n} = (a^m + b^n)(a^{4m} - a^{3m}b^n + a^{2m}b^{2n} - a^mb^{3n} + b^{4n}).$$

$$73. a^{5m} - b^{5n} = (a^m - b^n)(a^{4m} + a^{3m}b^n + a^{2m}b^{2n} + a^mb^{3n} + b^{4n}).$$

$$75. \quad \quad \quad cx + dx = c^2 + cd. \\ (c + d)x = c(c + d). \\ x = c.$$

$$76. \quad \quad \quad ma - mc + bc = ab. \\ m(a - c) = b(a - c). \\ m = b.$$

$$77. 5ay - 3by - 5a^2 + 3ab = 0. \\ (5a - 3b)y = a(5a - 3b). \\ y = a.$$

$$78. \quad \quad \quad az - 3ad = bz - 3bd. \\ (a - b)z = 3d(a - b). \\ z = 3d.$$

$$79. \quad \quad \quad cx - 2dx = c^2 - 4d^2. \\ (c - 2d)x = (c - 2d)(c + 2d). \\ x = c + 2d.$$

$$80. \quad \quad \quad r(2a - 7c) = 4a^2 - 28ac + 49c^2. \\ r(2a - 7c) = (2a - 7c)^2. \\ r = 2a - 7c.$$

$$81. \quad \quad \quad m(3a - c) - 9ad = 2ec - 6ae - 3cd. \\ m(3a - c) = 9ad - 3cd - 6ae + 2ec. \\ m(3a - c) = 3d(3a - c) - 2e(3a - c). \\ m = 3d - 2e.$$

$$82. \quad \quad \quad ax - 3bx - a^2 = 3b^2 - 4ab. \\ (a - 3b)x = a^2 - 4ab + 3b^2. \\ (a - 3b)x = (a - 3b)(a - b). \\ x = a - b.$$

83. $y + 21d^2 + 4d = 7dy + 1.$
 $y(1 - 7d) = 1 - 4d - 21d^2.$
 $= (1 - 7d)(1 + 3d).$
 $y = 1 + 3d.$
84. $d(1 - 3a) + x + 3ac = c + 3ax.$
 $(1 - 3a)x = c - 3ac - d(1 - 3a).$
 $= c(1 - 3a) - d(1 - 3a).$
 $x = c - d.$
85. $az + ae - 2ec + 2cd = 2cz + ad.$
 $az - 2cz = ad - 2cd - ae + 2ec.$
 $(a - 2c)z = d(a - 2c) - e(a - 2c).$
 $z = d - e.$
86. $5x - 2cx - a(5 - 2c) = 5a + 5 - 2c - 2ac.$
 $(5 - 2c)x = a(5 - 2c) + a(5 - 2c) + (5 - 2c).$
 $x = 2a + 1.$
87. $a^2x - ax + x - a^3 - 1 = 0.$
 $(a^2 - a + 1)x = a^3 + 1.$
 $x = a + 1.$
88. $c^3 - c^2x - 2cdx - 8d^3 = 4d^2x.$
 $(c^2 + 2cd + 4d^2)x = c^3 - 8d^3.$
 $x = c - 2d.$
89. $m(a - 2)(a^2 + 4) = a^4 - 16.$
 $m = a + 2.$
90. $16s - 8as + 4a^2s - 2a^3s + a^4s - 32 = a^5.$
 $(a^4 - 2a^3 + 4a^2 - 8a + 16)s = a^5 + 32$
 $s = a + 2.$

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|----|--|----|--|
| 1. | $x^2 - 4 = 0.$
$(x + 2)(x - 2) = 0.$
$x = -2, 2.$ | 5. | $x^2 + 5 = 6x.$
$x^2 - 6x + 5 = 0.$
$(x - 5)(x - 1) = 0.$
$x = 5, 1.$ |
| 2. | $x^2 - 5x = 0.$
$x(x - 5) = 0.$
$x = 0, 5.$ | 6. | $2x^2 + x = 6.$
$2x^2 + x - 6 = 0.$
$(2x - 3)(x + 2) = 0.$
$x = \frac{3}{2}, -2.$ |
| 3. | $x^2 - 4a^2 = 0.$
$(x + 2a)(x - 2a) = 0.$
$x = -2a, 2a.$ | | |
| 4. | $y^3 = 64y.$
$y(y + 8)(y - 8) = 0.$
$y = 0, -8, 8.$ | 7. | $m^2 - am - 2a^2 = 0.$
$(m - 2a)(m + a) = 0.$
$m = 2a, -a.$ |

8. $y^3 - y + 2 = 2y^2$.
 $y^3 - 2y^2 - y + 2 = 0$.
 $(y^2 - 1)(y - 2) = 0$.
 $(y + 1)(y - 1)(y - 2) = 0$.
 $y = -1, +1, 2$.
9. $x^2 - 9a^2 - x + 3a = 0$.
 $(x - 3a)(x + 3a - 1) = 0$.
 $x = 3a, 1 - 3a$.
10. $x^3 - a^2x + 2ax^2 - 2a^3 = 0$.
 $(x + 2a)(x + a)(x - a) = 0$.
 $x = -2a, -a, a$.
11. $y^3 - 7y - 6 = 0$.
 $(y + 1)(y - 3)(y + 2) = 0$.
 $y = -1, 3, -2$.
12. $y^4 - 13y^2 + 36 = 0$.
 $(y^2 - 9)(y^2 - 4) = 0$.
 $(y - 3)(y + 3)(y - 2)(y + 2) = 0$.
 $y = \pm 3, \pm 2$.
13. $x^5 - 5x^3 = -4x$.
 $x^5 - 5x^3 + 4x = 0$.
 $x(x + 2)(x - 2)(x + 1)(x - 1) = 0$.
 $x = 0, \pm 2, \pm 1$.
14. $z^3 + 12z - 6z^2 - 8 = 0$.
 $z^3 - 8 - 6z(z - 2) = 0$.
 $(z - 2)(z^2 - 4z + 4) = 0$.
 $(z - 2)^3 = 0$.
 $z = 2, 2, 2$.
15. $3x^3 + 72x + 33x^2 = 0$.
 $3x(x^2 + 11x + 24) = 0$.
 $3x(x + 8)(x + 3) = 0$.
 $x = 0, -8, -3$.
16. $8x^2 + 71x^3 - 9x^4 = 0$.
 $x^2(9x + 1)(x - 8) = 0$.
 $x = 0, 0, -\frac{1}{9}, 8$.
17. $(2x - 3)^2 - (5x + 6)^2 = 0$.
 $(2x - 3 + 5x + 6)(2x - 3 - 5x - 6) = 0$.
 $(7x + 3)(-3x - 9) = 0$.
 $x = -\frac{3}{7}, -3$.
18. $(x - 3)^2 - 2(x - 3) = 8$.
 $(x - 7)(x - 1) = 0$.
 $x = 7, 1$.
19. $x^3 + 50 - 25x - 2x^2 = 0$.
 $x^2(x - 2) - 25(x - 2) = 0$.
 $(x - 2)(x - 5)(x + 5) = 0$.
 $x = 2, 5, -5$.
20. $x^3 - ax^2 - 4a^2x + 4a^3 = 0$.
 $(x - a)(x^2 - 4a^2) = 0$.
 $x = a, \pm 2a$.
21. $y^3 + 3y^2 + 3y + 1 = 0$.
 $(y + 1)^3 = 0$.
 $y = -1, -1, -1$.
22. $acx^2 + bcx + adx + bd = 0$.
 $(cx + d)(ax + b) = 0$.
 $x = -\frac{d}{c}, -\frac{b}{a}$.
23. $x^2 - 7x - 8 = x + 1$.
 $x^2 - 8x - 9 = 0$.
 $(x - 9)(x + 1) = 0$.
 $x = 9, -1$.
24. $3x^2 - 11x + 6 = 3x - 2$.
 $3x^2 - 14x + 8 = 0$.
 $(3x - 2)(x - 4) = 0$.
 $x = \frac{2}{3}, 4$.
25. $6x^2 + 7x - 3 = 3x - 1$.
 $6x^2 + 4x - 2 = 0$.
 $2(3x - 1)(x + 1) = 0$.
 $x = \frac{1}{3}, -1$.
26. See p. 140.

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1. $12 = 2^2 \cdot 3.$
 $18 = 2 \cdot 3^2.$
 $24 = 2^3 \cdot 3.$
 $\text{H.C.F.} = 2 \cdot 3 = 6.$
2. $15 = 3 \cdot 5.$
 $25 = 5^2.$
 $40 = 5 \cdot 2^3.$
 $\text{H.C.F.} = 5.$
3. $12 a^2 = 2^2 \cdot 3 a^2.$
 $30 a^5 = 2 \cdot 3 \cdot 5 a^5.$
 $36 a^4 = 2^2 \cdot 3^2 a^4.$
 $\text{H.C.F.} = 2 \cdot 3 a^2 = 6 a^2.$
4. $28 a^2 b^4 = 2^2 \cdot 7 a^2 b^4.$
 $42 a b^5 = 2 \cdot 3 \cdot 7 a b^5.$
 $70 a^2 b^3 = 2 \cdot 5 \cdot 7 a^2 b^3.$
 $\text{H.C.F.} = 2 \cdot 7 a b^3 = 14 a b^3.$
5. $66 c^4 x = 2 \cdot 3 \cdot 11 c^4 x.$
 $132 c^2 x^3 = 2^2 \cdot 3 \cdot 11 c^2 x^3.$
 $165 c^4 x^2 = 3 \cdot 5 \cdot 11 c^4 x^2.$
 $\text{H.C.F.} = 3 \cdot 11 c^2 x = 33 c^2 x.$
6. $a^2 + 2 ab + b^2 = (a + b)^2.$
 $a^2 - b^2 = (a + b)(a - b).$
 $\text{H.C.F.} = a + b.$
7. $3 a^2 - 3 b^2 = 3(a + b)(a - b).$
 $9(a - b)^2 = 3^2(a - b)^2.$
 $3 a^3 - 3 b^3 = 3(a - b)(a^2 + ab + b^2).$
 $\text{H.C.F.} = 3(a - b).$
8. $ax^2 - 2axy + ay^2 = a(x - y)^2.$
 $a^2x^2 - a^2y^2 = a^2(x - y)(x + y).$
 $2ax^3 - 2ay^3 = 2a(x - y)(x^2 + xy + y^2).$
 $\text{H.C.F.} = a(x - y).$
9. $2 a^2 m^2 - 2 a^2 n^2 = 2 a^2(m + n)(m - n).$
 $4 am^2 - 12 amn + 8 an^2 = 2^2 a(m - 2n)(m - n).$
 $10 am - 10 an = 2 \cdot 5 a(m - n).$
 $\text{H.C.F.} = 2 a(m - n).$
10. $25 x^3 - 25 x^2 y = 5^2 x^2(x - y).$
 $10 x^3 - 20 x^2 y - 30 xy^2 = 2 \cdot 5 x(x + y)(x - 3y).$
 $5 x^4 - 5 xy^3 = 5 x(x - y)(x^2 + xy + y^2).$
 $\text{H.C.F.} = 5 x.$
11. $24 a^6 - 6 a^4 b^2 = 2 \cdot 3 a^4(2a + b)(2a - b).$
 $48 a^6 + 24 a^5 b = 2^3 \cdot 3 a^5(2a + b).$
 $48 a^5 - 48 a^4 b - 36 a^3 b^2 = 2^2 \cdot 3 a^3(2a - 3b)(2a + b).$
 $\text{H.C.F.} = 6 a^3(2a + b).$
12. $5 x^7 - 160 x^2 = 5 x^2(x - 2)(x^4 + 2x^3 + 4x^2 + 8x + 16).$
 $15 x^5 - 60 x^3 = 3 \cdot 5 x^3(x + 2)(x - 2).$
 $25 x^7 - 200 x^4 = 5^2 x^4(x - 2)(x^2 + 2x + 4).$
 $\text{H.C.F.} = 5 x^2(x - 2).$

13. $18a^4 - 2a^2b^2 = 2a^2(3a + b)(3a - b).$
 $12a^6 - 8a^5b - 4a^4b^2 = 2^2a^4(3a + b)(a - b).$
 $30a^4b^2 + 10a^3b^3 = 2 \cdot 5a^3b^2(3a + b).$
H.C.F. = $2a^2(3a + b).$
14. $a^5 - 3a^4b + 2a^3b^2 = a^3(a - 2b)(a - b).$
 $a^5 - 2a^4b - a^3b^2 + 2a^2b^3 = a^2(a - 2b)(a - b)(a + b).$
 $a^7 - 5a^5b^2 + 4a^3b^4 = a^3(a - 2b)(a + 2b)(a - b)(a + b).$
H.C.F. = $a^2(a - 2b)(a - b) = a^2(a^2 - 3ab + 2b^2).$

Page 310 (First set)

1. $\frac{2a^3b^2}{10a^2b^3} = \frac{a}{5b}.$ 2. $\frac{15xy^3}{245x^3y} = \frac{3y^2}{49x^2}.$
3. $\frac{x^2 - 9}{2x^2 + 6x} = \frac{(x + 3)(x - 3)}{2x(x + 3)} = \frac{x - 3}{2x}.$
4. $\frac{5x^2 + 5xy}{25x^4 - 25x^2y^2} = \frac{5x(x + y)}{25x^2(x + y)(x - y)} = \frac{1}{5x(x - y)}.$
5. $\frac{48x^4 - 6xy^3}{32x^4 - 32x^3y + 8x^2y^2} = \frac{6x(2x - y)(4x^2 + 2xy + y^2)}{8x^2(2x - y)(2x - y)}$
 $= \frac{3(4x^2 + 2xy + y^2)}{4x(2x - y)}.$
6. $\frac{3x^3 - 375}{2x^2 + 10x + 50} = \frac{3(x - 5)(x^2 + 5x + 25)}{2(x^2 + 5x + 25)} = \frac{3(x - 5)}{2}.$
7. $\frac{2c^2 + 2cd - 4d^2}{5c^2 - 5d^2} = \frac{2(c + 2d)(c - d)}{5(c + d)(c - d)} = \frac{2(c + 2d)}{5(c + d)}.$
8. $\frac{2c^6 - 64cd^5}{16c^4 - 40c^3d + 16c^2d^2} = \frac{2c(c - 2d)(c^4 + 2c^3d + 4c^2d^2 + 8cd^3 + 16d^4)}{8c^2(c - 2d)(2c - d)}$
 $= \frac{c^4 + 2c^3d + 4c^2d^2 + 8cd^3 + 16d^4}{4c(2c - d)}.$
9. $\frac{ad - 3ax + 2cd - 6cx}{ad - 3ax - cd + 3cx} = \frac{(a + 2c)(d - 3x)}{(a - c)(d - 3x)} = \frac{a + 2c}{a - c}.$
10. $\frac{4x^2 + 6x - 40}{4x - 15a + 6ax - 10} = \frac{2(2x - 5)(x + 4)}{(2x - 5)(2 + 3a)} = \frac{2(x + 4)}{2 + 3a}.$
11. $\frac{5x^4 - 40x}{3x^5 - 96} = \frac{5x(x - 2)(x^2 + 2x + 4)}{3(x - 2)(x^4 + 2x^3 + 4x^2 + 8x + 16)}$
 $= \frac{5x(x^2 + 2x + 4)}{3(x^4 + 2x^3 + 4x^2 + 8x + 16)}.$
12. $\frac{12a^3 + 10a^2 - 12a}{4 + 9a^2 - 12a} = \frac{2a(3a - 2)(2a + 3)}{(3a - 2)(3a - 2)} = \frac{2a(2a + 3)}{3a - 2}.$

$$\begin{aligned}
 13. \quad & \frac{32x^2 - x^7}{32 + 16x + 4x^3 + 8x^2 + 2x^4} = \frac{x^2(2-x)(16 + 8x + 4x^2 + 2x^3 + x^4)}{2(16 + 8x + 4x^2 + 2x^3 + x^4)} \\
 & = \frac{x^2(2-x)}{2}. \\
 14. \quad & \frac{(\frac{2}{3})^2 - 1}{\frac{2}{3} + 1} = \frac{(\frac{2}{3} + 1)(\frac{2}{3} - 1)}{(\frac{2}{3} + 1)} = \frac{2}{3} - 1 = -\frac{1}{3}. \\
 15. \quad & \frac{(\frac{3}{5})^3 - 1}{\frac{3}{5} - 1} = \frac{(\frac{3}{5} - 1)[(\frac{3}{5})^2 + \frac{3}{5} + 1]}{\frac{3}{5} - 1} = \left(\frac{3}{5}\right)^2 + \frac{3}{5} + 1 = \frac{49}{25}. \\
 16. \quad & \frac{(\frac{3}{2})^2 - (\frac{3}{2})(\frac{2}{5}) + (\frac{2}{5})^2}{(\frac{3}{2})^3 + (\frac{2}{5})^3} = \frac{(\frac{3}{2})^2 - (\frac{3}{2})(\frac{2}{5}) + (\frac{2}{5})^2}{[(\frac{3}{2}) + (\frac{2}{5})][(\frac{3}{2})^2 - (\frac{3}{2})(\frac{2}{5}) + (\frac{2}{5})^2]} \\
 & = \frac{1}{\frac{3}{2} + \frac{2}{5}} = \frac{1}{\frac{19}{10}} = \frac{10}{19}.
 \end{aligned}$$

Page 310 (Second set)

$$\begin{aligned}
 1. \quad & \frac{2}{3x} = \frac{4x}{6x^2}, \quad 2. \quad \frac{2x}{x-2} = \frac{2x(x+2)}{(x-2)(x+2)}, \quad 3. \quad \frac{3}{a^2-9} = \frac{3}{a^2-9}, \\
 & \frac{5}{2x^2} = \frac{15}{6x^2}, \quad \frac{3}{x+2} = \frac{3(x-2)}{(x-2)(x+2)}, \quad \frac{2a}{a-3} = \frac{2a(a+3)}{a^2-9}. \\
 4. \quad & \frac{2x-1}{x^2-6x+9} = \frac{2(2x-1)}{2(x-3)^2}, \quad 5. \quad \frac{x+1}{x^2-5x+6} = \frac{(x+1)(x+2)}{(x-2)(x-3)(x+2)}, \\
 & \frac{-x+5}{-2x+6} = \frac{(x-5)(x-3)}{2(x-3)^2}, \quad \frac{-x}{-x^2+4} = \frac{x(x-3)}{(x-2)(x+2)(x-3)}. \\
 6. \quad & \frac{2a}{a+2} = \frac{2a(3a+1)}{(a+2)(3a+1)}, \\
 & \frac{a+3}{3a+1} = \frac{(a+3)(a+2)}{(3a+1)(a+2)}, \\
 & \frac{2a^2+5}{3a^2+7a+2} = \frac{2a^2+5}{(3a+1)(a+2)}. \\
 7. \quad & \frac{a}{2x} = \frac{3a(x+2)}{6x(x+2)}, \\
 & \frac{2a}{3x} = \frac{4a(x+2)}{6x(x+2)}, \\
 & \frac{5a}{x^2+2x} = \frac{30a}{6x(x+2)}. \\
 8. \quad & \frac{2x+1}{x^2-2x+4} = \frac{(2x+1)(x+2)}{(x+2)(x^2-2x+4)}, \\
 & \frac{5}{x^3+8} = \frac{5}{(x+2)(x^2-2x+4)}, \\
 & \frac{1}{x+2} = \frac{x^2-2x+4}{(x+2)(x^2-2x+4)}.
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \frac{2x}{x^3-8} &= \frac{2x^2(x+2)}{x(x-2)(x+2)(x^2+2x+4)}, \\
 \frac{5x}{2-x} &= \frac{-5x^2(x+2)(x^2+2x+4)}{x(x-2)(x+2)(x^2+2x+4)}, \\
 \frac{4}{4-x^2} &= \frac{-4x(x^2+2x+4)}{x(x-2)(x+2)(x^2+2x+4)}, \\
 \frac{3x+2}{x^3+2x^2+4x} &= \frac{(3x+2)(x-2)(x+2)}{x(x-2)(x+2)(x^2+2x+4)}. \\
 10. \quad \frac{2x-9}{x^5-32} &= \frac{x^2(2x-9)}{x^2(x^5-32)}, \\
 \frac{-2}{-x^3+2x^2} &= \frac{2}{x^2(x-2)} = \frac{2(x^4+2x^3+4x^2+8x+16)}{x^2(x^5-32)}, \\
 \frac{x-2}{x^5+2x^4+4x^3+8x^2+16x} &= \frac{x(x-2)^2}{x^2(x^5-32)}.
 \end{aligned}$$

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$$\begin{aligned}
 1. \quad \frac{2x}{5} + \frac{3x}{10} - \frac{2x}{15} &= \frac{12x+9x-4x}{30} = \frac{17x}{30}. \\
 2. \quad \frac{2c}{3a} - \frac{5c^2}{6a^2} + \frac{c}{9a} &= \frac{12ac-15c^2+2ac}{18a^2} = \frac{14ac-15c^2}{18a^2}. \\
 3. \quad \frac{2rs^3+r}{r^2s} - \frac{r-6s}{3rs^2} - \frac{s^2}{2r} &= \frac{(12rs^4+6rs)-(2r^2-12rs)-3rs^4}{6r^2s^2} \\
 &= \frac{9rs^4+18rs-2r^2}{6r^2s^2} \\
 &= \frac{9s^4+18s-2r}{6rs^2}. \\
 4. \quad \frac{5a}{a-3} - \frac{2}{3} &= \frac{15a-(2a-6)}{3(a-3)} = \frac{13a+6}{3(a-3)}. \\
 5. \quad \frac{c}{c-4} - \frac{2c}{c+3} &= \frac{(c^2+3c)-(2c^2-8c)}{(c-4)(c+3)} = \frac{11c-c^2}{(c-4)(c+3)}. \\
 6. \quad \frac{a}{a-5} + \frac{2a}{a+4} - \frac{1}{2} &= \frac{(2a^2+8a)+(4a^2-20a)-(a^2-a-20)}{2(a-5)(a+4)} \\
 &= \frac{5a^2-11a+20}{2(a-5)(a+4)}.
 \end{aligned}$$

$$7. \frac{2x+1}{x^2-9} - \frac{4}{x-3} = \frac{(2x+1) - 4(x+3)}{(x+3)(x-3)} = \frac{-2x-11}{x^2-9} = \frac{2x+11}{9-x^2}.$$

$$8. \frac{a-3}{a^2-4} - \frac{a-3}{2-a} + \frac{5}{a+2} = \frac{a-3}{a^2-4} + \frac{a-3}{a-2} + \frac{5}{a+2}$$

$$= \frac{a-3 + a^2-a-6 + 5a-10}{a^2-4}$$

$$= \frac{a^2+5a-19}{a^2-4}.$$

$$9. \frac{2c-3}{c(c-2)} - \frac{c^2-7}{8-c^3} = \frac{2c-3}{c(c-2)} + \frac{c^2-7}{c^3-8}$$

$$= \frac{2c^3+c^2+2c-12+c^3-7c}{c(c^3-8)}$$

$$= \frac{3c^3+c^2-5c-12}{c(c^3-8)}.$$

$$10. \frac{m}{m-1} - \frac{2}{m} - \frac{5m-2}{m^2-1} = \frac{m^3+m^2-2m^2+2-5m^2+2m}{m(m^2-1)}$$

$$= \frac{m^3-6m^2+2m+2}{m(m^2-1)}.$$

$$11. 2 + \frac{5x}{x-7} = \frac{2x-14+5x}{x-7} = \frac{7x-14}{x-7}.$$

$$12. x+3 - \frac{x^2}{x-3} = \frac{x^2-9-x^2}{x-3} = \frac{9}{3-x}.$$

$$13. x^2+2x+4 - \frac{x^3+8}{x-2} = \frac{x^3-8-x^3-8}{x-2} = \frac{16}{2-x}.$$

$$14. a - \frac{a-6}{2a-3} - 2 = a-2 - \frac{a-6}{2a-3}$$

$$= \frac{2a^2-7a+6-a+6}{2a-3}$$

$$= \frac{2a^2-8a+12}{2a-3}.$$

$$15. c^2 - \frac{c^4+d^4}{c^2+cd+d^2} - cd + d^2 = -\frac{c^4+d^4}{c^2+cd+d^2} + c^2 - cd + d^2$$

$$= \frac{-c^4-d^4+c^4+c^2d^2+d^4}{c^2+cd+d^2}$$

$$= \frac{c^2d^2}{c^2+cd+d^2}.$$

$$\begin{aligned}
 16. \quad & \frac{x+2}{2x-1} - \frac{x}{1-x} - \frac{3x^2+2x-2}{2x^2-3x+1} = \frac{x+2}{2x-1} + \frac{x}{x-1} - \frac{3x^2+2x-2}{(2x-1)(x-1)} \\
 & = \frac{x^2+x-2+2x^2-x-3x^2-2x+2}{(2x-1)(x-1)} \\
 & = -\frac{2x}{2x^2-3x+1}.
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{a-3}{a^2-2a-3} - \frac{2-a}{a^2-3a+2} - \frac{1}{1-a^2} \\
 & = \frac{a-3}{(a-3)(a+1)} + \frac{a-2}{(a-2)(a-1)} + \frac{1}{a^2-1} \\
 & = \frac{1}{a+1} + \frac{1}{a-1} + \frac{1}{a^2-1} \\
 & = \frac{a-1+a+1+1}{a^2-1} \\
 & = \frac{2a+1}{a^2-1}.
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{b+a}{b-a} - 2\left(\frac{b}{a} - \frac{b}{a-b}\right) = -\frac{a+b}{a-b} + \frac{2b^2}{a(a-b)} \\
 & = \frac{-a^2-ab+2b^2}{a(a-b)} \\
 & = -\frac{a^2+ab-2b^2}{a(a-b)} \\
 & = -\frac{(a-b)(a+2b)}{a(a-b)} = -\frac{a+2b}{a}.
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & b - 2a\left(1 - \frac{2c-b}{c-b}\right) = b - 2a\left(\frac{-c}{c-b}\right) \\
 & = b + \frac{2ac}{c-b} = \frac{bc-b^2+2ac}{c-b}.
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & \frac{x^2-(m-n)^2}{(x+n)^2-m^2} + \frac{m^2-(x-n)^2}{(x+m)^2-n^2} + \frac{n^2-(x-m)^2}{(m+n)^2-x^2} \\
 & = \frac{(x+m-n)(x-m+n)}{(x+n+m)(x+n-m)} + \frac{(m+x-n)(m-x+n)}{(x+m+n)(x+m-n)} \\
 & \quad + \frac{(n+x-m)(n-x+m)}{(m+n+x)(m+n-x)} \\
 & = \frac{x+m-n+m-x+n+n+x-m}{m+n+x} \\
 & = \frac{x+m+n}{x+m+n} \\
 & = 1.
 \end{aligned}$$

21. See p. 159.

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1. $\frac{6a}{25x^2y} \cdot \frac{10xy^2}{4a^2x} = \frac{3y}{5ax^2}$.
2. $\frac{5c}{1} \cdot \frac{3d}{10cd^2} \cdot \frac{4c}{9d} = \frac{2c}{3d^2}$.
3. $4\frac{1}{2} \cdot \frac{2a^2}{3x^3} \cdot \frac{x^2}{15a} = \frac{9}{2} \cdot \frac{2a^2}{3x^3} \cdot \frac{x^2}{15a} = \frac{a}{5x}$.
4. $\frac{10a}{1} \cdot \frac{7x}{2a^4} \cdot \frac{a}{21x^3} = \frac{5}{3a^2x^2}$.
5. $\frac{a^2-4}{2a^2} \cdot \frac{4a}{2a-4} = \frac{(a+2)(a-2)}{2a^2} \cdot \frac{4a}{2(a-2)} = \frac{a+2}{a}$.
6. $\frac{a^2-1}{2a} \cdot \frac{6a^3}{a^2-2a+1} \cdot \frac{5a-5}{3(1-a)}$
 $= \frac{(a+1)(a-1)}{2a} \cdot \frac{6a^3}{(a-1)(a-1)} \cdot \frac{5(a-1)}{3(1-a)}$
 $= \frac{5a^2(a+1)}{1-a}$.
7. $\left(2a + \frac{1}{a}\right) \left(\frac{a^2}{4a^4-1}\right) \left(a - \frac{1}{2a}\right) = \frac{2a^2+1}{a} \cdot \frac{a^2}{(2a^2+1)(2a^2-1)} \cdot \frac{2a^2-1}{2a} = \frac{1}{2}$.
8. $\frac{35}{42} \div \frac{25}{30} = \frac{5 \cdot 7}{2 \cdot 3 \cdot 7} \cdot \frac{3 \cdot 2 \cdot 5}{5^2} = 1$.
9. $\frac{14x}{15ax^2} \div \frac{84a}{60a^2x^3} = \frac{2 \cdot 7 \cdot x}{3 \cdot 5 \cdot ax^2} \cdot \frac{2^2 \cdot 3 \cdot 5a^2x^3}{2^2 \cdot 3 \cdot 7a} = \frac{2x^2}{3}$.
10. $\frac{120c^2d}{42c^3x} \div \frac{100cd^3}{147c^2x^2} = \frac{2^3 \cdot 3 \cdot 5c^2d}{2 \cdot 3 \cdot 7c^3x} \cdot \frac{3 \cdot 7^2c^2x^2}{2^2 \cdot 5^2cd^3} = \frac{3 \cdot 7x}{5d^2} = \frac{21x}{5d^2}$.
11. See pp. 167 and 170.
12. See p. 167.
13. See p. 170.
14. $\frac{24x^2}{99xy^2} \cdot \frac{8y^2z}{22x^2y} \div \frac{48x^3z}{121x^2y^3} = \frac{24x^2}{99xy^2} \cdot \frac{8y^2z}{22x^2y} \cdot \frac{121x^2y^3}{48x^3z} = \frac{2y^2}{9x^2}$.
15. $\frac{15rs^2}{14s^2t} \div \frac{300rt^3}{98r^2s} \cdot \frac{160st^4}{28r^2s^3} = \frac{15rs^2}{14s^2t} \cdot \frac{98r^2s}{300rt^3} \cdot \frac{160st^4}{28r^2s^3} = \frac{2}{s}$.
16. $\frac{22m^2n^2}{35x^3} \div \frac{52m^5n}{125x^2} \div \frac{275n^2x}{39m^3x} = \frac{22m^2n^2}{35x^3} \cdot \frac{125x^2}{52m^5n} \cdot \frac{39m^3x}{275n^2x} = \frac{3}{14nx}$.
17. $\frac{5x^2}{x-y} \cdot \frac{(x-y)^2}{20x^2} \div \frac{1}{x(x-y)} = \frac{5x^2}{x-y} \cdot \frac{(x-y)^2}{20x^2} \cdot \frac{x(x-y)}{1} = \frac{x(x-y)^2}{4}$.
18. $\frac{x+3}{x^2-4} \cdot \frac{x^2-4x+4}{2x+6} \div \frac{x^2-2x}{x^2+2x} = \frac{x+3}{(x+2)(x-2)} \cdot \frac{(x-2)^2}{2(x+3)} \cdot \frac{x(x+2)}{x(x-2)} = \frac{1}{2}$.
19. $\frac{x^2+2x+4}{x^2-2x+4} \div \frac{x^3-8}{x^2+2x} \div \frac{x^2-2x}{x^3+8}$
 $= \frac{(x^2+2x+4)}{(x^2-2x+4)} \cdot \frac{x(x+2)}{(x-2)(x^2+2x+4)} \cdot \frac{(x+2)(x^2-2x+4)}{x(x-2)}$
 $= \frac{(x+2)^2}{(x-2)^2}$.

20. $\frac{2x^2+9x+9}{2x^2-9x+9} \div \frac{x^2-9}{4x^3-9x} = \frac{x^2-6x+9}{2x^2-3x} = \frac{(2x+3)(x+3)}{(2x-3)(x-3)} \cdot \frac{x(2x+3)(2x-3)}{x(2x-3)} = \frac{(2x+3)^2}{2x-3}.$
21. $\frac{x^3-6x^2+11x-6}{x^4-1} \div \frac{x^2-5x+6}{x^2+2x+1} = \frac{x^2+1}{x^2+2x+1} \cdot \frac{(x-1)(x-2)(x-3)}{(x^2+1)^2} = \frac{x^2+1}{(x-3)(x-2)} \cdot \frac{x^2+1}{x+1} = 1.$
22. $\left(\frac{2x}{3}\right)^2 \cdot \left(\frac{5}{4x}\right)^3 \left(\frac{6x}{25}\right)^2 = \frac{2^2x^2}{3^2} \cdot \frac{5^3}{2^6x^3} \cdot \frac{2^2 \cdot 3^2x^2}{5^4} = \frac{x}{20}.$
23. $\left(\frac{3a}{2b}\right)^3 \div \left(\frac{3a^2x}{5b^2}\right) \div \frac{(10x)^3}{64} = \frac{3^3a^3}{2^3b^3} \cdot \frac{5b^2}{3a^2x} \cdot \frac{2^6}{2^3 \cdot 5^3x^3} = \frac{3^2a}{5^2bx^4} = \frac{9a}{25bx^4}.$
24. $\left(\frac{2a}{5x^2}\right)^3 - 3\left(\frac{2a}{5x^2}\right)^2 \left(\frac{5x^3}{4a^2}\right) + 3\left(\frac{2a}{5x^2}\right) \left(\frac{5x^3}{4a^2}\right)^2 = \frac{2^3a^3}{5^3x^6} - \frac{3 \cdot 2^2a^2 \cdot 5 \cdot x^3}{5^2x^4 \cdot 2^2a^2} + \frac{3 \cdot 2a \cdot 5^2x^6}{5x^2 \cdot 2^4a^4}$
 $= \frac{8a^3}{125x^6} - \frac{3}{5x} + \frac{15x^4}{8a^3}$
 $= \frac{64a^6 - 600a^3x^5 + 1875x^{10}}{1000a^3x^6}.$
25. $\left(\frac{a^2}{2b}\right)^6 + 6\left(\frac{a^2}{2b}\right)^5 \left(\frac{2b}{a}\right) + 15\left(\frac{a^2}{2b}\right)^4 \left(\frac{2b}{a}\right)^2 + 30\left(\frac{a^2}{2b}\right)^3 \left(\frac{2b}{a}\right)^3 = \frac{a^{12}}{64b^6} + \frac{3a^9}{8b^4} + \frac{15a^6}{4b^2} + 30a^3$
 $= \frac{a^{12} + 24a^9b^2 + 240a^6b^4 + 1920a^3b^6}{64b^6}.$
26. $\frac{1-a^3}{1+a^2} \div \left(1 - \frac{2a^6}{a^6+1}\right) \div \left(\frac{1-a^2+a^4}{a^3+1}\right)$
 $= \frac{(1-a)(1+a+a^2)}{1+a^2} \cdot \frac{(1+a^2)(1-a^2+a^4)}{(1+a)(1-a+a^2)(1+a+a^2)} \cdot \frac{(a+1)(1-a+a^2)}{1-a^2+a^4}$
 $= 1.$

$$\begin{aligned}
 27. \quad & \frac{a}{a^2 - 4} \div \left(\frac{a}{3a^2 + 5a + 2} \right) \left(3a - \frac{2}{a} - 5 \right) \\
 &= \frac{a}{(a+2)(a-2)} \cdot \frac{(3a+2)(a+1)}{a} \cdot \frac{(3a+1)(a-2)}{a} \\
 &= \frac{(3a+2)(a+1)(3a+1)}{a(a+2)}.
 \end{aligned}$$

$$28. \quad \frac{x+1 + \frac{4}{x+3}}{x-5 + \frac{12}{x+3}} = \frac{\frac{x^2+4x+7}{x+3}}{\frac{x^2-2x-3}{x+3}} = \frac{x^2+4x+7}{x^2-2x-3}.$$

$$29. \quad \frac{2 + \frac{x+6}{x-2}}{3 + \frac{5}{x-2}} = \frac{\frac{3x+2}{x-2}}{\frac{3x-1}{x-2}} = \frac{3x+2}{3x-1}.$$

$$30. \quad \frac{x-1 - \frac{3}{x}}{x - \frac{x}{x-3}} = \frac{\frac{x^2-x-3}{x}}{\frac{x^2-x-3}{x-3}} = \frac{x-3}{x}.$$

$$31. \quad \frac{\frac{a}{b} + 1 + \frac{b}{a}}{\frac{b^2}{a^2} + 1 + \frac{a^2}{b^2}} = \frac{\frac{a^2+ab+b^2}{ab}}{\frac{a^4+a^2b^2+b^4}{a^2b^2}} = \frac{ab(a^2+ab+b^2)}{(a^2+ab+b^2)(a^2-ab+b^2)} = \frac{ab}{a^2-ab+b^2}.$$

$$32. \quad \frac{\frac{a}{a+1} + \frac{1-a}{a}}{\frac{a}{a+1} - \frac{1-a}{a}} = \frac{\frac{a^2+1-a^2}{a(a+1)}}{\frac{a^2-1+a^2}{a(a+1)}} = \frac{1}{2a^2-1}.$$

$$33. \quad \frac{\frac{a-b}{a+b} + \frac{a+b}{a-b}}{1 + \frac{2a^2b^2+2b^4}{a^4-b^4}} = \frac{\frac{2a^2+2b^2}{a^2-b^2}}{\frac{a^4+2a^2b^2+b^4}{a^4-b^4}} = 2.$$

$$34. \quad \left(\frac{x+y}{x-y} + 2 \right) \div \left(\frac{x+y}{x-y} - 2 \right) = \frac{3x-y}{x-y} \cdot \frac{x-y}{3y-x} = \frac{3x-y}{3y-x}.$$

$$35. \quad \frac{\frac{2}{b} - \frac{1}{a+b} + \frac{1}{a-b}}{\frac{a+b}{a-b} - \frac{a-b}{a+b}} = \frac{\frac{2a^2-2b^2-ab+b^2+ab+b^2}{b(a^2-b^2)}}{\frac{a^2+2ab+b^2-a^2+2ab-b^2}{a^2-b^2}} = \frac{2a^2}{b} \cdot \frac{1}{4ab} = \frac{a}{2b^2}.$$

$$36. \frac{a^2 + b^2}{a^2 - b^2} - \frac{\frac{b}{a+b}}{1 + \frac{2b-a}{a-b}} = \frac{a^2 + b^2}{a^2 - b^2} - \frac{a-b}{a+b} = \frac{a^2 + b^2 - (a-b)^2}{a^2 - b^2} = \frac{2ab}{a^2 - b^2}.$$

$$\begin{aligned} 37. \frac{2}{\frac{1}{b^2} \left(\frac{1}{c} - \frac{b}{2a} \right)} - b^2 c \left(1 - \frac{bc}{a - \frac{bc}{2}} \right) &= \frac{2}{\frac{1}{b^2 c} - \frac{1}{2ab}} - b^2 c \left(1 - \frac{2bc}{2a - bc} \right) \\ &= \frac{2}{\frac{2ab^2c}{2ab^2c}} - b^2 c \left(\frac{2a - 3bc}{2a - bc} \right) \\ &= \frac{4ab^2c}{2a - bc} - \frac{2ab^2c - 3b^3c^2}{2a - bc} \\ &= \frac{2ab^2c + 3b^3c^2}{2a - bc}. \end{aligned}$$

$$\begin{aligned} 38. \frac{\frac{2b}{c} - 2}{\frac{2a}{c} - 2} - \frac{b}{ac} \left(\frac{\frac{4ab}{c} - \frac{b}{a}}{\frac{1}{c} - \frac{1}{b}} \right) &= \frac{\frac{2b}{c} - 2}{\frac{2a}{c} - 2} - \frac{b}{ac} \left(\frac{\frac{4ab - bc}{ac}}{\frac{b - c}{bc}} \right) \\ &= \frac{\frac{4ab}{c} - 2}{\frac{2a}{c} - 2} - \frac{b}{ac} \left(\frac{4a^2bc}{b(4a - c)} - \frac{abc}{b - c} \right) \\ &= \frac{\frac{4ab}{c} - 2}{\frac{2a}{c} - 2} - \frac{4ab}{4a - c} + \frac{b^2}{b - c} \\ &= \frac{8ab}{c - 4a} + \frac{b^2}{b - c} \\ &= \frac{8ab^2 - 8abc + b^2c - 4ab^2}{(c - 4a)(b - c)} \\ &= \frac{4ab^2 - 8abc + b^2c}{(c - 4a)(b - c)}. \end{aligned}$$

39. If $a = x - \frac{1}{x}$ and $b = x + \frac{1}{x}$, then

$$\frac{a^3 - b^3}{a^2 + b^2} = \frac{\left(x - \frac{1}{x}\right)^3 - \left(x + \frac{1}{x}\right)^3}{\left(x - \frac{1}{x}\right)^2 + \left(x + \frac{1}{x}\right)^2} = \frac{-6x - \frac{2}{x^3}}{2x^2 + \frac{2}{x^2}} = -\frac{3x + \frac{1}{x^3}}{x^2 + \frac{1}{x^2}} = -\frac{3x^4 + 1}{x(x^4 + 1)}.$$

$$40. 1 - \frac{1}{1 - \frac{1}{1 + \frac{1}{2}}} = 1 - \frac{1}{1 - \frac{2}{3}} = 1 - 3 = -2.$$

$$41. 1 - \frac{1}{1 + \frac{2}{1 - \frac{2}{3}}} = 1 - \frac{1}{1 + 6} = 1 - \frac{1}{7} = \frac{6}{7}.$$

$$42. 1 + \frac{1}{1 - \frac{1}{1 + \frac{1}{a}}} = 1 + \frac{1}{1 - \frac{1}{\frac{a+1}{a}}} = 1 + \frac{1}{1 - \frac{a}{a+1}} = 1 + \frac{1}{\frac{1}{a+1}} \\ = 1 + a + 1 \\ = a + 2.$$

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1. $\frac{5x}{2} - 2 = 2x.$ (1) 7. $\frac{2x+1}{4} - \frac{1-2x}{3} = 9\frac{1}{4}.$ (1)
- (1) · 2, $5x - 4 = 4x.$ (1) · 12, $6x + 3 - 4 + 8x = 111.$
- $x = 4.$ $14x = 112.$
2. $\frac{2x}{5} + 1 = x - \frac{7}{5}.$ (1) $x = 8.$
- (1) · 5, $2x + 5 = 5x - 7.$ 8. $\frac{3}{4x} + \frac{7}{16} = \frac{4}{3x}.$ (1)
- $-3x = -12.$ (1) · 48x, $36 + 21x = 64.$
- $x = 4.$ $21x = 28.$
3. $\frac{11x^2}{9} - \frac{2x}{3} = x^2.$ (1) $x = \frac{4}{3}.$
- (1) · 9, $11x^2 - 6x = 9x^2.$ 9. $3(x+2) - 4(2x-3) + 2 = 0.$
- $2x^2 - 6x = 0.$ $3x + 6 - 8x + 12 + 2 = 0.$
- $x = 0, 3.$ $-5x = -20.$
4. $\frac{3x}{2} - \frac{16}{3} = x - \frac{25}{6}.$ (1) $x = 4.$
- (1) · 6, $9x - 32 = 6x - 25.$ 10. $\frac{x+5}{x-2} = \frac{10}{3}.$ (1)
- $3x = 7.$ (1) · 3(x-2), $3x + 15 = 10x - 20.$
- $x = \frac{7}{3}.$ $-7x = -35.$
5. $\frac{5x}{6} - \frac{21}{5} = \frac{5x-2}{10}.$ (1) $x = 5.$
- (1) · 30, $25x - 126 = 15x - 6.$ 11. $\frac{8}{7x+3} = \frac{3}{3x+1}.$ (1)
- $10x = 120.$ (1) · (7x+3)(3x+1), $24x + 8 = 21x + 9.$
- $x = 12.$ $3x = 1.$
6. $\frac{2(x-4)}{3} - 1 = \frac{x+5}{3}.$ (1) $x = \frac{1}{3}.$
- (1) · 3, $2x - 8 - 3 = x + 5.$
- $x = 16.$

$$12. \quad \frac{2x-7}{3x-4} + \frac{3}{2} = 0. \quad (1)$$

$$(1) \cdot 2(3x-4), \quad 4x-14+9x-12=0.$$

$$13x=26.$$

$$x=2.$$

$$13. \quad (x+5)(x+1)-(x-3)(x-2)=10.$$

$$x^2+6x+5-x^2+5x-6=10.$$

$$11x=11.$$

$$x=1.$$

$$14. \quad \frac{3}{2x-5} + \frac{1}{2x-1} = 0. \quad (1)$$

$$(1) \cdot (2x-5)(2x-1), \quad 6x-3+2x-5=0.$$

$$x=1.$$

$$15. \quad \frac{2x-5}{2x+7} = \frac{3x-14}{3x-2}. \quad (1)$$

$$(1) \cdot (2x+7)(3x-2), \quad 6x^2-19x+10=6x^2-7x-98.$$

$$-12x=-108.$$

$$x=9.$$

$$16. \quad 5x+3 = \frac{(5x+7)(2x-3)}{2x}. \quad (1)$$

$$(1) \cdot 2x \quad 10x^2+6x=10x^2-x-21.$$

$$x=-3.$$

$$17. \quad \frac{1}{x} - \frac{1-x}{6-x^2} = 0. \quad (1)$$

$$(1) \cdot x(6-x^2), \quad 6-x^2-x+x^2=0.$$

$$x=6.$$

$$18. \quad \frac{x+3}{2x+1} = \frac{3x+4}{6x-2}. \quad (1)$$

$$(1) \cdot (2x+1)(6x-2), \quad 6x^2+16x-6=6x^2+11x+4.$$

$$x=2.$$

$$19. \quad \frac{x-3}{x+7} = 1 - \frac{5}{x+1}. \quad (1)$$

$$(1) \cdot (x+7)(x+1), \quad x^2-2x-3=x^2+8x+7-5x-35.$$

$$x=5.$$

$$20. \quad \frac{3(1-8x)}{5} - \frac{1+8x}{8x-1} = \frac{1-34x}{5}. \quad (1)$$

$$(1) \cdot 5(8x-1), \\ -192x^2 + 48x - 3 - 10x - 80x^2 = -272x^2 + 42x - 1. \\ x = -\frac{1}{2}.$$

$$21. \quad \frac{3}{2x+1} + \frac{2x+1}{1-2x} = 1 - \frac{8x^2}{4x^2-1}. \quad (1)$$

$$(1) \cdot (4x^2-1), \\ 6x-3-4x^2-4x-1 = 4x^2-1-8x^2. \\ x = \frac{3}{2}.$$

$$22. \quad \frac{1+2x}{2x-1} - \frac{3x+1}{x+1} = \frac{8+3x-4x^2}{2x^2+x-1}. \quad (1)$$

$$(1) \cdot (2x-1)(x+1), \\ 2x^2+3x+1-6x^2+x+1 = 8+3x-4x^2. \\ x = 6.$$

$$23. \quad \frac{x}{a} - a = \frac{x}{b} - b. \quad (1)$$

$$(1) \cdot ab, \\ bx - a^2b = ax - ab^2. \\ bx - ax = a^2b - ab^2. \\ x = -ab.$$

$$24. \quad \frac{x+a}{a} - \frac{x+b}{b} = b-a. \quad (1)$$

$$(1) \cdot ab, \\ bx + ab - ax - ab = ab^2 - a^2b. \\ (b-a)x = ab(b-a). \\ x = ab.$$

$$25. \quad \frac{x^2-x}{a-1} + \frac{x^2-x}{a+1} = 2a(x-1). \quad (1)$$

$$(1) \cdot (a-1)(a+1), \\ 2ax^2 - 2a^3x = 2a - 2a^3. \\ x^2 - 1 - a^2(x-1) = 0. \\ (x-1)(x+1-a^2) = 0. \\ x = a^2 - 1, 1.$$

$$26. \quad \frac{x}{a+2} - \frac{x+16}{a-2} = 4a. \quad (1)$$

$$(1) \cdot (a^2-4), \\ ax - 2x - ax - 2x - 16a - 32 = 4a^3 - 16a. \\ x = -a^3 - 8.$$

$$27. \quad \frac{2x+1}{x+3} + \frac{3x-7}{2-x} = \frac{9-3x-x^2}{x^2+x-6}. \quad (1)$$

$$(1) \cdot (x^2+x-6), \\ 2x^2-3x-2-3x^2-2x+21 = 9-3x-x^2. \\ x = 5.$$

$$28. \quad \frac{x-a}{2x-a} - \frac{3x-c}{6x-c} = 0. \quad (1)$$

$$\begin{aligned} (1) \cdot (2x-a)(6x-c), \\ 6x^2 - cx - 6ax + ac = 6x^2 - 3ax - 2cx + ac. \\ cx - 3ax = 0. \\ (c-3a)x = 0. \\ x = 0. \end{aligned}$$

$$29. \quad 11.3 - \frac{5-2x}{.5} = 2.3 - (5-7x) + \frac{x+2}{.2}. \quad (1)$$

$$\begin{aligned} (1) \cdot 10, \\ 113 - 100 + 40x = 23 - 50 + 70x + 50x + 100. \\ -80x = 60. \\ x = -\frac{3}{4}. \end{aligned}$$

$$30. \quad \frac{3x-1}{.25} + \frac{x-4}{.5} = 3(3x-14). \quad (1)$$

$$\begin{aligned} (1) \cdot 1, \\ 12x - 4 + 2x - 8 = 9x - 42. \\ 5x = -30. \\ x = -6. \end{aligned}$$

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1. Let x = the first part.
Then $\frac{11}{6}x$ = the second part,
and $\frac{25}{9}x$ = the third part.

$$\begin{aligned} x + \frac{11x}{6} + \frac{25x}{9} &= 1010. \\ 18x + 33x + 50x &= 18,180. \\ x &= 180, \end{aligned}$$

and $\frac{11}{6}x = 330$,
and $\frac{25}{9}x = 500$.

2. Let x = the divisor.
Then $\frac{352}{x} = 15 + \frac{7}{x}$.
 $352 = 15x + 7$.
 $x = 23$.

3. Let x = the number to be subtracted.

$$\begin{aligned} \text{Then } \frac{32-x}{39-x} &= \frac{2}{3}. \\ 96 - 3x &= 78 - 2x. \\ x &= 18. \end{aligned}$$

4. Let x = one part.
Then $96 - x$ = the other part.

$$\begin{aligned} 56 - \frac{2}{3}x &= 96 - x - 16. \\ x &= 72, \\ \text{and } 96 - x &= 24. \end{aligned}$$

5. Let x = the number of years.
 $x + 12 = \frac{6}{5}(x + 8)$.
 $5x + 60 = 6x + 48$.
 $x = 12$.

6. Let $x =$ the number.
 Then $x + 1$ and $x + 2 =$ the next two consecutive numbers.

$$x^2 - 4 = \frac{2}{3}(x + 1)(x + 2).$$

$$3x^2 - 12 = 2x^2 + 6x + 4.$$

$$x^2 - 6x - 16 = 0.$$

$$x = 8, -2.$$

7. Let $x =$ the number of yards in the width
 of the rectangle.
 Then $2\frac{1}{2}x =$ the number of yards in the length
 of the rectangle.

$$(x + 1\frac{1}{2})(2\frac{1}{2}x - 10) = x \cdot 2\frac{1}{2}x - 12\frac{5}{2}.$$

$$\frac{5}{2}x^2 - \frac{25x}{4} - 15 = \frac{5}{2}x^2 - 140.$$

$$x = 20,$$

and

$$2\frac{1}{2}x = 50.$$

8. Let $x =$ the number of days required by A
 and B together.

$$\frac{1}{x} = \frac{1}{10} + \frac{1}{12}.$$

$$60 = 6x + 5x.$$

$$x = 5\frac{5}{11}.$$

9. Let $x =$ the number of days A requires
 to finish.

$$\frac{5}{10} + \frac{5}{15} + \frac{x}{10} = 1.$$

$$x = 1\frac{2}{3}.$$

10. Let $x =$ the number of hours to fill the tank.

$$\frac{1}{4} - \frac{1}{6} = \frac{1}{x}.$$

$$3x - 2x = 12.$$

$$x = 12.$$

11. Let $x =$ the number of hours to empty
 the tank.

$$\frac{1}{6} + \frac{1}{8} - \frac{1}{4} = \frac{1}{x}.$$

$$4x + 3x - 6x = 24.$$

$$x = 24.$$

12. Let x = the number of miles in the diameter of the moon.

Then $3\frac{2}{3}x$ = the number of miles in the diameter of the earth.

$$3\frac{2}{3}x - x = 5760.$$

$$x = 2160,$$

and $3\frac{2}{3}x = 7920.$

13. Let x = the number of miles in the diameter of the earth.

Then $10\frac{1}{11}x$ = the number of miles in the diameter of Jupiter.

$$x + 10\frac{1}{11}x = 94,320.$$

$$x = 7920,$$

and $10\frac{1}{11}x = 86,400.$

14. Let x = the number of hours of rowing upstream.

Then $10 - x$ = the number of hours of rowing downstream.

$$(4 + 1\frac{1}{2})(10 - x) = x(4 - 1\frac{1}{2}).$$

$$55 - 5\frac{1}{2}x = 2\frac{1}{2}x.$$

$$x = 6\frac{7}{8}.$$

15. Let x = the number of miles per hour the freight train travels.

Then $8x$ = the number of miles it travels.

$$8x = 40 \cdot 5\frac{1}{4} = 210.$$

$$x = 26\frac{1}{4}.$$

16. Let x = the number of dollars invested at 5%.

Then $8000 - x$ = the number of dollars invested at 4%.

$$\frac{5}{100}x + \frac{4}{100}(8000 - x) = 345.$$

$$5x + 32,000 - 4x = 34,500.$$

$$x = 2500,$$

and $8000 - x = 5500.$

17. Let x = the number of dollars taxed 4%.

$$\frac{2}{100}(10,000 - 2000) + \frac{4}{100}x = 180.$$

$$16,000 + 4x = 18,000.$$

$$x = 500.$$

$$\text{Income} = 10,000 + 500 = 10,500.$$

18. Let x = the number of gallons of water to be added.

$$\frac{\frac{95}{100} \cdot 1}{1 + x} = \frac{1}{10}.$$

$$\frac{95}{100(1 + x)} = \frac{1}{10}.$$

$$95 = 10 + 10x.$$

$$x = 8.5.$$

19. Let x = the number of pounds of 25-cent coffee.

Then $10 - x$ = the number of pounds of 35-cent coffee.

$$25x + 35(10 - x) = 32 \cdot 10.$$

$$25x + 350 - 35x = 320.$$

$$x = 3,$$

and $10 - x = 7.$

Weight of 25-cent coffee used is to weight of 35-cent coffee as 3 : 7.

20. Let x = the number of gallons of water.

$$\frac{20}{100} \cdot 25 = \frac{18}{100}(25 + x).$$

$$500 = 450 + 18x.$$

$$x = 2\frac{7}{9}.$$

21. Let x = the number of pounds of tin to be added.

$$\frac{\frac{16}{100} \cdot 410 + x}{410 + x} = \frac{18}{100}.$$

$$6560 + 100x = 7380 + 18x.$$

$$x = 10.$$

22. Let x = the number of gallons of 90-per-cent-pure alcohol.

Then
$$\frac{\frac{90}{100}x + \frac{96}{100} \cdot 12}{x + 12} = \frac{93}{100}.$$

$$90x + 1152 = 93x + 1116.$$

$$x = 12.$$

23. Let x = the number of tons of pure iron.

$$\frac{\frac{93}{100} \cdot 10 + x}{10 + x} = \frac{98}{100}.$$

$$930 + 100x = 980 + 98x.$$

$$x = 25.$$

24. Let x = the number of pounds on the shorter arm.

$$6 \cdot 70 = 5 \cdot x.$$

$$x = 84.$$

25. Let x = the number of pounds on the longer arm.

$$7 \cdot 36 = 12 \cdot x.$$

$$x = 21$$

26. Let x = the number of tons stress on one support.

Then $2 - x$ = the number of tons stress on the other support.

$$4x = 8(2 - x).$$

$$x = 1\frac{1}{3},$$

and

$$2 - x = \frac{2}{3}.$$

28. Let x = the number of spaces the minute hand travels.

$$x - \frac{x}{12} = 45.$$

$$12x - x = 540.$$

$$x = 49\frac{1}{11}, \text{ or } 49\frac{1}{11} \text{ minutes after 9 o'clock.}$$

29. Let x = the number of spaces the minute hand travels.

$$x - \frac{x}{12} = 45.$$

$$x = 49\frac{1}{11}, \text{ or } 49\frac{1}{11} \text{ minutes after 3 o'clock.}$$

Also

$$x - \frac{x}{12} = 15.$$

$$x = 16\frac{4}{11}, \text{ or } 16\frac{4}{11} \text{ minutes after 3 o'clock.}$$

30. Let x = the number of spaces the minute hand travels.

$$x - \frac{x}{12} = 35.$$

$$12x - x = 420.$$

$$x = 38\frac{2}{11}, \text{ or } 38\frac{2}{11} \text{ minutes after 5 o'clock.}$$

Also

$$x - \frac{x}{12} = 15.$$

$$x = 16\frac{4}{11}, \text{ or } 16\frac{4}{11} \text{ minutes after 5 o'clock.}$$

31. Let x = the number of spaces the minute hand travels.

$$x - \frac{x}{12} = 35.$$

$$x = 38\frac{2}{11}, \text{ or } 38\frac{2}{11} \text{ minutes after 4 o'clock.}$$

Also

$$x - \frac{x}{12} = 5.$$

$$x = 5\frac{5}{11}, \text{ or } 5\frac{5}{11} \text{ minutes after 4 o'clock.}$$

32. If $x =$ the number of inches average height, the sum of the heights of n boys is nx inches.

$$nx \text{ inches} = \frac{nx}{36} \text{ yards.}$$

33. One package weighs $\frac{p}{3m+5}$ pounds.

$$\text{Then } n \text{ packages would weigh } n \cdot \frac{p}{3m+5} = \frac{np}{3m+5} \text{ pounds.}$$

34. If a man can do a piece of work in t hours, in one hour he will do $\frac{1}{t}$ part of the work; and n men working at the same rate would do $n \cdot \frac{1}{t}$, or $\frac{n}{t}$, part of the work in one hour. k men in h hours would do $k \cdot h \cdot \frac{1}{t}$, or $\frac{hk}{t}$, part of the work.

35. Let $t =$ the number of hours for x men to do it.

$$t = \frac{nh}{x}.$$

36. Let $g =$ the number of cents gain on n dozen.

$$g = 12n \left(b - \frac{d}{12} \right) \\ = n(12b - d).$$

37. Let $l =$ the number of dollars loss.

$$l = \frac{\frac{mc}{100}}{100} - 10 \\ = \frac{mc}{10000} - 10.$$

38. Let $p =$ the cost in cents of x yards.

$$p = \frac{xc}{n}.$$

39. Let $n =$ the number of yards.

$$n = \frac{100d}{\frac{x}{y}} = \frac{100dy}{x}.$$

40. Let $p =$ the per cent of gain.

$$p = \frac{s-b}{b}.$$

41. Let $x =$ number of days both will take.

$$\frac{1}{d} + \frac{1}{n} = \frac{1}{x}. \\ nx + dx = nd. \\ x = \frac{nd}{n+d}.$$

42. Let $d =$ the number of 10-hour days.

$$d = \frac{n}{10 \cdot \frac{m}{h}} = \frac{hn}{10m}.$$

43. $m = \frac{3600 \frac{y}{t}}{1760} = \frac{45y}{22t}.$

44. Let $m =$ the number of men required.

$$m = \frac{nd}{t}.$$

45. Let $x =$ the number of minutes under fire.

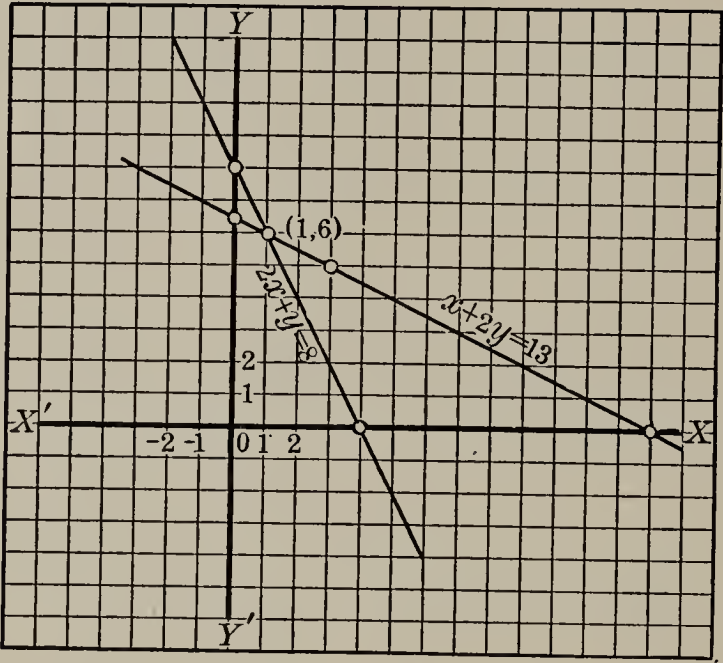
$$x = \frac{f}{6080k} \times 60 = \frac{3f}{304k}.$$

1. For $2x + y = 8$.

For $x + 2y = 13$.

If $x =$	0	4	1
then $y =$	8	0	6

If $x =$	0	13	3
then $y =$	$6\frac{1}{2}$	0	5



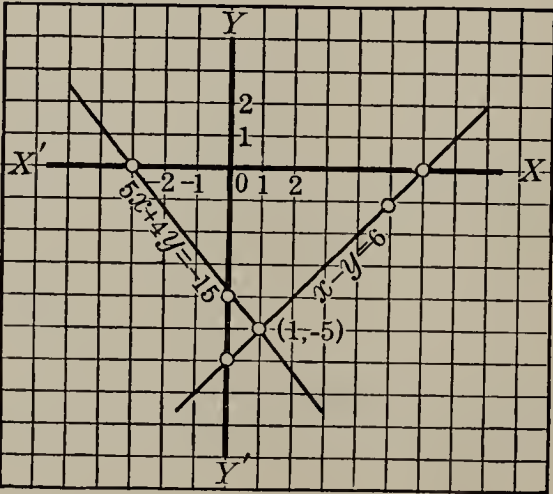
$x = 1, y = 6$.

2. For $x - y = 6$.

For $5x + 4y = -15$.

If $x =$	0	6	5
then $y =$	-6	0	-1

If $x =$	0	-3	1
then $y =$	$-3\frac{3}{4}$	0	-5



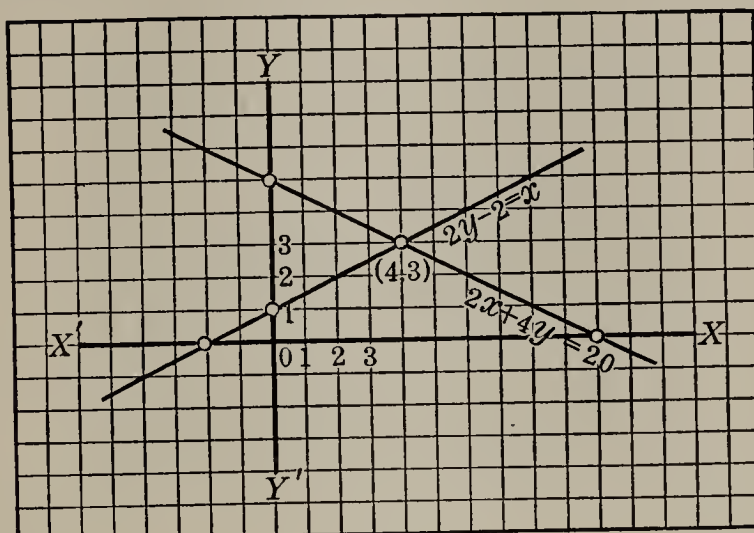
$x = 1, y = -5$.

3. For $2x + 4y = 20$.

If $x =$	0	10	4
then $y =$	5	0	3

For $2y - 2 = x$.

If $x =$	0	-2	4
then $y =$	1	0	3



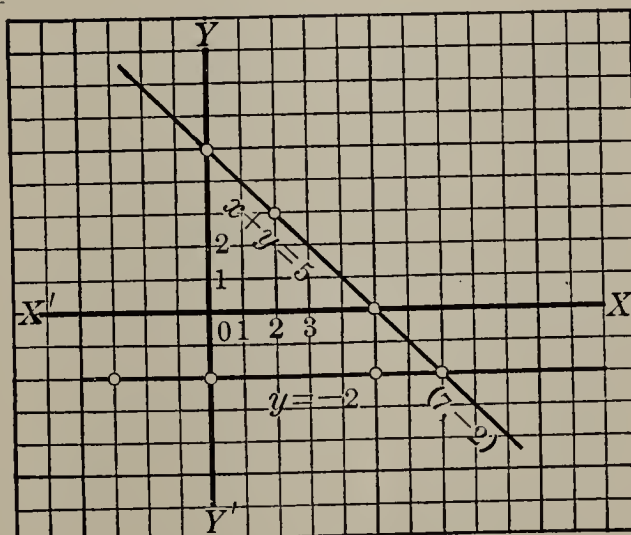
$$x = 4, y = 3.$$

4. For $x + y = 5$.

If $x =$	0	5	2
then $y =$	5	0	3

For $y + 2 = 0$, or $0x + y = -2$.

If $x =$	4	0	-3
then $y =$	-2	-2	-2



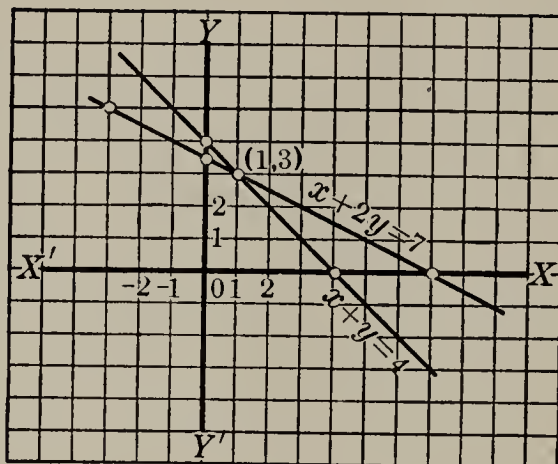
$$x = 7, y = -2.$$

5. For $x + y = 4$.

If $x =$	0	4	1
then $y =$	4	0	3

For $x + 2y = 7$.

If $x =$	0	7	-3
then $y =$	$\frac{7}{2}$	0	5



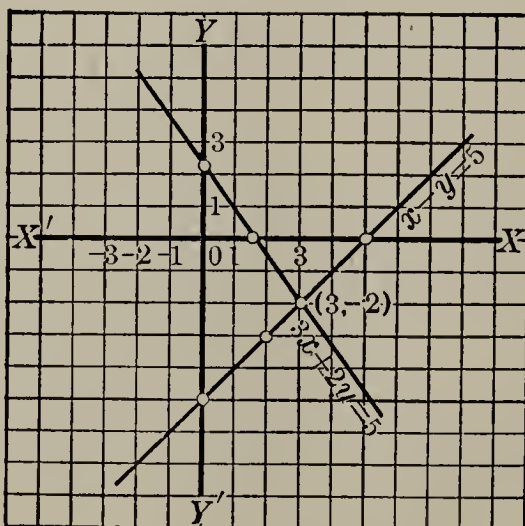
$$x = 1, y = 3.$$

6. For $x - y = 5$.

If $x =$	0	5	2
then $y =$	-5	0	-3

For $3x + 2y = 5$.

If $x =$	0	$1\frac{2}{3}$	3
then $y =$	$2\frac{1}{2}$	0	-2



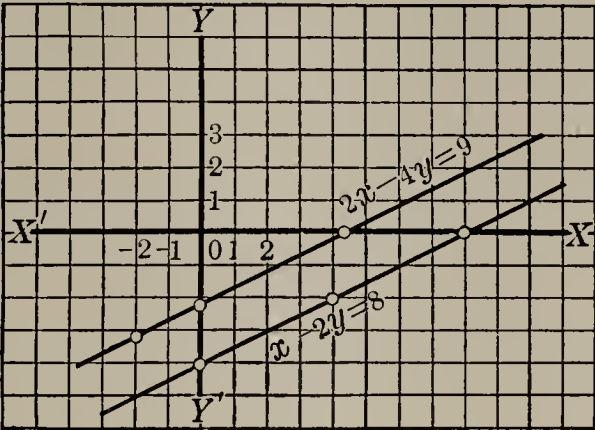
$$x = 3, y = -2.$$

7. For $2x - 4y = 9$.

For $x - 2y = 8$.

If $x =$	0	$\frac{9}{2}$	-2
then $y =$	$-\frac{9}{4}$	0	$-3\frac{1}{4}$

If $x =$	0	8	4
then $y =$	-4	0	-2

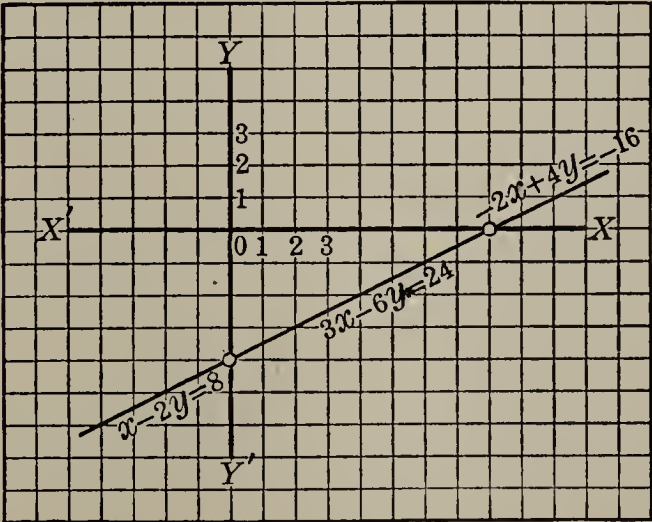


The lines are parallel, and the equations are inconsistent.

8. Yes.

9. For $x - 2y = 8$, for $3x - 6y = 24$, and for $-2x + 4y = -16$.

If $x =$	0	8
then $y =$	-4	0



The three graphs are identical.

An infinite number of sets of roots will satisfy the three equations.

The conclusion that any two dependent equations have the same graph seems warranted.

10. The coördinates of the origin are (0, 0).

11. Yes, if the graphs of the two equations are parallel lines, as in Example 7, above, or in $x + 2y = 5$ and $x + 2y = 7$.

12. The form of the equation of a line parallel to the x -axis is one in which x does not occur, as $y = a$. The form of the equation of a line parallel to the y -axis is one in which y does not occur, as $x = b$.

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$$1. \quad 3x - 8y = 20, \quad (1)$$

$$x - 6y = 0. \quad (2)$$

$$\text{From (2),} \quad x = 6y. \quad (3)$$

$$\text{Then in (1),} \quad 18y - 8y = 20.$$

$$y = 2.$$

$$\text{Then in (2),} \quad x - 12 = 0.$$

$$x = 12.$$

$$2. \quad 2x + 5y = 8, \quad (1) \quad 5. \quad 3x - 2y = 18, \quad (1)$$

$$x - 10y = 9. \quad (2) \quad 4y + 3x = 0. \quad (2)$$

$$\text{From (2),} \quad x = 10y + 9. \quad (3) \quad \text{From (2),} \quad y = -\frac{3}{4}x. \quad (3)$$

Then in (1),

Then in (1),

$$2(10y + 9) + 5y = 8.$$

$$3x + \frac{3}{2}x = 18.$$

$$25y = -10.$$

$$9x = 36.$$

$$y = -\frac{2}{5}.$$

$$x = 4.$$

$$\text{Then in (3),} \quad x = -4 + 9.$$

$$\text{Then in (3),} \quad y = -3.$$

$$x = 5.$$

$$3. \quad 2(x + y) + 3y = 4, \quad (1) \quad 6. \quad \frac{8m - 3n}{2} + 6n = -9, \quad (1)$$

$$5 = x + y. \quad (2) \quad 4m - 1 = 3n. \quad (2)$$

Substituting 5 for $x + y$ in (1),

Clearing (1) of fractions,

$$10 + 3y = 4.$$

$$8m + 9n = -18. \quad (3)$$

$$y = -2.$$

$$\text{Then in (2),} \quad 5 = x - 2.$$

$$\text{From (2),} \quad m = \frac{3n + 1}{4}. \quad (4)$$

$$x = 7.$$

Then in (3),

$$4. \quad 16x + 7 = 15y, \quad (1) \quad 2(3n + 1) + 9n = -18.$$

$$4x + 5y = 0. \quad (2) \quad 15n = -20.$$

$$\text{From (2),} \quad y = -\frac{4}{5}x. \quad (3) \quad n = -\frac{4}{3}.$$

Then in (1),

Then from (4),

$$16x + 7 = -12x.$$

$$m = \frac{-4 + 1}{4}$$

$$x = -\frac{1}{4}.$$

$$= -\frac{3}{4}.$$

$$\text{Then in (3),} \quad y = \frac{1}{5}.$$

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$$1. \quad x + y = 4, \quad (1)$$

$$x - y = 2. \quad (2)$$

$$\text{From (2),} \quad x = y + 2. \quad (3)$$

$$\text{Substituting (3) in (1),} \quad y + 2 + y = 4.$$

$$2y = 2.$$

$$y = 1.$$

$$\text{Then from (2),} \quad x - 1 = 2.$$

$$x = 3.$$

$$2. \quad x + 2y = 3, \quad (1)$$

$$x + y = 2. \quad (2)$$

$$\text{From (2),} \quad x = 2 - y. \quad (3)$$

$$\text{Substituting (3) in (1),} \quad 2 - y + 2y = 3.$$

$$2 + y = 3.$$

$$y = 1.$$

$$\text{Then from (2),} \quad x + 1 = 2.$$

$$x = 1.$$

$$3. \quad x - y = 5, \quad (1) \quad 5. \quad x + 3y = 0, \quad (1)$$

$$2x + y = 4. \quad (2) \quad 2x - 3y = 9. \quad (2)$$

$$\text{From (1),} \quad x = 5 + y. \quad (3) \quad \text{From (1),} \quad x = -3y \quad (3)$$

$$\text{Substituting (3) in (2),} \quad \text{Substituting (3) in (2),}$$

$$10 + 3y = 4. \quad -6y - 3y = 9.$$

$$3y = -6. \quad -9y = 9.$$

$$y = -2. \quad y = -1.$$

$$\text{Then from (1),} \quad \text{Then from (1),}$$

$$x + 2 = 5. \quad x - 3 = 0.$$

$$x = 3. \quad x = 3.$$

$$6. \quad 5x - 6y = 7, \quad (1)$$

$$4x - 3y = 2. \quad (2)$$

$$\text{From (2),} \quad x = \frac{2 + 3y}{4}. \quad (3)$$

$$\text{Substituting (3) in (1),}$$

$$\frac{10 + 15y}{4} - 6y = 7.$$

$$10 + 15y - 24y = 28.$$

$$-9y = 18.$$

$$y = -2.$$

$$\text{Then from (2),}$$

$$4x + 6 = 2.$$

$$4x = -4.$$

$$x = -1.$$

$$4. \quad 2x + 3y = 8, \quad (1)$$

$$x - y = 1. \quad (2)$$

$$\text{From (2),} \quad x = y + 1. \quad (3)$$

$$\text{Substituting (3) in (1),}$$

$$5y + 2 = 8.$$

$$5y = 6.$$

$$y = \frac{6}{5}.$$

$$\text{Then from (2),}$$

$$x - \frac{6}{5} = 1.$$

$$x = \frac{5}{5} + \frac{6}{5}.$$

$$x = \frac{11}{5}.$$

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$$1. \quad \frac{2x}{3} + y = -7, \quad (1)$$

$$5x - 3y = 105. \quad (2)$$

Clearing (1) of fractions,

$$2x + 3y = -21. \quad (3)$$

$$(2) + (3), \quad 7x = 84.$$

$$x = 12.$$

$$\text{Then from (2),} \quad y = -15.$$

$$2. \quad \frac{3r}{4} - \frac{7}{2} = \frac{s}{12}, \quad (1)$$

$$r + 8 = -2s. \quad (2)$$

Clearing (1) of fractions,

$$9r - 42 = s. \quad (3)$$

$$2 \cdot (3), \quad 18r - 84 = 2s. \quad (4)$$

$$(2) + (4),$$

$$19r - 76 = 0.$$

$$r = 4.$$

$$\text{Then from (2),} \quad s = -6.$$

$$3. \quad \frac{3y+1}{4} - \frac{z+22}{12} = 3, \quad (1)$$

$$z - 2y = 1. \quad (2)$$

$$\text{From (2),} \quad z = 2y + 1.$$

In (1),

$$\frac{3y+1}{4} - \frac{2y+23}{12} = 3.$$

$$9y + 3 - 2y - 23 = 36.$$

$$7y = 56.$$

$$y = 8.$$

$$\text{Then from (2),} \quad z = 17.$$

$$4. \quad .5x + .7y = \frac{3}{5}, \quad (1)$$

$$.8x - .2y = 3\frac{3}{5}. \quad (2)$$

$$(1) \cdot 8, \quad 4x + 5.6y = 4.8. \quad (3)$$

$$(2) \cdot 5, \quad 4x - y = 18. \quad (4)$$

$$(3) - (4), \quad 6.6y = -13.2.$$

$$y = -2.$$

$$\text{Then in (4),} \quad x = 4.$$

$$5. \quad \frac{m+2n}{5} - \frac{2m-n}{10} = \frac{5}{2}, \quad (1)$$

$$\frac{m+n}{4} - \frac{m-n}{7} = \frac{n}{2}. \quad (2)$$

Clearing (1) and (2) of fractions,

$$2(m+2n) - (2m-n) = 25. \quad (3)$$

$$7(m+n) - 4(m-n) = 14n. \quad (4)$$

Simplifying (3) and (4),

$$5n = 25.$$

$$n = 5. \quad (5)$$

$$3m - 3n = 0. \quad (6)$$

Substituting from (5), in (6),

$$3m = 15.$$

$$m = 5.$$

$$6. \quad \frac{1}{x} + \frac{1}{y} = -\frac{1}{6}, \quad (1)$$

$$\frac{2}{x} - \frac{3}{y} = \frac{4}{3}. \quad (2)$$

$$3 \cdot (1) + (2), \quad \frac{5}{x} = \frac{5}{6}.$$

$$x = 6.$$

$$2 \cdot (1) - (2), \quad \frac{5}{y} = -\frac{5}{3}.$$

$$y = -3.$$

$$7. \quad \frac{2r+4s}{2r-s} = \frac{38}{3}, \quad (1)$$

$$\frac{5}{r} - \frac{7}{s} = 0. \quad (2)$$

Clearing of fractions,

$$6r + 12s = 76r - 38s. \quad (3)$$

$$5s - 7r = 0. \quad (4)$$

From (3),

$$70r - 50s = 0,$$

$$\text{or} \quad 7r - 5s = 0. \quad (5)$$

But (5) is equal to $-1 \cdot (4)$;
hence the equations are dependent.

$$8. \quad \frac{3}{x-3} - \frac{1}{y} = 0, \quad (1)$$

$$\frac{x-6y}{5} = \frac{2}{5}. \quad (2)$$

$$\text{From (1), } 3y - x + 3 = 0. \quad (3)$$

From (2),

$$5x - 30y - 10 = 0. \quad (4)$$

$$5 \cdot (3) + (4), \quad -15y + 5 = 0.$$

$$y = \frac{1}{3}.$$

$$\text{Then from (3), } x = 4.$$

$$9. \quad \frac{3(x+y)}{3} + \frac{x-y}{-11} = 0, \quad (1)$$

$$2x + y = 7. \quad (2)$$

From (1),

$$\frac{3(x+y)}{3} + \frac{x-y}{-11} = 0.$$

$$-33(x+y) + 3(x-y) = 0.$$

$$-30x - 36y = 0.$$

$$y = -\frac{5}{6}x. \quad (3)$$

$$\text{Then in (2), } 2x - \frac{5}{6}x = 7.$$

$$12x - 5x = 7x = 42.$$

$$x = 6.$$

$$\text{Then from (3), } y = -5.$$

$$10. \quad \frac{3x+z}{z-x} + 2 = \frac{12}{x-z}, \quad (1)$$

$$\frac{x+7}{z} + \frac{17+x}{5-z} = 0. \quad (2)$$

From (1),

$$3x + z + 2(z-x) = -12.$$

$$x + 3z = -12. \quad (3)$$

From (2),

$$(x+7)(5-z) + (17+x)z = 0.$$

$$5x + 10z = -35.$$

$$x + 2z = -7. \quad (4)$$

$$(3) - (4), \quad z = -5.$$

$$\text{Then in (4), } x = 3.$$

$$11. \quad 5x + 4y = 10a + 4, \quad (1)$$

$$x - 2ay = 0. \quad (2)$$

$$\text{From (2), } x = 2ay. \quad (3)$$

Then in (1),

$$(10a + 4)y = 10a + 4.$$

$$y = 1.$$

$$\text{Then from (3), } x = 2a.$$

$$12. \quad 2ax - 3by = 7, \quad (1)$$

$$5ax + 7by = 3. \quad (2)$$

$$5 \cdot (1) - 2 \cdot (2), \quad -29by = 29.$$

$$y = -\frac{1}{b}.$$

$$\text{Then from (1), } 2ax = 4.$$

$$x = \frac{2}{a}.$$

$$13. \quad \frac{x}{a} - \frac{y}{c} = 6, \quad (1)$$

$$\frac{x}{2a} + \frac{y}{3c} = 13. \quad (2)$$

Clearing of fractions,

$$cx - ay = 6ac. \quad (3)$$

$$3cx + 2ay = 78ac. \quad (4)$$

$$2 \cdot (3) + (4), \quad 5cx = 90ac.$$

$$x = 18a.$$

$$\text{Then in (3), } ay = 12ac.$$

$$y = 12c.$$

$$14. \quad \frac{a}{x} + \frac{3a}{y} = 1, \quad (1)$$

$$\frac{3a}{x} + \frac{a}{y} = \frac{1}{2}. \quad (2)$$

$$3 \cdot (1) - (2), \quad \frac{8a}{y} = \frac{5}{2}.$$

$$y = \frac{16a}{5}.$$

$$3 \cdot (2) - (1), \quad \frac{8a}{x} = \frac{1}{2}.$$

$$x = 16a.$$

$$15. \quad \frac{x}{a} + \frac{y}{b} = \frac{a+b}{ab}, \quad (1)$$

$$x - y = \frac{a^2 - b^2}{ab}. \quad (2)$$

$$\text{From (1),} \quad bx + ay = a + b. \quad (3)$$

$$\text{From (2),} \quad abx - aby = a^2 - b^2. \quad (4)$$

$$a \cdot (3) - (4), \quad (a^2 + ab)y = ab + b^2.$$

$$a(a+b)y = b(a+b).$$

$$y = \frac{b}{a}.$$

$$b \cdot (3) + (4), \quad (b^2 + ab)x = ab + a^2.$$

$$b(b+a)x = a(b+a).$$

$$x = \frac{a}{b}.$$

$$16. \quad 2ax - 4by = 7c, \quad (1)$$

$$3ax - 6by = 5c. \quad (2)$$

$$3 \cdot (1) - 2 \cdot (2), \quad 0 + 0 = 11c, \text{ and the equations are inconsistent.}$$

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$$1. \quad x + 3y - 5z = 2, \quad (1) \quad 3. \quad x + 2y + z = -1, \quad (1)$$

$$2x - y - z = 1, \quad (2) \quad 2x - y + z = -20, \quad (2)$$

$$3x + 5y - 7z = -10. \quad (3) \quad -x - y - 5z = 13. \quad (3)$$

$$2 \cdot (1) - (2), \quad 7y - 9z = 3. \quad (4) \quad (1) + (3), \quad y - 4z = 12. \quad (4)$$

$$3 \cdot (1) - (3), \quad 4y - 8z = 16. \quad (5) \quad (2) + 2 \cdot (3),$$

$$4 \cdot (4) - 7 \cdot (5), \quad 20z = -100. \quad -3y - 9z = 6. \quad (5)$$

$$z = -5. \quad 3 \cdot (4) + (5), \quad -21z = 42.$$

$$\text{Then in (5),} \quad y = -6, \quad z = -2.$$

$$\text{and in (1),} \quad x = -5. \quad \text{Then in (4),} \quad y = 4,$$

$$\text{and in (1),} \quad x = -7.$$

$$2. \quad x - y + 3z = 0, \quad (1)$$

$$5x + 2y + z = 14, \quad (2)$$

$$2x + 3y + 4z = -14. \quad (3)$$

$$(1) + 3 \cdot (2), \quad 5x + 13z = -14. \quad (4)$$

$$(3) + 2 \cdot (2), \quad 7x + 7z = 14. \quad (5)$$

$$7 \cdot (4) - 5 \cdot (5), \quad 56z = -168.$$

$$z = -3.$$

$$\text{Then in (4),} \quad x = 5,$$

$$\text{and in (2),} \quad y = -4.$$

$$4. \quad x + y + z = 1, \quad (1)$$

$$x + y - z = 2, \quad (2)$$

$$x - y + z = 3. \quad (3)$$

$$(2) + (3), \quad 2x = 5 \text{ or } x = \frac{5}{2}.$$

$$(1) - (2), \quad 2z = -1 \text{ or } z = -\frac{1}{2}.$$

$$\text{Substituting these values in (1),}$$

$$y = -1.$$

$$\begin{aligned} 5. \quad x + 2y + z &= 1, & (1) \\ 2x + y - z &= 0, & (2) \\ x + 2y + z &= 0. & (3) \end{aligned}$$

Comparison of (1) and (3) shows at once that these equations are inconsistent.

$$\begin{aligned} 6. \quad 2x - y + 5z &= 0, & (1) \\ 8x + 7y + z &= 38, & (2) \\ x - 5y - z &= 7. & (3) \\ (1) + 5 \cdot (3), \quad 7x - 26y &= 35. & (4) \\ (2) + (3), \quad 9x + 2y &= 45. & (5) \\ (4) + 13 \cdot (5), \quad 124x &= 620. \end{aligned}$$

$$x = 5.$$

$$\begin{aligned} \text{Then in (5),} \quad y &= 0, \\ \text{and in (2),} \quad z &= -2. \end{aligned}$$

$$\begin{aligned} 7. \quad 4x - 3y &= z, & (1) \\ z &= x + y, & (2) \\ 2x &= 3y + 1. & (3) \end{aligned}$$

$$\begin{aligned} \text{Substituting } x + y \text{ for } z \text{ from (2)} \\ \text{in (1),} \quad 3x - 4y &= 0. \\ x &= \frac{4}{3}y. & (4) \end{aligned}$$

$$\begin{aligned} \text{Substituting in (3) the value of } x, \\ \frac{8}{3}y &= 3y + 1. \end{aligned}$$

$$\begin{aligned} \text{Then in (4),} \quad x &= -4, \\ \text{and in (2),} \quad z &= -7. \end{aligned}$$

$$\begin{aligned} 8. \quad x - 2y &= 10, & (1) \\ 3y + 4z &= -1, & (2) \\ 5x - z &= 18. & (3) \end{aligned}$$

$$\text{In (3),} \quad z = 5x - 18. \quad (4)$$

$$\begin{aligned} \text{Substituting in (2),} \\ 20x + 3y &= 71. & (5) \end{aligned}$$

$$\begin{aligned} (5) - 20 \cdot (1), \quad 43y &= -129. \\ y &= -3. \end{aligned}$$

$$\begin{aligned} \text{Then in (1),} \quad x &= 4, \\ \text{and in (4),} \quad z &= 2. \end{aligned}$$

$$\begin{aligned} 9. \quad x + y &= 3a, & (1) \\ x + z &= 4a, & (2) \\ y + z &= 5a. & (3) \end{aligned}$$

$$(2) - (1), \quad z - y = a. \quad (4)$$

$$(3) + (4), \quad 2z = 6a.$$

$$z = 3a.$$

$$\begin{aligned} \text{Then in (3),} \quad y &= 2a, \\ \text{and in (1),} \quad x &= a. \end{aligned}$$

$$\begin{aligned} 10. \quad .3x + .2y + .4z &= 1.9, & (1) \\ .02x &= .1 - .01y - .02z, & (2) \\ x + y + z &= 6. & (3) \end{aligned}$$

Clearing (1) and (2) of decimals,

$$3x + 2y + 4z = 19, \quad (4)$$

$$2x + y + 2z = 10. \quad (5)$$

$$2 \cdot (3) - (5), \quad y = 2.$$

Substituting in (3) and (4),

$$x + z = 4, \quad (6)$$

$$3x + 4z = 15. \quad (7)$$

$$(7) - 3 \cdot (6), \quad z = 3.$$

$$\text{Then in (6),} \quad x = 1.$$

$$11. \quad \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 3, \quad (1)$$

$$\frac{a}{x} - \frac{b}{y} + \frac{2c}{z} = 2, \quad (2)$$

$$\frac{5a}{x} - \frac{3b}{y} - \frac{2c}{z} = 0. \quad (3)$$

$$(1) + (2), \quad \frac{2a}{x} + \frac{3c}{z} = 5. \quad (4)$$

$$3 \cdot (1) + (3), \quad \frac{8a}{x} + \frac{c}{z} = 9. \quad (5)$$

$$3 \cdot (5) - (4), \quad \frac{22a}{x} = 22.$$

$$x = a.$$

$$\begin{aligned} \text{Then in (5),} \quad z &= c, \\ \text{and in (1),} \quad y &= b. \end{aligned}$$

$$12. \quad \frac{x+y}{3} - \frac{z}{5} = \frac{2}{3}, \quad (1)$$

$$\frac{y+z}{4} - \frac{x}{2} = \frac{5}{12}, \quad (2)$$

$$3x + 2y = 12 - 3z. \quad (3)$$

Clearing of fractions in (1)
and (2),

$$5x + 5y - 3z = 10, \quad (4)$$

$$-6x + 3y + 3z = 5. \quad (5)$$

$$(4) + (5), \quad -x + 8y = 15. \quad (6)$$

$$(4) + (3), \quad 8x + 7y = 22. \quad (7)$$

$$8 \cdot (6) + (7), \quad 71y = 142.$$

$$y = 2.$$

$$\text{Then in (6),} \quad x = 1,$$

$$\text{and in (3),} \quad z = \frac{5}{3}.$$

$$13. \quad 2x + y + z + w = 1, \quad (1)$$

$$x - y - z + 2w = 4, \quad (2)$$

$$x + 2y - z - w = 0, \quad (3)$$

$$x - y + 2z - w = 1. \quad (4)$$

$$(1) + (2), \quad 3x + 3w = 5. \quad (5)$$

$$2 \cdot (1) - (3),$$

$$3x + 3z + 3w = 2. \quad (6)$$

$$(1) + (4), \quad 3x + 3z = 2. \quad (7)$$

$$(6) - (5), \quad 3z = -3.$$

$$z = -1.$$

$$(6) - (7), \quad 3w = 0.$$

$$w = 0.$$

$$\text{Then in (5),} \quad 3x = 5,$$

$$x = \frac{5}{3},$$

$$\text{and in (1),} \quad y = -\frac{4}{3}.$$

$$14. \quad x + y + z = 1, \quad (1)$$

$$x - y - w = -1, \quad (2)$$

$$x - z - w = -5, \quad (3)$$

$$y - z + w = 0. \quad (4)$$

$$(1) + (2), \quad 2x + z - w = 0. \quad (5)$$

$$(2) + (4), \quad x - z = -1. \quad (6)$$

$$(6) - (3), \quad w = 4.$$

$$\text{Then (5) becomes} \quad 2x + z = 4. \quad (7)$$

$$(6) + (7), \quad 3x = 3,$$

$$x = 1,$$

$$\text{and in (7),} \quad z = 2.$$

$$\text{Then in (1),} \quad y = -2.$$

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1. Let x and y respectively represent the numbers.

$$\text{Then} \quad x + y = 109,$$

$$\text{and} \quad x - y = 49.$$

$$\text{Solving,} \quad x = 79, y = 30.$$

2. Let

x = tons capacity of a truck,

and

y = tons capacity of a dray.

$$\text{Then} \quad 10x + 6y = 39,$$

$$\text{and} \quad 8x + 10y = 39.$$

$$\text{Solving,} \quad x = 3, y = 1\frac{1}{2}.$$

3. Let x and y = the number of days in which one man and one boy, respectively, can do the work.

Then
$$\frac{1}{x} + \frac{1}{y} = \frac{1}{18},$$

and
$$\frac{5}{x} + \frac{9}{y} = \frac{1}{3}.$$

Solving, $x = 24, y = 72.$

4. Let x and y = the amounts at 5% and 6% respectively.

Then
$$1\frac{5}{100}x + 1\frac{6}{100}y = 100,$$

and
$$1\frac{6}{100}x + 1\frac{5}{100}y = 98.$$

Solving, $x = 800, y = 1000.$

5. Substituting the values of x and y which are to satisfy, we get the system in a and b :

$$2a + 3b = 2,$$

$$6a + 5b = 2.$$

Solving, $a = -\frac{1}{2}, b = 1.$

6. Let x and y = the number of ounce and $\frac{3}{8}$ -ounce balls respectively.

Then
$$x + y = 23,$$

and
$$x + \frac{3}{8}y = 18.$$

Solving, $x = 15, y = 8.$

7. Let d and q = the number of dimes and quarters respectively.

Then
$$d + q = 36,$$

and
$$10d + 25q = 450.$$

Solving, $d = 30, q = 6.$

8. Let x = the distance walked, in miles,

and y = the distance by stage, in miles.

Then
$$x + y = 20,$$

and
$$\frac{x}{4} + \frac{y}{12} = 3,$$

since $\frac{\text{Distance}}{\text{Rate}} = \text{Time}.$

Solving, $x = 8, y = 12.$

9. Let x = the faster rate, in miles per hour,

and y = the slower rate, in miles per hour.

Then in the first case the distances covered are x and y miles; in the second, $5x$ and $5y$ miles.

Therefore
$$x + y = 25,$$

and
$$5x - 5y = 25.$$

Solving, $x = 15, y = 10.$

OR

10. Let d = the distance in miles,
 and r = the rate in miles per hour.
 Then $\frac{d}{r} = 3$,
 and $\frac{\frac{3}{2}d}{r + 50} = 3$.
 Solving, $d = 300, r = 100$.

11. Let x, y , and z = the respective numbers.
 Then $x + y + z = 108$,
 and $\frac{1}{3}x + \frac{1}{4}y + \frac{1}{6}z = 25$,
 and $3x + 4y + 6z = 504$.
 Solving, $x = 24, y = 36, z = 48$.

12. Let x, y , and z = the respective numbers.
 Then $x + y + z = 217$,
 and $\frac{x}{y} = 5$,
 and $\frac{y}{z} = 5$.
 Solving, $x = 175, y = 35, z = 7$.

13. Let x, y , and z = the three digits, from left to right.
 Then $x + y + z = 14$,
 and $100x + 10z + y + 27 = 100x + 10y + z$,
 and $100y + 10x + z + 180 = 100x + 10y + z$.
 Solving, $x = 7, y = 5, z = 2$,
 and the required number is 752.

14. Let the sides be x, y, z , and u respectively.
 Then $x + y = 20$, (1) $x + y = 20$, (5)
 $y + z = 27$, (2) $y + z = 27$, (6)
 $y + u = 23$, (3) or $y + u = 29$, (7)
 $z + x = 29$, (4) $z + x = 23$. (8)
 Then (2) - (1), $z - x = 7$, (9)
 and (9) + (4), $2z = 36$,
 or $z = 18$.
 Then from (4), $x = 11$,
 from (1), $y = 9$,
 and from (3), $u = 14$.
 Hence $x = 11, y = 9, z = 18, u = 14$.

In the second case, (6) – (5),

$$z - x = 7, \quad (10)$$

$$(10) + (8), \quad 2z = 30,$$

$$\text{or} \quad z = 15.$$

$$\text{Then from (8),} \quad x = 8,$$

$$\text{from (5),} \quad y = 12,$$

$$\text{and from (7),} \quad u = 17.$$

$$\text{Hence} \quad x = 8, y = 12, z = 15, u = 17.$$

15. Let x and y = the respective costs of the chairs, in cents.

$$\text{Then} \quad x + y = 100h,$$

$$\text{and} \quad x - y = m.$$

$$\text{Solving,} \quad x = \frac{100h + m}{2} \text{ (cents),}$$

$$y = \frac{100h - m}{2} \text{ (cents).}$$

$$16. \quad x + y = a,$$

$$\text{and} \quad x - y = b.$$

$$\text{Solving,} \quad x = \frac{a + b}{2}, y = \frac{a - b}{2}.$$

$$17. \quad x + y = a + b,$$

$$\text{and} \quad x - y = a - b.$$

$$\text{Solving,} \quad x = a, y = b.$$

18. Let x and y = the respective distances in miles.

$$\text{Then} \quad x + y = k,$$

$$\text{and} \quad x - y = c.$$

$$\text{Solving,} \quad x = \frac{k + c}{2}, y = \frac{k - c}{2}.$$

19. Let d and q = the number of dimes and quarters respectively.

$$\text{Then} \quad d + q = c,$$

$$\text{and} \quad 10d + 25q = 100a + b.$$

$$\text{Solving,} \quad q = \frac{100a + b - 10c}{15},$$

$$\text{and} \quad d = \frac{25c - b - 100a}{15}.$$

20. Let q and n = the number of quarters and nickels respectively.

$$\text{Then} \quad 25q + 5n = 100a,$$

$$\text{and} \quad q - n = b.$$

$$\text{Solving,} \quad q = \frac{20a + b}{6},$$

$$\text{and} \quad n = \frac{20a - 5b}{6}.$$

21. Let x = rate in miles per hour in still water,
and y = rate of river, in miles per hour.

Then
$$\frac{m}{x+y} = t,$$

and
$$\frac{m}{x-y} = a.$$

Solving,
$$x = \frac{m(a+t)}{2at},$$

and
$$y = \frac{m(a-t)}{2at}.$$

22. Solving the equations of the Hint,

$$x = \frac{1}{4}^5, y = \frac{2}{4}^5.$$

23. Let x and y = the percentage of purity of the
respective solutions.

Then
$$\frac{\left(\frac{x}{100}\right)(8) + \left(\frac{y}{100}\right)(12)}{20} = \frac{84}{100},$$

and
$$\frac{\left(\frac{x}{100}\right)(3) + \left(\frac{y}{100}\right)(2)}{5} = \frac{86}{100}.$$

Solving,
$$x = 90\%, y = 80\%.$$

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1. $(x + x^{\frac{1}{2}} + x^{-\frac{1}{2}})2x^{\frac{3}{2}} = 2x^{\frac{5}{2}} + 2x^2 + 2x.$
2. $(a^{\frac{1}{2}}x + ax^{\frac{1}{2}} - a^{\frac{1}{2}}x^{-\frac{1}{2}})ax^{\frac{1}{2}} = a^{\frac{3}{2}}x^{\frac{3}{2}} + a^2x - a^{\frac{3}{2}}.$
3. $(x^2 - 5ax + 6a^2)a^{\frac{2}{3}}x^{-\frac{2}{3}} = a^{\frac{2}{3}}x^{\frac{4}{3}} - 5a^{\frac{5}{3}}x^{\frac{1}{3}} + 6a^{\frac{8}{3}}x^{-\frac{2}{3}}.$
4. $(x^{\frac{1}{3}} + y^{\frac{1}{3}})(x^{\frac{1}{3}} - y^{\frac{1}{3}}) = x^{\frac{2}{3}} - y^{\frac{2}{3}}.$
5. $(a^{-2} + 3)(a^{-2} - 5) = a^{-4} - 2a^{-2} - 15.$
6. $(x^{2a} - 3x^a)(x^2 - 2x) = x^{2a+2} - 3x^{a+2} - 2x^{2a+1} + 6x^{a+1}.$
7. $(a^{-1} - a)^2 = a^{-2} - 2 + a^2.$
8. $(a^3 - 2a^{-2})^3 = a^9 - 6a^4 + 12a^{-1} - 8a^{-6}.$
9. $(a^{-1} - 2a + 3a^{-2})^2 = a^{-2} + 4a^2 + 9a^{-4} - 4 + 6a^{-3} - 12a^{-1}.$
10. $(e^x + e^{-x})^2 = e^{2x} + 2 + e^{-2x}.$
11. $(a^{-1} + b^{-2})(a^{-1} - b^{-2}) = a^{-2} - b^{-4}.$
12. $(e^{2x} - 2 + e^{-2x})^2 = e^{4x} + 4 + e^{-4x} - 4e^{2x} + 2 - 4e^{-2x}$
 $= e^{4x} - 4e^{2x} + 6 - 4e^{-2x} + e^{-4x}.$
13. $(x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} - 3x^{\frac{3}{2}})^2 = x^{-1} + 4x + 9x^3 + 4 - 6x - 12x^2$
 $= x^{-1} + 4 - 2x - 12x^2 + 9x^3.$

$$\begin{aligned}
 14. (e^{2x} - 3e^{-x})^4 &= (e^{4x} - 6e^x + 9e^{-2x})^2 \\
 &= e^{8x} + 36e^{2x} + 81e^{-4x} - 12e^{5x} + 18e^{2x} - 108e^{-x} \\
 &= e^{8x} - 12e^{5x} + 54e^{2x} - 108e^{-x} + 81e^{-4x}.
 \end{aligned}$$

$$\begin{aligned}
 15. & a^{\frac{4}{5}} - a^{\frac{2}{5}}x + 4x^2 \\
 & \frac{a^{\frac{2}{5}} + 2x}{a^{\frac{6}{5}} - a^{\frac{4}{5}}x + 4a^{\frac{2}{5}}x^2} \\
 & \quad + 2a^{\frac{4}{5}}x - 2a^{\frac{2}{5}}x^2 + 8x^3 \\
 & \frac{a^{\frac{6}{5}} + a^{\frac{4}{5}}x + 2a^{\frac{2}{5}}x^2 + 8x^3}{a^{\frac{6}{5}} + a^{\frac{4}{5}}x + 2a^{\frac{2}{5}}x^2 + 8x^3}
 \end{aligned}$$

$$16. (x^{\frac{1}{3}} + 2y^{\frac{1}{3}})(x^{\frac{2}{3}} - 2x^{\frac{1}{3}}y^{\frac{1}{3}} + 4y^{\frac{2}{3}}) = x + 8y.$$

$$17. (a^{\frac{1}{2}} - a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}})(a^{\frac{1}{2}} + a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}}) = a + a^{\frac{1}{2}}b^{\frac{1}{2}} + b.$$

$$18. x + x^{\frac{1}{2}}y^{-\frac{1}{2}} + y^{-1}$$

$$\frac{x - x^{\frac{1}{2}}y^{-\frac{1}{2}} + y^{-1}}{x^2 + x^{\frac{3}{2}}y^{-\frac{1}{2}} + xy^{-1}}$$

$$- x^{\frac{3}{2}}y^{-\frac{1}{2}} - xy^{-1} - x^{\frac{1}{2}}y^{-\frac{3}{2}}$$

$$+ xy^{-1} + x^{\frac{1}{2}}y^{-\frac{3}{2}} + y^{-2}$$

$$\frac{x^2 + xy^{-1} + y^{-2}}{x^2 + xy^{-1} + y^{-2}}$$

$$20. (5c^{-\frac{5}{2}} + d^{-1} - ac^{-\frac{2}{5}})^2$$

$$= 25c^{-5} + d^{-2} + a^2c^{-\frac{4}{5}} + 10c^{-\frac{5}{2}}d^{-1} - 10ac^{-\frac{2}{5}}d^{-1} - 2ac^{-\frac{2}{5}}d^{-1}.$$

$$21. (a^{\frac{1}{2}} + b^{\frac{1}{2}}c^{-1})(a^{\frac{1}{2}} - b^{\frac{1}{2}}c^{-1})^2 = (a - bc^{-2})(a^{\frac{1}{2}} - b^{\frac{1}{2}}c^{-1}).$$

$$\frac{a - bc^{-2}}{a^{\frac{1}{2}} - b^{\frac{1}{2}}c^{-1}}$$

$$\frac{a^{\frac{3}{2}} - a^{\frac{1}{2}}bc^{-2} - ab^{\frac{1}{2}}c^{-1} + b^{\frac{3}{2}}c^{-3}}{a^{\frac{3}{2}} - a^{\frac{1}{2}}bc^{-2} - ab^{\frac{1}{2}}c^{-1} + b^{\frac{3}{2}}c^{-3}}$$

$$19. a^{\frac{5}{3}} - 5a^{\frac{1}{2}}$$

$$\frac{a^{\frac{5}{3}} - 5a^{\frac{1}{2}}}{a^{\frac{10}{3}} - 5a^{\frac{13}{6}}}$$

$$- 5a^{\frac{13}{6}} + 25a$$

$$\frac{a^{\frac{10}{3}} - 10a^{\frac{13}{6}} + 25a}{a^{\frac{10}{3}} - 10a^{\frac{13}{6}} + 25a}$$

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$$1. x^4 \div x^6 = x^{-2}.$$

$$2. x^3 \div x^{\frac{1}{3}} = x^{\frac{8}{3}}.$$

$$3. ax^{\frac{2}{3}} \div a^{\frac{1}{2}}x^{\frac{1}{2}} = a^{\frac{1}{2}}x^{\frac{1}{6}}.$$

$$4. (ax - a^2x^3) \div a^2x^{\frac{1}{2}} = a^{-1}x^{\frac{1}{2}} - x^{\frac{5}{2}}.$$

$$5. (x^a - 2x^{2a-1} + 3x^{3a-2}) \div x^{2a-1} = x^{-a+1} - 2 + 3x^{a-1}.$$

$$6. (6a^{3+4n} - 9a^{n-2} + 12a^{2-n}) \div 3a^{n-2} = 2a^{3n+5} - 3 + 4a^{4-2n}.$$

$$7. x$$

$$+ y \left| \frac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}}} \right|$$

$$\frac{x + x^{\frac{2}{3}}y^{\frac{1}{3}}}{x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}}}$$

$$- x^{\frac{2}{3}}y^{\frac{1}{3}}$$

$$- x^{\frac{2}{3}}y^{\frac{1}{3}} - x^{\frac{1}{3}}y^{\frac{2}{3}}$$

$$\frac{x^{\frac{1}{3}}y^{\frac{2}{3}}}{x^{\frac{1}{3}}y^{\frac{2}{3}} + y}$$

$$x^{\frac{1}{3}}y^{\frac{2}{3}} + y$$

$$\begin{array}{r}
 8. \quad x \qquad \qquad \qquad - 8y \left| \begin{array}{l} x^{\frac{1}{3}} - 2y^{\frac{1}{3}} \\ \hline x^{\frac{2}{3}} + 2x^{\frac{1}{3}}y^{\frac{1}{3}} + 4y^{\frac{2}{3}} \end{array} \right. \\
 \hline
 x - 2x^{\frac{2}{3}}y^{\frac{1}{3}} \\
 2x^{\frac{2}{3}}y^{\frac{1}{3}} \\
 \hline
 2x^{\frac{2}{3}}y^{\frac{1}{3}} - 4x^{\frac{1}{3}}y^{\frac{2}{3}} \\
 \hline
 4x^{\frac{1}{3}}y^{\frac{2}{3}} \\
 \hline
 4x^{\frac{1}{3}}y^{\frac{2}{3}} - 8y \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 9. \quad a^{\frac{3}{2}} \qquad \qquad \qquad + b \left| \begin{array}{l} a^{\frac{1}{2}} + b^{\frac{1}{3}} \\ \hline a - a^{\frac{1}{2}}b^{\frac{1}{3}} + b^{\frac{2}{3}} \end{array} \right. \\
 \hline
 a^{\frac{3}{2}} + ab^{\frac{1}{3}} \\
 - ab^{\frac{1}{3}} \\
 \hline
 - ab^{\frac{1}{3}} - a^{\frac{1}{2}}b^{\frac{2}{3}} \\
 \hline
 a^{\frac{1}{2}}b^{\frac{2}{3}} \\
 \hline
 a^{\frac{1}{2}}b^{\frac{2}{3}} + b \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 10. \quad a^2 \qquad \qquad + ab^{-1} \qquad \qquad + b^{-2} \left| \begin{array}{l} a - a^{\frac{1}{2}}b^{-\frac{1}{2}} + b^{-1} \\ \hline a + a^{\frac{1}{2}}b^{-\frac{1}{2}} + b^{-1} \end{array} \right. \\
 \hline
 a^2 - a^{\frac{3}{2}}b^{-\frac{1}{2}} + ab^{-1} \\
 \hline
 a^{\frac{3}{2}}b^{-\frac{1}{2}} \\
 \hline
 a^{\frac{3}{2}}b^{-\frac{1}{2}} - ab^{-1} + a^{\frac{1}{2}}b^{-\frac{3}{2}} \\
 \hline
 ab^{-1} - a^{\frac{1}{2}}b^{-\frac{3}{2}} \\
 \hline
 ab^{-1} - a^{\frac{1}{2}}b^{-\frac{3}{2}} + b^{-2} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 11. \quad e^{2x} + 2 + e^{-2x} \left| \begin{array}{l} e^x + e^{-x} \\ \hline e^x + e^{-x} \end{array} \right. \\
 \hline
 e^{2x} + 1 \\
 \hline
 1 \\
 \hline
 1 + e^{-2x} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 12. \quad e^{3x} + 3e^x + 3e^{-x} + e^{-3x} \left| \begin{array}{l} e^x + e^{-x} \\ \hline e^{2x} + 2 + e^{-2x} \end{array} \right. \\
 \hline
 e^{3x} + e^x \\
 \hline
 2e^x \\
 \hline
 2e^x + 2e^{-x} \\
 \hline
 e^{-x} \\
 \hline
 e^{-x} + e^{-3x} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 13. \quad a + a^{\frac{3}{5}}b^{\frac{2}{3}} + 2a^{\frac{2}{5}}b^{\frac{1}{3}} + 2b \left| \begin{array}{l} a^{\frac{3}{5}} + 2b^{\frac{1}{3}} \\ \hline a^{\frac{2}{5}} + b^{\frac{2}{3}} \end{array} \right. \\
 \hline
 a \qquad \qquad + 2a^{\frac{2}{5}}b^{\frac{1}{3}} \\
 \hline
 a^{\frac{3}{5}}b^{\frac{2}{3}} \\
 \hline
 a^{\frac{3}{5}}b^{\frac{2}{3}} \qquad \qquad + 2b \\
 \hline
 \end{array}$$

$$14. \frac{m^4 + 7m^2 + 8 - 7m^{-2} + m^{-4}}{m^4 + 5m^2 - 1} \bigg| \frac{m^2 + 5 - m^{-2}}{m^2 + 2 - m^{-2}}$$

$$\begin{array}{r} 2m^2 + 9 \\ 2m^2 + 10 - 2m^{-2} \\ \hline -1 - 5m^{-2} \\ -1 - 5m^{-2} + m^{-4} \end{array}$$

$$15. \frac{9x^{5n-4}}{9x^{5n-4} + 6x^{4n-3}} - \frac{x^{3n-2} + 2x^{2n-1}}{3x^{3n-2} - 2x^{2n-1} + x^n} \bigg| \frac{3x^{2n-2} + 2x^{n-1}}{3x^{3n-2} - 2x^{2n-1} + x^n}$$

$$\begin{array}{r} -6x^{4n-3} \\ -6x^{4n-3} - 4x^{3n-2} \\ \hline 3x^{3n-2} \\ 3x^{3n-2} + 2x^{2n-1} \end{array}$$

$$16. \frac{-16a^{\frac{19}{5}}b^{\frac{3}{4}} + 40ab - 25a^{-\frac{9}{5}}b^{\frac{5}{4}}}{-16a^{\frac{19}{5}}b^{\frac{3}{4}} + 20ab} \bigg| \frac{-4a^{\frac{12}{5}}b^{-\frac{5}{8}} + 5a^{-\frac{2}{5}}b^{-\frac{3}{8}}}{4a^{\frac{7}{5}}b^{\frac{11}{8}} - 5a^{-\frac{7}{5}}b^{\frac{13}{8}}}$$

$$\begin{array}{r} 20ab \\ 20ab - 25a^{-\frac{9}{5}}b^{\frac{5}{4}} \end{array}$$

$$17. \frac{a^{\frac{3}{2}} - ab^{\frac{1}{2}} + a^{\frac{1}{2}}b - b^{\frac{3}{2}}}{a^{\frac{3}{2}} - ab^{\frac{1}{2}}} \bigg| \frac{a^{\frac{1}{2}} - b^{\frac{1}{2}}}{a + b}$$

$$\begin{array}{r} a^{\frac{1}{2}}b \\ a^{\frac{1}{2}}b - b^{\frac{3}{2}} \end{array}$$

$$18. \frac{x^6}{x^6 + 2x^2 + 3x^{-2}} - \frac{4x^{-6} + 3x^{-10}}{x^2 + 2x^{-2} + 3x^{-6}} \bigg| \frac{x^2 + 2x^{-2} + 3x^{-6}}{x^4 - 2 + x^{-4}}$$

$$\begin{array}{r} -2x^2 - 3x^{-2} \\ -2x^2 - 4x^{-2} - 6x^{-6} \\ \hline x^{-2} + 2x^{-6} \\ x^{-2} + 2x^{-6} + 3x^{-10} \end{array}$$

$$19. (x^{\frac{1}{5}} - y^{\frac{1}{5}}) \div (x^{\frac{1}{10}} + y^{\frac{1}{10}}) = x^{\frac{1}{10}} - y^{\frac{1}{10}}.$$

$$\frac{x^{\frac{3}{5}}}{x^{\frac{3}{5}} - x^{\frac{1}{2}}y^{\frac{1}{10}}} - y^{\frac{3}{5}} \bigg| \frac{x^{\frac{1}{10}} - y^{\frac{1}{10}}}{x^{\frac{1}{2}} + x^{\frac{2}{5}}y^{\frac{1}{10}} + x^{\frac{3}{10}}y^{\frac{1}{5}} + x^{\frac{1}{5}}y^{\frac{3}{10}} + x^{\frac{1}{10}}y^{\frac{2}{5}} + y^{\frac{1}{2}}}$$

$$x^{\frac{1}{2}}y^{\frac{1}{10}}$$

$$x^{\frac{1}{2}}y^{\frac{1}{10}} - x^{\frac{2}{5}}y^{\frac{1}{5}}$$

$$x^{\frac{2}{5}}y^{\frac{1}{5}}$$

$$x^{\frac{2}{5}}y^{\frac{1}{5}} - x^{\frac{3}{10}}y^{\frac{3}{10}}$$

$$x^{\frac{3}{10}}y^{\frac{3}{10}}$$

$$x^{\frac{3}{10}}y^{\frac{3}{10}} - x^{\frac{1}{5}}y^{\frac{2}{5}}$$

$$x^{\frac{1}{5}}y^{\frac{2}{5}}$$

$$x^{\frac{1}{5}}y^{\frac{2}{5}} - x^{\frac{1}{10}}y^{\frac{1}{2}}$$

$$x^{\frac{1}{10}}y^{\frac{1}{2}} - y^{\frac{3}{5}}$$

$$x^{\frac{1}{10}}y^{\frac{1}{2}} - y^{\frac{3}{5}}$$

20. $9m \qquad -13 \qquad +4m^{-1} \qquad \frac{3m^{\frac{1}{2}}-5+2m^{-\frac{1}{2}}}{3m^{\frac{1}{2}}+5+2m^{-\frac{1}{2}}}$

$\frac{15m^{\frac{1}{2}}-19}{15m^{\frac{1}{2}}-25+10m^{-\frac{1}{2}}}$
 $\frac{6-10m^{-\frac{1}{2}}}{6-10m^{-\frac{1}{2}}+4m^{-1}}$

21. $9x^{2a} \qquad -19x^{-a} \qquad +25x^{-4a} \qquad \frac{3x^a-7x^{-\frac{a}{2}}+5x^{-2a}}{3x^a+7x^{-\frac{a}{2}}+5x^{-2a}}$

$\frac{9x^{2a}-21x^{\frac{a}{2}}+15x^{-a}}{21x^{\frac{a}{2}}-34x^{-a}}$

$\frac{21x^{\frac{a}{2}}-49x^{-a}+35x^{-\frac{5a}{2}}}{15x^{-a}-35x^{-\frac{5a}{2}}}$
 $\frac{15x^{-a}-35x^{-\frac{5a}{2}}+25x^{-4a}}{15x^{-a}-35x^{-\frac{5a}{2}}+25x^{-4a}}$

22. $\left(\frac{x^{\frac{3}{2}}}{8}+\frac{27y^{\frac{3}{2}}}{64}\right)\div\left[\left(\frac{3x^{\frac{1}{2}}}{2}+\frac{9y^{\frac{1}{2}}}{4}\right)(64x-96x^{\frac{1}{2}}y^{\frac{1}{2}}+144y)\right]$

$=\frac{(2x^{\frac{1}{2}}+3y^{\frac{1}{2}})(4x-6x^{\frac{1}{2}}y^{\frac{1}{2}}+9y)}{64} \cdot \frac{4}{3(2x^{\frac{1}{2}}+3y^{\frac{1}{2}})} \cdot \frac{1}{16(4x-6x^{\frac{1}{2}}y^{\frac{1}{2}}+9y)}$

$=\frac{1}{768}.$

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1. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$.
2. $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$.
3. $2^{-2} \cdot 3^{-4} = \frac{1}{2^2 \cdot 3^4} = \frac{1}{324}$.
4. $7 \cdot 7^0 \cdot 0 = 0$.
5. $(\frac{1}{2})^{-3} = 2^3 = 8$.
6. $(\frac{2}{3})^{-2} \cdot 4^0 = (\frac{3}{2})^2 = \frac{9}{4}$.
7. $\frac{2}{3^{-2}} = 2 \cdot 3^2 = 18$.
8. $\frac{12}{4^{-1}} = 12 \cdot 4 = 48$.
9. $5 \cdot 2^0 - (5 \cdot 2)^0 = 5 - 1 = 4$.
10. $\frac{4^{-2} \cdot 3^{-2}}{6^{-2}} = \frac{6^2}{4^2 \cdot 3^2} = \frac{1}{4}$.
11. $32^{-\frac{2}{5}} = \frac{1}{\sqrt[5]{(32)^2}} = \frac{1}{4}$.
12. $4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{2}$.
13. $16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{2}$.
14. $8^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{8^2}} = \frac{1}{4}$.
15. $25^{\frac{3}{2}} = 5^3 = 125$.
16. $(-64)^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{(-64)^2}} = \frac{1}{16}$.
17. $(-125)^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{(-125)^2}} = \frac{1}{25}$.
18. $\sqrt[3]{27^{-2}} = \sqrt[3]{\frac{1}{27^2}} = \frac{1}{9}$.
19. $(\sqrt[3]{-8})^2 = 4$.
20. $(\frac{1}{2})^{-4} \cdot (\frac{1}{3})^{-3} \cdot (\frac{1}{2})^0 = 2^4 \cdot 3^3 \cdot 1 = 432$.
21. $(\frac{1}{6})^{-2} = 6^2 = 36$.
22. $(.04)^{\frac{3}{2}} = (.2)^3 = .008$.
23. $(.027)^{-\frac{2}{3}} = \frac{1}{(\sqrt[3]{.027})^2} = \frac{1}{.09} = \frac{100}{9}$.
24. $(.00032)^{\frac{2}{5}} = (.2)^2 = .04$.
25. $\frac{2^{-1}}{2^{-2} - 2^{-3}} = \frac{\frac{1}{2}}{\frac{1}{4} - \frac{1}{8}} = \frac{1}{2} \cdot \frac{8}{1} = 4$.
26. $\frac{3^{-2} - 2^{-2}}{3^{-1} - 2^{-1}} = \frac{\frac{1}{3^2} - \frac{1}{2^2}}{\frac{1}{3} - \frac{1}{2}} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$.
27. $\frac{3^{-3} - 2^{-3}}{3^{-1} - 2^{-1}} = \frac{\frac{1}{27} - \frac{1}{8}}{\frac{1}{3} - \frac{1}{2}} = -\frac{19}{216} \cdot -\frac{6}{1} = \frac{19}{36}$.
28. $\frac{2}{a^{-2} - b^{-2}} = \frac{2}{\frac{1}{a^2} - \frac{1}{b^2}} = \frac{2a^2b^2}{b^2 - a^2}$.
29. $\frac{3}{a^{-1} + b^{-1}} = \frac{3}{\frac{1}{a} + \frac{1}{b}} = \frac{3}{\frac{b+a}{ab}} = \frac{3ab}{a+b}$.

$$30. \frac{a}{a^{-2} - b^{-2}} = \frac{a}{\frac{1}{a^2} - \frac{1}{b^2}} = \frac{a^3 b^2}{b^2 - a^2}.$$

$$31. \frac{5se^2}{s^{-2} + e^{-2}} = \frac{5se^2}{\frac{1}{s^2} + \frac{1}{e^2}} = \frac{5se^2}{\frac{e^2 + s^2}{e^2 s^2}} = \frac{5s^3 e^4}{e^2 + s^2}.$$

$$32. \frac{a^{-2}}{a^{-2} + b^{-2}} = \frac{\frac{1}{a^2}}{\frac{1}{a^2} + \frac{1}{b^2}} = \frac{1}{a^2} \cdot \frac{a^2 b^2}{a^2 + b^2} = \frac{b^2}{a^2 + b^2}.$$

$$33. \frac{a^{-3} b^{-3}}{a^{-3} + b^{-3}} = \frac{\frac{1}{a^3 b^3}}{\frac{1}{a^3} + \frac{1}{b^3}} = \frac{1}{a^3 + b^3}.$$

$$34. \frac{a^{-2} - b^{-2}}{a^{-1} + b^{-1}} = a^{-1} - b^{-1} = \frac{1}{a} - \frac{1}{b} = \frac{b - a}{ab}.$$

$$35. \frac{a^{-1} + b^{-1}}{a^{-3} + b^{-3}} = \frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a^3} + \frac{1}{b^3}} = \frac{a + b}{ab} \cdot \frac{a^3 b^3}{a^3 + b^3} = \frac{a^2 b^2}{a^2 - ab + b^2}.$$

$$36. \frac{a^{-3} - 27^{-1}}{a^{-1} - 3^{-1}} = \frac{\frac{1}{a^3} - \frac{1}{27}}{\frac{1}{a} - \frac{1}{3}} = \frac{27 - a^3}{27a^3} \cdot \frac{3a}{3 - a} = \frac{9 + 3a + a^2}{9a^2}.$$

$$37. \frac{2xy}{z^2} = 2xyz^{-2}.$$

$$38. \frac{3x}{a^{-2}b^4} = 3a^2b^{-4}x.$$

$$39. \frac{4sc^{-3}}{2^{-1}s^{-2}} = 8s^3c^{-3}.$$

$$40. \frac{12a^2b^3}{4xy^2} = 3a^2b^3x^{-1}y^{-2}.$$

$$41. \frac{5ac^{-2}}{c(x-y)^2} = 5ac^{-3}(x-y)^{-2}.$$

$$42. \frac{1}{5a^2(c+d)^{-3}} = 5^{-1}a^{-2}(c+d)^3.$$

$$43. \frac{42m^{-n}n^{2m}}{56m^{-2}nn^{-3m}} = 3 \cdot 4^{-1}m^nn^{5m}.$$

$$44. \frac{r^{-1}s^2}{r^{-2}s^3(s-r)^3} = rs^{-1}(s-r)^{-3}.$$

$$45. (2^3)^3 = (8)^3 = 512.$$

$$46. (2^3)^{-2} = 2^{-6} = \frac{1}{64}.$$

$$47. (2^{-3})^{-2} = 2^6 = 64.$$

$$48. \left[\left(\frac{3}{2}\right)^{-3}\right]^2 = \left(\frac{3}{2}\right)^{-6} = \left(\frac{2}{3}\right)^6 = \frac{64}{729}.$$

$$49. (x^{\frac{1}{2}})^4 = x^2.$$

$$50. (x^{\frac{1}{3}})^{\frac{1}{2}} = x^{\frac{1}{6}}.$$

$$51. (x^{-\frac{1}{2}})^{-\frac{1}{3}} = x^{\frac{1}{6}}.$$

$$52. (5a^2)^{-3} = \frac{1}{125a^6}.$$

$$53. (c^{-2}d)^2 = c^{-4}d^2.$$

$$54. (5^0 \cdot 2^6 \cdot 3^3)^{\frac{1}{3}} = 2^2 \cdot 3 = 12.$$

$$55. (2x^5)^0 \cdot 8 \cdot 4^{\frac{1}{2}} = 16.$$

$$56. (a^2b)^3(a+b) = a^7b^3 + a^6b^4.$$

$$57. (a^3)^{2x} \cdot (a^2)^{3x} = a^{12x}.$$

$$58. (a^{x+1})^2 \cdot (a^{1-x})^2 = a^4.$$

$$59. (a^3)^{x+y} \cdot (a^2)^{y-x} = a^{x+5y}.$$

$$60. (x^2 + xy^{-2})x^{-2} = 1 + x^{-1}y^{-2}.$$

$$61. 2^2 \cdot 4^2 = 2^2 \cdot 2^4 = 2^6.$$

$$62. 2^n \cdot 4^n = 2^{3n}.$$

$$63. \frac{2^n \cdot 8^{2n}}{4^{3n}} = \frac{2^n \cdot 2^{6n}}{2^{6n}} = 2^n.$$

$$64. 2^n \cdot 4^{n+1} \div 2^n = 2^n \cdot 2^{2n+2} \div 2^n \\ = 2^{2n+2}.$$

$$65. \frac{4^{n+1}}{2^n(4^n-1)^n} \div \frac{8^{n+1}}{(4^{n+1})^{n-1}} + 5 \\ = \frac{2^{2n+2}}{2^n \cdot 2^{2n^2-2n}} \cdot \frac{2^{2n^2-2}}{2^{3n+3}} + 5 \\ = \frac{2^{2n^2+2n}}{2^{2n^2+2n+3}} + 5 \\ = \frac{1}{2^3} + 5 \\ = 5\frac{1}{8}.$$

$$66. 3^6 \cdot 9^n = 81^2.$$

$$3^6 \cdot 3^{2n} = 3^8.$$

$$3^{2n+6} = 3^8.$$

$$2n + 6 = 8.$$

$$n = 1.$$

$$67. 9^n \cdot 3^3 = 27^n.$$

$$3^{2n} \cdot 3^3 = 3^{3n}.$$

$$3^{2n+3} = 3^{3n}.$$

$$2n + 3 = 3n.$$

$$n = 3.$$

$$68. 2^{2n+2} \cdot 4^{n+2} = 8^{2n}.$$

$$2^{2n+2} \cdot 2^{2n+4} = 2^{6n}.$$

$$4n + 6 = 6n.$$

$$n = 3.$$

$$69. 81 \cdot 27^n = (9^n)^{\frac{n}{2}}.$$

$$3^4 \cdot 3^{3n} = 3^{n^2}.$$

$$n^2 = 3n + 4.$$

$$(n-4)(n+1) = 0.$$

$$n = 4, -1.$$

$$70. (25^n)^n = \frac{5^{7n}}{(125)^2}.$$

$$5^{2n^2} = 5^{7n-6}.$$

$$2n^2 = 7n - 6.$$

$$\text{Whence } n = 2, \frac{3}{2}.$$

$$71. 2^{6n+3} \cdot 4^{3n+6} = (8^n)^n.$$

$$2^{6n+3} \cdot 2^{6n+12} = 2^{3n^2}.$$

$$3n^2 - 12n - 15 = 0.$$

$$\text{Whence } n = 5, -1.$$

$$72. x^{\frac{3}{2}} = 8.$$

$$x^3 = 64.$$

$$x = 4.$$

NOTE. In Exercises 72-80, inclusive, only *real* values of x are obtained.

$$73. x^{-\frac{1}{2}} = 2.$$

$$x^{\frac{1}{2}} = \frac{1}{2}.$$

$$x = \frac{1}{4}.$$

$$74. x^{\frac{2}{3}} = 4.$$

$$x^{\frac{1}{3}} = \pm 2.$$

$$x = \pm 8.$$

$$\begin{array}{lll}
 75. & x^{-\frac{2}{3}} = 4. & 77. & x^{\frac{3}{2}} \neq 343. \\
 & x^{-\frac{1}{3}} = \pm 2. & & x^{\frac{1}{2}} = 7. \\
 & x = \pm \frac{1}{8}. & & x = 49. \\
 76. & x^{-\frac{2}{3}} = 32. & 78. & \frac{1}{2} x^{-\frac{2}{3}} = 2. \\
 & x^{-\frac{2}{3}} = 2^5. & & x^{\frac{2}{3}} = \frac{1}{4}. \\
 & x^{\frac{2}{3}} = \frac{1}{2^5}. & & x^{\frac{1}{3}} = \pm \frac{1}{2}. \\
 & x^2 = \frac{1}{2^{15}}. & & x = \pm \frac{1}{8}. \\
 & x = \pm \sqrt{\frac{1}{2^{15}}} = \pm \frac{\sqrt{2}}{256}. & 79. & (ax^{\frac{1}{2}})^{-6} = 27. \\
 & & & a^{-6} x^{-3} = 27. \\
 & & & a^{-2} x^{-1} = 3. \\
 & & & x = \frac{1}{3a^2}.
 \end{array}$$

$$80. \quad \frac{\sqrt[3]{x^{\frac{7}{5}}}}{\sqrt[3]{x^{\frac{1}{5}}}} = \frac{\sqrt[5]{25}}{\sqrt[5]{16}}.$$

$$x^{\frac{2}{5}} = \frac{5^{\frac{2}{5}}}{2^{\frac{4}{5}}}.$$

$$x^{\frac{1}{5}} = \pm \frac{5^{\frac{1}{5}}}{2^{\frac{2}{5}}}.$$

$$x = \pm \frac{5}{2^2}$$

$$= \pm \frac{5}{4}.$$

$$81. \quad \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x - e^{-x})^2}$$

$$= \frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{e^{2x} - 2 + e^{-2x}} = \frac{4}{e^{2x} - 2 + e^{-2x}}.$$

$$82. \quad x \cdot \frac{1}{2} (a^2 - x^2)^{-\frac{1}{2}} (-2x) - (a^2 - x^2)^{\frac{1}{2}} = \frac{-x^2 - a^2 + x^2}{(a^2 - x^2)^{\frac{1}{2}}} = \frac{a^2 \sqrt{a^2 - x^2}}{x^2 - a^2}.$$

$$83. \quad \frac{x^3 \cdot a \cdot x^0 - a \cdot 3x^2}{(x^3)^2} \div \frac{a}{x^3} = \frac{ax^3 - 3ax^2}{x^6} \cdot \frac{x^3}{a} = \frac{x-3}{x}.$$

$$\begin{aligned}
 84. \quad & \frac{x^4(3x^2) - (x^3 + 5)4x^3}{(x^4)^2} \div \frac{x^3 + 20}{x^4} = \frac{3x^6 - 4x^6 - 20x^3}{x^8} \cdot \frac{x^4}{x^3 + 20} \\
 & = \frac{-x^6 - 20x^3}{x^4(x^3 + 20)} \\
 & = -\frac{1}{x}.
 \end{aligned}$$

$$\begin{aligned}
 85. \quad & \frac{(x^2 - 1)ax^{a-1} - x^a \cdot 2x}{(x^2 - 1)^2} = \frac{ax^{a+1} - ax^{a-1} - 2x^{a+1}}{(x^2 - 1)^2} \cdot \frac{x^2 - 1}{x^a} \\
 & = \frac{x^2 - 1}{x^a} \\
 & = \frac{ax - ax^{-1} - 2x}{x^2 - 1}.
 \end{aligned}$$

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$$1. \quad x^4 + 4x^3 - 2x^2 - 12x + 9 \overline{) x^2 + 2x - 3}$$

$$\begin{array}{r}
 x^4 \\
 2x^2 + 2x \quad \overline{) 4x^3} \\
 \quad \quad \quad \overline{) 4x^3 + 4x^2} \\
 2x^2 + 4x - 3 \quad \overline{) -6x^2} \\
 \quad \quad \quad \overline{) -6x^2 - 12x + 9}
 \end{array}$$

$$\begin{array}{r}
 2. \quad a^6 - 10a^4 - 4a^3 + 25a^2 + 20a + 4 \overline{) a^3 - 5a - 2} \\
 \quad \quad \quad \overline{a^6} \\
 \quad \quad \quad 2a^3 - 5a \overline{) -10a^4} \\
 \quad \quad \quad \quad \quad \quad \overline{-10a^4} \quad + 25a^2 \\
 \quad \quad \quad 2a^3 - 10a - 2 \overline{) -4a^3} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \overline{-4a^3} \quad + 20a + 4
 \end{array}$$

$$\begin{array}{r}
 3. \quad t^6 + 4t^5 + 4t^4 - 6t^3 - 12t^2 + 9 \overline{) t^3 + 2t^2 - 3} \\
 \quad \quad \quad \overline{t^6} \\
 \quad \quad \quad 2t^3 + 2t^2 \overline{) 4t^5} \\
 \quad \quad \quad \quad \quad \quad \overline{4t^5 + 4t^4} \\
 \quad \quad \quad 2t^3 + 4t^2 - 3 \overline{) -6t^3} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \overline{-6t^3 - 12t^2 + 9}
 \end{array}$$

$$\begin{array}{r}
 4. \quad 4a^4 + 12a^3t + 5a^2t^2 - 6at^3 + t^4 \overline{) 2a^2 + 3at - t^2} \\
 \quad \quad \quad \overline{4a^4} \\
 \quad \quad \quad 4a^2 + 3at \overline{) 12a^3t} \\
 \quad \quad \quad \quad \quad \quad \overline{12a^3t + 9a^2t^2} \\
 \quad \quad \quad 4a^2 + 6at - t^2 \overline{) -4a^2t^2} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \overline{-4a^2t^2 - 6at^3 + t^4}
 \end{array}$$

$$\begin{array}{r}
 5. \quad 4a^8 + 12a^4 - 7 - 24a^{-4} + 16a^{-8} \overline{) 2a^4 + 3 - 4a^{-4}} \\
 \quad \quad \quad \overline{4a^8} \\
 \quad \quad \quad 4a^4 + 3 \overline{) 12a^4} \\
 \quad \quad \quad \quad \quad \quad \overline{12a^4 + 9} \\
 \quad \quad \quad 4a^4 + 6 - 4a^{-4} \overline{) -16} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \overline{-16 - 24a^{-4} + 16a^{-8}}
 \end{array}$$

$$\begin{array}{r}
 6. \quad 49c^{-6} - 28c^{-4} + 74c^{-2} - 20 + 25c^2 \overline{) 7c^{-3} - 2c^{-1} + 5c} \\
 \quad \quad \quad \overline{49c^{-6}} \\
 \quad \quad \quad 14c^{-3} - 2c^{-1} \overline{) -28c^{-4}} \\
 \quad \quad \quad \quad \quad \quad \overline{-28c^{-4} + 4c^{-2}} \\
 \quad \quad \quad 14c^{-3} - 4c^{-1} + 5c \overline{) 70c^{-2}} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \overline{70c^{-2} - 20 + 25c^2}
 \end{array}$$

$$\begin{array}{r}
 7. \quad 9x^4 - 6x^{\frac{7}{2}} + x^3 - 66x^{\frac{5}{2}} + 22x^2 + 121x \overline{) 3x^2 - x^{\frac{3}{2}} - 11x^{\frac{1}{2}}} \\
 \quad \quad \quad \overline{9x^4} \\
 \quad \quad \quad 6x^2 - x^{\frac{3}{2}} \overline{) -6x^{\frac{7}{2}}} \\
 \quad \quad \quad \quad \quad \quad \overline{-6x^{\frac{7}{2}} + x^3} \\
 \quad \quad \quad 6x^2 - 2x^{\frac{3}{2}} - 11x^{\frac{1}{2}} \overline{) -66x^{\frac{5}{2}}} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \overline{-66x^{\frac{5}{2}} + 22x^2 + 121x}
 \end{array}$$

$$\begin{array}{r}
 8. \quad \frac{25x^3 - 10x^2 + 90x^{\frac{7}{4}} + x - 18x^{\frac{3}{4}} + 81x^{\frac{1}{2}}}{25x^3} \left| \frac{5x^{\frac{3}{2}} - x^{\frac{1}{2}} + 9x^{\frac{1}{4}}}{25x^3} \right. \\
 \hline
 10x^{\frac{3}{2}} - x^{\frac{1}{2}} \left| \begin{array}{l} -10x^2 \\ -10x^2 \end{array} \right. + x \\
 \hline
 10x^{\frac{3}{2}} + 1 + 2x^{\frac{1}{2}} + 9x^{\frac{1}{4}} \left| \begin{array}{l} 90x^{\frac{7}{4}} - 18x^{\frac{3}{4}} + 81x^{\frac{1}{2}} \\ 90x^{\frac{7}{4}} - 18x^{\frac{3}{4}} + 81x^{\frac{1}{2}} \end{array} \right. \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 9. \quad \frac{x^4 + 6x^3 + \frac{29x^2}{3} + 2x + \frac{1}{9}}{x^4} \left| \frac{x^2 + 3x + \frac{1}{3}}{x^4} \right. \\
 \hline
 2x^2 + 3x \left| \begin{array}{l} 6x^3 \\ 6x^3 + 9x^2 \end{array} \right. \\
 \hline
 2x^2 + 6x + \frac{1}{3} \left| \begin{array}{l} 2x^2 \\ 3 \\ \frac{2x^2}{3} + 2x + \frac{1}{9} \end{array} \right. \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 11. \quad \frac{4a^4 + \frac{4a^3}{3} - \frac{35a^2}{9} - \frac{2a}{3} + 1}{4a^4} \left| \frac{2a^2 + \frac{a}{3} - 1}{4a^4} \right. \\
 \hline
 4a^2 + \frac{a}{3} \left| \begin{array}{l} 4a^3 \\ 3 \\ \frac{4a^3}{3} + \frac{a^2}{9} \end{array} \right. \\
 \hline
 4a^2 + \frac{2a}{3} - 1 \left| \begin{array}{l} -4a^2 \\ -4a^2 - \frac{2a}{3} + 1 \end{array} \right. \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 12. \quad \frac{\frac{25x^4}{4} + \frac{10x^3}{3} - \frac{127x^2}{18} - 2x + 2\frac{1}{4}}{\frac{25x^4}{4}} \left| \frac{\frac{5x^2}{2} + \frac{2x}{3} - \frac{3}{2}}{\frac{25x^4}{4}} \right. \\
 \hline
 5x^2 + \frac{2x}{3} \left| \begin{array}{l} 10x^3 \\ 3 \\ \frac{10x^3}{3} + \frac{4x^2}{9} \end{array} \right. \\
 \hline
 5x^2 + \frac{4x}{3} - \frac{3}{2} \left| \begin{array}{l} -\frac{15x^2}{2} \\ -\frac{15x^2}{2} - 2x + 2\frac{1}{4} \end{array} \right. \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 13. \quad \frac{9}{4}a^4 - 2a^3 + \frac{67}{9}a^2 - \frac{28}{9}a + \frac{49}{9} \left| \frac{3a^2}{2} - \frac{2a}{3} + \frac{7}{3} \right. \\
 \hline
 \frac{9a^4}{4} \\
 \hline
 3a^2 - \frac{2a}{3} \left| \begin{array}{l} -2a^3 \\ -2a^3 + \frac{4a^2}{9} \end{array} \right. \\
 \hline
 3a^2 - \frac{4a}{3} + \frac{7}{3} \left| \begin{array}{l} 7a^2 \\ 7a^2 - \frac{28a}{9} + \frac{49}{9} \end{array} \right.
 \end{array}$$

$$\begin{array}{r}
 14. \quad \frac{100a^2}{9} - 20 + \frac{9}{a^2} \left| \frac{10a}{3} - \frac{3}{a} \right. \\
 \hline
 \frac{100a^2}{9} \\
 \hline
 \frac{20a}{3} - \frac{3}{a} \left| \begin{array}{l} -20 \\ -20 + \frac{9}{a^2} \end{array} \right.
 \end{array}$$

$$\begin{array}{r}
 15. \quad \frac{x^2}{a^2} + \frac{5x}{a} + 4\frac{1}{4} - \frac{5a}{x} + \frac{a^2}{x^2} \left| \frac{x}{a} + \frac{5}{2} - \frac{a}{x} \right. \\
 \hline
 \frac{x^2}{a^2} \\
 \hline
 \frac{2x}{a} + \frac{5}{2} \left| \begin{array}{l} \frac{5x}{a} \\ \frac{5x}{a} + \frac{25}{4} \end{array} \right. \\
 \hline
 \frac{2x}{a} + 5 - \frac{a}{x} \left| \begin{array}{l} -2 \\ -2 - \frac{5a}{x} + \frac{a^2}{x^2} \end{array} \right.
 \end{array}$$

$$\begin{array}{r}
 16. \quad \frac{a^2}{b^2} + 2a^2 + a^2b^2 + 2 + 2b^2 + \frac{b^2}{a^2} \left| \frac{a}{b} + ab + \frac{b}{a} \right. \\
 \hline
 \frac{a^2}{b^2} \\
 \hline
 \frac{2a}{b} + ab \left| \begin{array}{l} 2a^2 \\ 2a^2 + a^2b^2 \end{array} \right. \\
 \hline
 \frac{2a}{b} + 2ab + \frac{b}{a} \left| \begin{array}{l} 2 \\ 2 + 2b^2 + \frac{b^2}{a^2} \end{array} \right.
 \end{array}$$

$$\begin{array}{r}
 17. \quad \frac{a^4}{25c^4} + \frac{2a^2}{c^3} + \frac{117}{5c^2} - \frac{40}{a^2c} + \frac{16}{a^4} \left| \frac{a^2}{5c^2} + \frac{5}{c} - \frac{4}{a^2} \right. \\
 \hline
 \frac{a^4}{25c^4} \\
 \frac{2a^2}{5c^2} + \frac{5}{c} \quad \left| \frac{2a^2}{c^3} \right. \\
 \hline
 \frac{2a^2}{5c^2} + \frac{10}{c} - \frac{4}{a^2} \quad \left| \begin{array}{l} \frac{2a^2}{c^3} + \frac{25}{c^2} \\ - \frac{8}{5c^2} \\ - \frac{8}{5c^2} - \frac{40}{a^2c} + \frac{16}{a^4} \end{array} \right.
 \end{array}$$

$$\begin{array}{r}
 18. \quad \frac{a^2}{4c^4} - \frac{3a}{c^2} + 9 + \frac{2}{5ac} - \frac{12c}{5a^2} + \frac{4c^2}{25a^4} \left| \frac{a}{2c^2} - 3 + \frac{2c}{5a^2} \right. \\
 \hline
 \frac{a^2}{4c^4} \\
 \frac{a}{c^2} - 3 \quad \left| \begin{array}{l} - \frac{3a}{c^2} \\ - \frac{3a}{c^2} + 9 \end{array} \right. \\
 \hline
 \frac{a}{c^2} - 6 + \frac{2c}{5a^2} \quad \left| \begin{array}{l} \frac{2}{5ac} \\ \frac{2}{5ac} - \frac{12c}{5a^2} + \frac{4c^2}{25a^4} \end{array} \right.
 \end{array}$$

$$\begin{array}{r}
 19. \quad \frac{a^4}{25x^2} - \frac{6a}{5} + \frac{1}{5x} + \frac{9x^2}{a^2} - \frac{3x}{a^3} + \frac{1}{4a^4} \left| \frac{a^2}{5x} - \frac{3x}{a} + \frac{1}{2a^2} \right. \\
 \hline
 \frac{a^4}{25x^2} \\
 \frac{2a^2}{5x} - \frac{3x}{a} \quad \left| \begin{array}{l} \frac{6a}{5} \\ - \frac{6a}{5} + \frac{9x^2}{a^2} \end{array} \right. \\
 \hline
 \frac{2a^2}{5x} - \frac{6x}{a} + \frac{1}{2a^2} \quad \left| \begin{array}{l} \frac{1}{5x} \\ \frac{1}{5x} - \frac{3x}{a^3} + \frac{1}{4a^4} \end{array} \right.
 \end{array}$$

20.

$$\begin{array}{r} 1 + 2x \overline{) 1 + x - \frac{x^2}{2} + \frac{x^3}{2}} \\ 1 \phantom{+ x - \frac{x^2}{2} + \frac{x^3}{2}} \\ \hline 2 + x \phantom{+ x - \frac{x^2}{2} + \frac{x^3}{2}} \phantom{+ x - \frac{x^2}{2} + \frac{x^3}{2}} \\ \overline{) 2x} \\ 2x + x^2 \\ \hline 2 + 2x - \frac{x^2}{2} \phantom{+ x - \frac{x^2}{2} + \frac{x^3}{2}} \\ \phantom{2 + 2x - \frac{x^2}{2}} \overline{) -x^2} \\ \phantom{2 + 2x - \frac{x^2}{2}} -x^2 - x^3 + \frac{x^4}{4} \\ \hline 2 + 2x - x^2 + \frac{x^3}{2} \phantom{+ x - \frac{x^2}{2} + \frac{x^3}{2}} \phantom{+ x - \frac{x^2}{2} + \frac{x^3}{2}} \\ \phantom{2 + 2x - x^2 + \frac{x^3}{2}} \overline{) x^3 - \frac{x^4}{4}} \end{array}$$

21.

$$\begin{array}{r} \frac{25}{9} + a^3 \overline{) \frac{5}{3} + \frac{3a^3}{10} - \frac{27a^6}{1000} + \frac{243a^9}{50000}} \\ \frac{25}{9} \\ \hline \frac{10}{3} + \frac{3a^3}{10} \phantom{+ x - \frac{x^2}{2} + \frac{x^3}{2}} \phantom{+ x - \frac{x^2}{2} + \frac{x^3}{2}} \\ \phantom{\frac{10}{3} + \frac{3a^3}{10}} \overline{) a^3} \\ \phantom{\frac{10}{3} + \frac{3a^3}{10}} a^3 + \frac{9a^6}{100} \\ \hline \frac{10}{3} + \frac{3a^3}{5} - \frac{27a^6}{1000} \phantom{+ x - \frac{x^2}{2} + \frac{x^3}{2}} \\ \phantom{\frac{10}{3} + \frac{3a^3}{5} - \frac{27a^6}{1000}} \overline{) -\frac{9a^6}{100}} \\ \phantom{\frac{10}{3} + \frac{3a^3}{5} - \frac{27a^6}{1000}} -\frac{9a^6}{100} - \frac{81a^9}{5000} + \frac{729a^{12}}{1000000} \\ \hline \frac{10}{3} + \frac{3a^3}{5} - \frac{27a^6}{500} + \frac{243a^9}{50000} \phantom{+ x - \frac{x^2}{2} + \frac{x^3}{2}} \\ \phantom{\frac{10}{3} + \frac{3a^3}{5} - \frac{27a^6}{500} + \frac{243a^9}{50000}} \overline{) \frac{81a^9}{5000} - \frac{729a^{12}}{1000000}} \end{array}$$

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1.

$$\begin{array}{r} 62'41 \overline{) 79} \\ 49 \\ \hline 149 \overline{) 13 \ 41} \\ 13 \ 41 \\ \hline \end{array}$$

3.

$$\begin{array}{r} 1'51'29 \overline{) 123} \\ 1 \\ \hline 22 \overline{) 51} \\ 44 \\ \hline 243 \overline{) 7 \ 29} \\ 7 \ 29 \\ \hline \end{array}$$

5.

$$\begin{array}{r} 1.46'41 \overline{) 1.21} \\ 1 \\ \hline 22 \overline{) 46} \\ 44 \\ \hline 241 \overline{) 2 \ 41} \\ 2 \ 41 \\ \hline \end{array}$$

2.

$$\begin{array}{r} 92'16 \overline{) 96} \\ 81 \\ \hline 186 \overline{) 11 \ 16} \\ 11 \ 16 \\ \hline \end{array}$$

4.

$$\begin{array}{r} 5'61'69 \overline{) 237} \\ 4 \\ \hline 43 \overline{) 1 \ 61} \\ 1 \ 29 \\ \hline 467 \overline{) 32 \ 69} \\ 32 \ 69 \\ \hline \end{array}$$

6.

$$\begin{array}{r} 2'16.09 \overline{) 14.7} \\ 1 \\ \hline 24 \overline{) 1 \ 16} \\ 96 \\ \hline 287 \overline{) 20 \ 09} \\ 20 \ 09 \\ \hline \end{array}$$

7. $98'80'36 \mid 994$

$$\begin{array}{r}
 81 \\
 189 \overline{) 1780} \\
 \underline{1701} \\
 1984 \overline{) 7936} \\
 \underline{7936}
 \end{array}$$

8. $7'04'90'25 \mid 2655$

$$\begin{array}{r}
 4 \\
 46 \overline{) 304} \\
 \underline{276} \\
 525 \overline{) 2890} \\
 \underline{2625} \\
 5305 \overline{) 26525} \\
 \underline{26525}
 \end{array}$$

9. $3.96'01 \mid 1.99$

$$\begin{array}{r}
 1 \\
 29 \overline{) 296} \\
 \underline{261} \\
 389 \overline{) 3501} \\
 \underline{3501}
 \end{array}$$

10. $7 \mid 2.645+$

$$\begin{array}{r}
 4 \\
 46 \overline{) 300} \\
 \underline{276} \\
 524 \overline{) 2400} \\
 \underline{2096} \\
 5285 \overline{) 30400} \\
 \underline{26425}
 \end{array}$$

11. $.63 \mid .793+$

$$\begin{array}{r}
 49 \\
 149 \overline{) 1400} \\
 \underline{1341} \\
 1583 \overline{) 5900} \\
 \underline{4749}
 \end{array}$$

12. $.01'23'50 \mid .111+$

$$\begin{array}{r}
 1 \\
 21 \overline{) 23} \\
 \underline{21} \\
 221 \overline{) 250} \\
 \underline{221}
 \end{array}$$

13. $.96'38'40 \mid .981+$

$$\begin{array}{r}
 81 \\
 188 \overline{) 1538} \\
 \underline{1504} \\
 1961 \overline{) 3240} \\
 \underline{1961}
 \end{array}$$

14. $\frac{5}{7} = .714285+.$

$.71'42'85 \mid .845+$

$$\begin{array}{r}
 64 \\
 164 \overline{) 742} \\
 \underline{656} \\
 1685 \overline{) 8685} \\
 \underline{8425}
 \end{array}$$

15. $4\frac{2}{9} = 4.222222+.$

$4.22'22'22 \mid 2.054+$

$$\begin{array}{r}
 4 \\
 405 \overline{) 2222} \\
 \underline{2025} \\
 4104 \overline{) 19722} \\
 \underline{16416}
 \end{array}$$

16. $1\frac{0}{7} = 1.428571+.$

$1.42'85'71 \mid 1.195+$

$$\begin{array}{r}
 1 \\
 21 \overline{) 42} \\
 \underline{21} \\
 229 \overline{) 2185} \\
 \underline{2061} \\
 2385 \overline{) 12471} \\
 \underline{11925}
 \end{array}$$

17. $\frac{4}{3} = .800000.$

$$\begin{array}{r} .80'00'00 \mid .894 + \\ 64 \\ 169 \mid 16 \ 00 \\ 15 \ 21 \\ 1784 \mid 79 \ 00 \\ 71 \ 36 \end{array}$$

18. $23\frac{6}{11} = 23.545454 + .$

$$\begin{array}{r} 23.54'54'54 \mid 4.852 + \\ 16 \\ 88 \mid 7 \ 54 \\ 7 \ 04 \\ 965 \mid 50 \ 54 \\ 48 \ 25 \\ 9702 \mid 2 \ 29 \ 54 \\ 1 \ 94 \ 04 \end{array}$$

19. $89\frac{1}{3} = 89.333333.$

$$\begin{array}{r} 89.33'33'33 \mid 9.451 + \\ 81 \\ 184 \mid 8 \ 33 \\ 7 \ 36 \\ 1885 \mid 97 \ 33 \\ 94 \ 25 \\ 18901 \mid 3 \ 08 \ 33 \\ 1 \ 89 \ 01 \end{array}$$

20. $\overline{136}^2 + \overline{273}^2$

$$= 18,496 + 74,529$$

$$= 93,025.$$

$$.9'30'25 \mid 305$$

$$\begin{array}{r} 9 \\ 605 \mid 30 \ 25 \\ 30 \ 25 \end{array}$$

21. $\sqrt{90^2 + 90^2} = 90\sqrt{2}$

$$= 90 \cdot 1.4142$$

$$= 127.28 \text{ feet.}$$

22. $\overline{207}^2 - \overline{83}^2 = 42,849 - 6889$

$$= 35,960.$$

$$3'59'60 \mid 189.63$$

$$\begin{array}{r} 1 \\ 28 \mid 2 \ 59 \\ 2 \ 24 \\ 369 \mid 35 \ 60 \\ 33 \ 21 \\ 3786 \mid 2 \ 39 \ 00 \\ 2 \ 27 \ 16 \\ 37923 \mid 11 \ 84 \ 00 \\ 11 \ 37 \ 69 \end{array}$$

23. $\overline{5471}^2 - \overline{4059}^2$

$$= 29,931,841 - 16,475,481$$

$$= 13,456,360.$$

$$13'45'63'60 \mid 3668.2 +$$

$$\begin{array}{r} 9 \\ 66 \mid 4 \ 45 \\ 3 \ 96 \\ 726 \mid 49 \ 63 \\ 43 \ 56 \\ 7328 \mid 6 \ 07 \ 60 \\ 5 \ 86 \ 24 \\ 73362 \mid 21 \ 36 \ 00 \\ 14 \ 67 \ 24 \end{array}$$

24. $\sqrt{(17)^2 - (\frac{17}{2})^2} = \frac{17}{2}\sqrt{3}$

$$= 8.5(1.7320)$$

$$= 14.722$$

$$= 14.7 \text{ inches.}$$

25. $s^2 - \left(\frac{s}{2}\right)^2 = (15)^2.$

$$s = 10\sqrt{3}$$

$$= 17.320 \text{ inches.}$$

26. $\sqrt{37 \cdot 2 \cdot 10 \cdot 25} = 10\sqrt{185}$

$$= 10(13.60147)$$

$$= 136.0147 \text{ square inches}$$

$$= .945 \text{ square feet.}$$

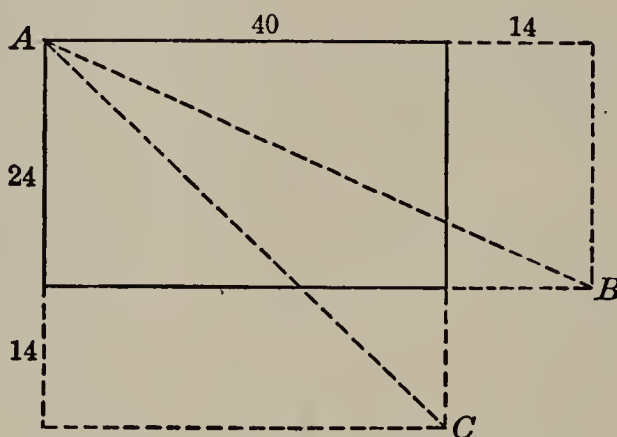
$$\begin{aligned}
 27. \quad \sqrt{33 \cdot 11 \cdot 11 \cdot 11} &= 121\sqrt{3} \\
 &= 121(1.73205) \\
 &= 209.57 \text{ square inches.}
 \end{aligned}$$

$$\begin{aligned}
 28. \quad \pi r^2 &= 70. \\
 r &= \sqrt{70 \div 3.1416} \\
 &= 4.72 \text{ feet.}
 \end{aligned}$$

$$\begin{aligned}
 29. \quad \text{Diagonal} &= \sqrt{15^2 + 22^2 + 28^2} \\
 &= 38.63 \text{ feet.}
 \end{aligned}$$

$$\begin{aligned}
 30. \quad \text{Diagonal} &= \sqrt{8^2 + 8^2 + 8^2} \\
 &= 8\sqrt{3} \\
 &= 13.85 \text{ feet.}
 \end{aligned}$$

31. Folding back one end wall and one side wall shows that the shortest line is either AB or AC .



$$AB = \sqrt{(40 + 14)^2 + (24)^2} = \sqrt{3492}.$$

$$AC = \sqrt{(24 + 14)^2 + (40)^2} = \sqrt{3044}.$$

Therefore $AC = 2\sqrt{761} = 2(27.586) = 55.172$. *Ans.*

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1. $\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$.
2. $\sqrt{28} = \sqrt{4 \cdot 7} = 2\sqrt{7}$.
3. $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$.
4. $\sqrt{52} = \sqrt{4 \cdot 13} = 2\sqrt{13}$.
5. $\sqrt{68} = \sqrt{4 \cdot 17} = 2\sqrt{17}$.
6. $\sqrt{108} = \sqrt{36 \cdot 3} = 6\sqrt{3}$.
7. $\sqrt{192} = \sqrt{64 \cdot 3} = 8\sqrt{3}$.
8. $2\sqrt{45} = 2\sqrt{9 \cdot 5} = 6\sqrt{5}$.
9. $\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = 2\sqrt[3]{2}$.
10. $4\sqrt[3]{54} = 4\sqrt[3]{27 \cdot 2} = 12\sqrt[3]{2}$.
11. $\sqrt[4]{48} = \sqrt[4]{16 \cdot 3} = 2\sqrt[4]{3}$.
12. $3\sqrt[5]{64} = 3\sqrt[5]{32 \cdot 2} = 6\sqrt[5]{2}$.

13. $\sqrt{a^3} = \sqrt{a^2 \cdot a} = a\sqrt{a}$.
14. $\sqrt{ax^3} = \sqrt{x^2(ax)} = x\sqrt{ax}$.
15. $\sqrt[3]{a^2x^5} = \sqrt[3]{x^3(a^2x^2)} = x\sqrt[3]{a^2x^2}$.
16. $5\sqrt[3]{16a^4} = 5\sqrt[3]{8a^3 \cdot 2a} = 10a\sqrt[3]{2a}$.
18. $\sqrt{\frac{2}{5}} = \sqrt{\frac{10}{25}} = \frac{1}{5}\sqrt{10}$.
19. $\sqrt{\frac{3}{7}} = \sqrt{\frac{21}{49}} = \frac{1}{7}\sqrt{21}$.
20. $\sqrt[3]{\frac{1}{4}} = \sqrt[3]{\frac{2}{8}} = \frac{1}{2}\sqrt[3]{2}$.
21. $\sqrt[3]{\frac{2}{9}} = \sqrt[3]{\frac{6}{27}} = \frac{1}{3}\sqrt[3]{6}$.
23. $\sqrt{\frac{1}{2x}} = \sqrt{\frac{2x}{4x^2}} = \frac{1}{2x}\sqrt{2x}$.
24. $\sqrt{\frac{1}{a^3}} = \sqrt{\frac{a}{a^4}} = \frac{1}{a^2}\sqrt{a}$.

$$25. \sqrt[3]{\frac{1}{a^2}} = \sqrt[3]{\frac{a}{a^3}} = \frac{1}{a} \sqrt[3]{a}.$$

$$26. \sqrt[3]{\frac{1}{ax^3}} = \sqrt[3]{\frac{a^2}{a^3x^3}} = \frac{1}{ax} \sqrt[3]{a^2}.$$

$$27. \sqrt[3]{-\frac{3}{4}} = \sqrt[3]{-\frac{6}{8}} = -\frac{1}{2} \sqrt[3]{6}.$$

$$31. \sqrt{s^3 + \left(\frac{s}{2}\right)^3} = \sqrt{\frac{9s^3}{8}} = \frac{3s}{4} \sqrt{2s}.$$

$$32. \sqrt{\left(\frac{a+1}{2}\right)^2 - a} = \sqrt{\frac{a^2 - 2a + 1}{4}} = \frac{a-1}{2}.$$

$$33. \sqrt{\left(\frac{e^x + e^{-x}}{2}\right)^2 - 1} = \sqrt{\frac{e^{2x} - 2 + e^{-2x}}{4}} = \frac{e^x - e^{-x}}{2}.$$

$$34. \sqrt{4 - 8\sqrt{3}} = \sqrt{4(1 - 2\sqrt{3})} = 2\sqrt{1 - 2\sqrt{3}}.$$

$$35. \sqrt{36 + 18\sqrt{5}} = 3\sqrt{4 + 2\sqrt{5}}.$$

$$36. \sqrt[3]{16 - 8\sqrt{3}} = 2\sqrt[3]{2 - \sqrt{3}}.$$

$$37. \sqrt[3]{54 - 9\sqrt{18}} = \sqrt[3]{54 - 27\sqrt{2}} = 3\sqrt[3]{2 - \sqrt{2}}.$$

$$38. \sqrt{R^2 + 3R^2\sqrt{5}} = R\sqrt{1 + 3\sqrt{5}}.$$

$$39. \sqrt{R^2 - \frac{R^2}{2}\sqrt{3}} = \sqrt{\frac{4R^2 - 2R^2\sqrt{3}}{4}} = \frac{R}{2}\sqrt{4 - 2\sqrt{3}}.$$

$$41. \sqrt[4]{9} = 3^{\frac{2}{4}} = 3^{\frac{1}{2}} = \sqrt{3}.$$

$$42. \sqrt[4]{a^2b^4} = a^{\frac{1}{2}}b = b\sqrt{a}.$$

$$43. \sqrt[4]{64a^2b^4} = 2^{\frac{6}{4}}a^{\frac{2}{4}}b \\ = b\sqrt{2^3a} \\ = 2b\sqrt{2a}.$$

$$44. \sqrt[6]{9a^2} = 3^{\frac{1}{3}}a^{\frac{1}{3}} = \sqrt[3]{3a}.$$

$$45. \sqrt[6]{\frac{4a^6}{25}} = a\sqrt[3]{\frac{2}{5}} = \frac{a}{5}\sqrt[3]{50}.$$

$$47. 3\sqrt{5} = \sqrt{45}.$$

$$48. 2\sqrt[3]{8} = \sqrt[3]{64}.$$

$$49. a\sqrt{a} = \sqrt{a^3}.$$

$$50. 2c\sqrt[3]{c^2} = \sqrt[3]{8c^5}.$$

$$51. 4\sqrt[3]{\frac{1}{4}} = \sqrt[3]{16}.$$

$$52. \frac{a}{3}\sqrt[3]{\frac{9}{a^2}} = \sqrt[3]{\frac{a}{3}}.$$

$$28. \sqrt{1 - \left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9}} = \frac{2}{3}\sqrt{2}.$$

$$29. \sqrt{3^2 - \left(\frac{3}{2}\right)^2} = \sqrt{\frac{27}{4}} = \frac{3}{2}\sqrt{3}.$$

$$30. \sqrt{s^2 - \left(\frac{s}{2}\right)^2} = \sqrt{\frac{3s^2}{4}} = \frac{s}{2}\sqrt{3}.$$

$$53. e^x \sqrt{e^x + e^{-x}} = \sqrt{e^{3x} + e^x}.$$

$$54. (a+1)\sqrt{\frac{1}{a^2-1}} = \sqrt{\frac{a+1}{a-1}}.$$

$$55. \frac{x-3a}{5}\sqrt[3]{\frac{125}{(x-3a)^2}} = \sqrt[3]{x-3a}.$$

$$57. \sqrt{\sqrt{a}} = (a^{\frac{1}{2}})^{\frac{1}{2}} = a^{\frac{1}{4}} = \sqrt[4]{a}.$$

$$58. \sqrt[3]{\sqrt{a}} = (a^{\frac{1}{2}})^{\frac{1}{3}} = \sqrt[6]{a}.$$

$$59. \sqrt{\sqrt[3]{a^2}} = (a^{\frac{2}{3}})^{\frac{1}{2}} = \sqrt[3]{a}.$$

$$60. \sqrt{\sqrt{x^3}} = (x^{\frac{3}{2}})^{\frac{1}{2}} = \sqrt[4]{x^3}.$$

$$61. \sqrt[3]{\sqrt{x^5}} = (x^{\frac{5}{2}})^{\frac{1}{3}} = \sqrt[6]{x^5}.$$

$$62. \sqrt[3]{\sqrt{8a^2x}} = ((2^3a^2x)^{\frac{1}{2}})^{\frac{1}{3}} \\ = \sqrt[6]{8a^2x}.$$

$$63. \sqrt{3\sqrt{3}} = (3 \cdot 3^{\frac{1}{2}})^{\frac{1}{2}} = \sqrt[4]{27}.$$

$$64. \sqrt[4]{\sqrt{8}} = ((8)^{\frac{1}{2}})^{\frac{1}{4}} = \sqrt[8]{8}.$$

$$65. 2 \sqrt[3]{2 \sqrt[3]{2}} = 2 \cdot 2^{\frac{1}{3}} \cdot 2^{\frac{1}{9}} = 2 \sqrt[9]{16}. \quad 67. \sqrt[n]{x^{\frac{a}{n}}} = \left(x^{\frac{a}{n}}\right)^{\frac{n}{m}} = \sqrt[m]{x^a}.$$

$$66. \sqrt[n]{\sqrt[n]{x^c}} = \left((x^c)^{\frac{1}{n}}\right)^{\frac{1}{n}} = \sqrt[n]{x^c}.$$

$$68. \sqrt{12 \cdot 2 \cdot 4 \cdot 6} = \sqrt{6^2 \cdot 2^2 \cdot 4} = 24.$$

$$69. \sqrt{28 \cdot 3 \cdot 4 \cdot 21} = \sqrt{7^2 \cdot 3^2 \cdot 4^2} = 84.$$

$$70. \sqrt{77 \cdot 44 \cdot 21 \cdot 12} = \sqrt{7 \cdot 11^2 \cdot 4 \cdot 3 \cdot 7 \cdot 4 \cdot 3} = 11 \cdot 4 \cdot 3 \cdot 7 = 924.$$

$$71. \sqrt{221 \cdot 36 \cdot 68 \cdot 117} = \sqrt{13 \cdot 17 \cdot 36 \cdot 4 \cdot 17 \cdot 9 \cdot 13} \\ = 13 \cdot 17 \cdot 6 \cdot 2 \cdot 3 \\ = 7956.$$

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$$2. \sqrt{20} + 3\sqrt{5} = 2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}.$$

$$3. \sqrt{50} + \sqrt{98} - \sqrt{32} = 5\sqrt{2} + 7\sqrt{2} - 4\sqrt{2} = 8\sqrt{2}.$$

$$4. \sqrt{12} + 5\sqrt{75} - 2\sqrt{27} = 2\sqrt{3} + 25\sqrt{3} - 6\sqrt{3} = 21\sqrt{3}.$$

$$5. \sqrt{75} + 3\sqrt{147} - \sqrt{12} = 5\sqrt{3} + 21\sqrt{3} - 2\sqrt{3} = 24\sqrt{3}.$$

$$6. 2\sqrt{54} + \sqrt{24} - \sqrt{96} = 6\sqrt{6} + 2\sqrt{6} - 4\sqrt{6} = 4\sqrt{6}.$$

$$7. \sqrt[3]{16} + \sqrt[3]{54} - 3\sqrt[3]{2} = 2\sqrt[3]{2} + 3\sqrt[3]{2} - 3\sqrt[3]{2} = 2\sqrt[3]{2}.$$

$$8. \sqrt[3]{192} - 4\sqrt[3]{24} + \sqrt[3]{375} = 4\sqrt[3]{3} - 8\sqrt[3]{3} + 5\sqrt[3]{3} = \sqrt[3]{3}.$$

$$9. \sqrt[3]{625} + \sqrt[3]{40} + \sqrt[3]{135} = 5\sqrt[3]{5} + 2\sqrt[3]{5} + 3\sqrt[3]{5} = 10\sqrt[3]{5}.$$

$$10. 10\sqrt{\frac{6}{5}} - \sqrt{\frac{3}{10}} + \sqrt{\frac{15}{2}} = 2\sqrt{30} - \frac{1}{10}\sqrt{30} + \frac{1}{2}\sqrt{30} = \frac{15}{5}\sqrt{30}.$$

$$11. a\sqrt{x^3} - \sqrt{a^2x} - 5\sqrt{a^2x} = ax\sqrt{x} - a\sqrt{x} - 5a\sqrt{x} = (ax - 6a)\sqrt{x}.$$

$$12. \sqrt{x^3} + \sqrt[4]{x^2} - 12\sqrt[6]{x^3} = x\sqrt{x} + \sqrt{x} - 12\sqrt{x} = (x - 11)\sqrt{x}.$$

$$13. \sqrt{\frac{a}{x^3}} - \sqrt{\frac{a}{x^5}} + \sqrt{\frac{5x^3}{a}} = \frac{1}{x^2}\sqrt{ax} - \frac{1}{x^3}\sqrt{ax} + \frac{x}{a}\sqrt{5ax}.$$

$$14. \sqrt{\frac{6}{7}} + \sqrt{\frac{3}{14}} - \sqrt{\frac{2}{21}} = \frac{1}{7}\sqrt{42} + \frac{1}{14}\sqrt{42} - \frac{1}{2}\sqrt{42} = -\frac{2}{7}\sqrt{42}.$$

$$15. \sqrt[4]{32x^5} + \sqrt[4]{1250x} - \sqrt[4]{512x} - \sqrt[4]{2x} = 2x\sqrt[4]{2x} + 5\sqrt[4]{2x} - 4\sqrt[4]{2x} - \sqrt[4]{2x} \\ = 2x\sqrt[4]{2x}.$$

$$16. \sqrt{(a+c)^3} - c\sqrt[4]{(a+c)^2} + 2c\sqrt[6]{(a+c)^3} = (a+c)\sqrt{a+c} - c\sqrt{a+c} + 2c\sqrt{a+c} \\ = (a+2c)\sqrt{a+c}.$$

$$17. \sqrt[3]{(a-c)^4} + c\sqrt[6]{a^2 - 2ac + c^2} + (a+c)\sqrt[3]{a-c} \\ = (a-c)\sqrt[3]{a-c} + c\sqrt[3]{a-c} + (a+c)\sqrt[3]{a-c} = (2a+c)\sqrt[3]{a-c}.$$

$$18. \sqrt{\frac{a}{c}} - \sqrt{\frac{c}{a}} + \sqrt{\frac{a^2 + c^2}{ac}} + 2 - \sqrt{\frac{a^2 + c^2}{ac}} - 2 \\ = \frac{1}{c}\sqrt{ac} - \frac{1}{a}\sqrt{ac} + \frac{a+c}{ac}\sqrt{ac} - \frac{a-c}{ac}\sqrt{ac} \\ = \frac{a+c}{ac}\sqrt{ac}.$$

$$19. \sqrt[3]{24} + \sqrt[3]{(3a+9)(a+3)^2} - \sqrt[3]{81} + a\sqrt[6]{9} - 4\sqrt[3]{3} \\ = 2\sqrt[3]{3} + (a+3)\sqrt[3]{3} - 3\sqrt[3]{3} + a\sqrt[3]{3} - 4\sqrt[3]{3} = (2a-2)\sqrt[3]{3}.$$

$$20. 2\sqrt{9a^3-9a^2b} - 3\sqrt{9ab^2-9b^3} + \sqrt{(a^2-b^2)(a+b)} \\ = 6a\sqrt{a-b} - 9b\sqrt{a-b} + (a+b)\sqrt{a-b} \\ = (7a-8b)\sqrt{a-b}.$$

$$21. (a-b)\sqrt{\frac{a+b}{a-b}} + \sqrt{25a^2-25b^2} + \frac{a+b}{a-b}\sqrt{\frac{36ab^2-36b^3}{a+b}} \\ = \sqrt{a^2-b^2} + 5\sqrt{a^2-b^2} + \frac{6b}{a-b}\sqrt{a^2-b^2} \\ = \frac{6a}{a-b}\sqrt{a^2-b^2}.$$

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$$1. \sqrt{12}\sqrt{18} = \sqrt{216} = 6\sqrt{6}.$$

$$2. \sqrt{\frac{2}{3}} \cdot \sqrt{\frac{3}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2}.$$

$$3. \sqrt{\frac{4}{5}} \cdot \sqrt{\frac{5}{6}} \cdot \sqrt{\frac{3}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2}.$$

$$4. (\sqrt{2} - 3\sqrt{5})\sqrt{5} = \sqrt{10} - 15.$$

$$5. \begin{array}{r} \sqrt{3} - 2\sqrt{2} \\ - \sqrt{3} + \sqrt{2} \\ \hline -3 + 2\sqrt{6} \\ -4 + \sqrt{6} \\ \hline -7 + 3\sqrt{6} \end{array}$$

$$6. \begin{array}{r} \sqrt{5} - 3\sqrt{2} \\ 2\sqrt{5} - \sqrt{3} \\ \hline 10 - 6\sqrt{10} - \sqrt{15} + 3\sqrt{6} \end{array}$$

$$7. \begin{array}{r} \sqrt{a} - \sqrt{ax} \\ \sqrt{a} + 2\sqrt{ax} \\ \hline a - a\sqrt{x} \\ + 2a\sqrt{x} - 2ax \\ \hline a + a\sqrt{x} - 2ax \end{array}$$

$$8. \begin{array}{r} 3\sqrt{5} - \sqrt{2} \\ 3\sqrt{5} + \sqrt{2} \\ \hline 45 - 3\sqrt{10} \\ -2 + 3\sqrt{10} \\ \hline 43 \end{array}$$

$$9. \begin{array}{r} 4\sqrt{5} + 2\sqrt{7} \\ 4\sqrt{5} - 2\sqrt{7} \\ \hline 80 + 8\sqrt{35} \\ -28 - 8\sqrt{35} \\ \hline 52 \end{array}$$

$$10. \begin{array}{r} \sqrt{3a} - \sqrt{2a} \\ \sqrt{3a} - \sqrt{2a} \\ \hline 3a - a\sqrt{6} \\ 2a - a\sqrt{6} \\ \hline 5a - 2a\sqrt{6} \end{array}$$

$$11. (2\sqrt{3x-1})^2 = 12x - 4.$$

$$12. \begin{array}{r} \sqrt{x} - \sqrt{x-2} \\ \sqrt{x} - \sqrt{x-2} \\ \hline x - \sqrt{x^2-2x} \\ x-2 - \sqrt{x^2-2x} \\ \hline 2x-2-2\sqrt{x^2-2x} \end{array}$$

$$13. \begin{array}{r} \sqrt{5} - \sqrt{3} - \sqrt{2} \\ \sqrt{5} + \sqrt{3} - \sqrt{2} \\ \hline 5 - \sqrt{15} - \sqrt{10} \\ -3 + \sqrt{15} - \sqrt{6} \\ \hline 2 - \sqrt{10} + \sqrt{6} \\ \hline 4 - 2\sqrt{10} \end{array}$$

$$\begin{array}{r}
 14. \quad \frac{3\sqrt{2} + 2\sqrt{3} + \sqrt{30}}{2\sqrt{2} + 2\sqrt{3} - 2\sqrt{5}} \\
 \frac{12 + 4\sqrt{6} + 4\sqrt{15}}{12 + 6\sqrt{6} \quad \quad \quad + 6\sqrt{10}} \\
 \frac{-10\sqrt{6} - 4\sqrt{15} \quad -6\sqrt{10}}{24}
 \end{array}$$

$$\begin{array}{r}
 15. \quad R - \frac{R}{2}\sqrt{3} \\
 \frac{2R + \frac{3R}{2}\sqrt{3}}{2R^2 - R^2\sqrt{3}} \\
 - \frac{9R^2}{4} + \frac{3R^2}{2}\sqrt{3} \\
 \frac{-\frac{R^2}{4} + \frac{R^2}{2}\sqrt{3}}{\quad}
 \end{array}$$

$$16. \left(\frac{R}{2}\right)^2 + \left(\frac{R}{2} - \frac{R}{2}\sqrt{3}\right)^2 = \frac{R^2}{4} + \frac{R^2}{4} - \frac{R^2}{2}\sqrt{3} + \frac{3R^2}{4} = \frac{5R^2}{4} - \frac{R^2}{2}\sqrt{3}.$$

$$\begin{aligned}
 17. \quad R^2 - \left(\frac{R}{2}\sqrt{5} - \frac{R}{2}\right)^2 &= R^2 - \frac{5R^2}{4} + \frac{R^2\sqrt{5}}{2} - \frac{R^2}{4} \\
 &= \frac{R^2}{2}\sqrt{5} - \frac{R^2}{2} \\
 &= \frac{R^2}{2}(\sqrt{5} - 1).
 \end{aligned}$$

$$18. (\sqrt{2} - \sqrt{x-3})^2 = 2 - 2\sqrt{2x-6} + x - 3 = x - 1 - 2\sqrt{2x-6}.$$

$$\begin{aligned}
 19. \quad (\sqrt{x-3} + \sqrt{x+5})^2 &= x - 3 + 2\sqrt{x^2 + 2x - 15} + x + 5 \\
 &= 2x + 2 + 2\sqrt{x^2 + 2x - 15}.
 \end{aligned}$$

$$\begin{aligned}
 20. \quad (2\sqrt{x} - 3\sqrt{2x+1})^2 &= 4x - 12\sqrt{2x^2+x} + 18x + 9 \\
 &= 22x + 9 - 12\sqrt{2x^2+x}.
 \end{aligned}$$

$$\begin{aligned}
 21. \quad (3\sqrt{x-1} + 2\sqrt{2-x})^2 &= 9x - 9 + 12\sqrt{-x^2 + 3x - 2} + 8 - 4x \\
 &= 5x - 1 + 12\sqrt{-x^2 + 3x - 2}.
 \end{aligned}$$

$$22. (a + \sqrt{a+b})(a - \sqrt{a+b}) = a^2 - a - b.$$

$$23. (\sqrt{2x-3} - \sqrt{3x})(\sqrt{2x-3} + \sqrt{3x}) = 2x - 3 - 3x = -x - 3.$$

$$24. \sqrt{2}, \sqrt[3]{3} = 2^{\frac{1}{2}}, 3^{\frac{1}{3}} = 2^{\frac{2}{6}}, 3^{\frac{2}{6}} = \sqrt[6]{8}, \sqrt[6]{9} \text{ respectively.}$$

$$25. \sqrt[4]{ax}, \sqrt{a} = \sqrt[4]{ax}, \sqrt[4]{a^2} \text{ respectively.}$$

$$26. \sqrt[3]{9}, \sqrt{8} = 9^{\frac{2}{6}}, 8^{\frac{3}{6}} = \sqrt[6]{81}, 2\sqrt[6]{8} \text{ respectively.}$$

$$27. \sqrt{3}\sqrt[3]{3} = \sqrt[6]{3^3 \cdot 3^2} = \sqrt[6]{243}.$$

$$28. \sqrt{a}\sqrt[3]{a} = \sqrt[6]{a^3 \cdot a^2} = \sqrt[6]{a^5}.$$

$$29. \sqrt[4]{a^3}\sqrt{a} = \sqrt[4]{a^3 \cdot a^2} = a\sqrt[4]{a}.$$

$$30. \sqrt[3]{12}\sqrt{\frac{1}{8}} = \sqrt[6]{144 \cdot \frac{1}{2}} = \sqrt[6]{\frac{9}{2}} = \frac{1}{2}\sqrt[6]{18}.$$

$$31. \sqrt[3]{a^2}\sqrt{a^3} = \sqrt[6]{a^4a^9} = a^2\sqrt[6]{a}.$$

$$32. \sqrt{\frac{x}{a}}\sqrt[3]{\frac{a}{x}} = \sqrt[6]{\frac{x^3}{a^3} \cdot \frac{a^2}{x^2}} = \sqrt[6]{\frac{x}{a}} = \frac{\sqrt[6]{a^5x}}{a}.$$

$$33. \sqrt[4]{2x^3}\sqrt{3x} = \sqrt[4]{2x^3 \cdot 9x^2} = x\sqrt[4]{18x}.$$

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1. $\sqrt{7}$.
2. $\sqrt{5}$.
3. $\sqrt{2}$.
4. $\sqrt[3]{2}$.
5. $\sqrt[3]{25}$.
6. $\sqrt[4]{2}$.
7. $\sqrt{3} + 2$.
8. $3\sqrt{6} + 2\sqrt{11}$.
9. $\sqrt{3a} + \sqrt{2x}$.
10. $\sqrt{x-3} + 2\sqrt{3x}$.
11. $\sqrt{a-b} - \sqrt{a+b}$.
12. $3\sqrt{2} + \sqrt{5}$.
13. $\sqrt{8} \div \sqrt{2} = \sqrt{4} = 2$.
14. $6\sqrt{10} \div \sqrt{5} = 6\sqrt{2}$.
15. $\sqrt{8} \div \sqrt{24} = \sqrt{\frac{1}{3}} = \frac{1}{3}\sqrt{3}$.
16. $\sqrt{2ax} \div \sqrt{3a^2x} = \sqrt{\frac{2}{3a}} = \frac{1}{3a}\sqrt{6a}$.
17. $(2\sqrt{10} - 3\sqrt{5}) \div 2\sqrt{5} = \sqrt{2} - \frac{3}{2}$.
18. $(\sqrt{ax^2} - \sqrt{a^2x}) \div \sqrt{ax} = \sqrt{x} - \sqrt{a}$.
19. $8 \div 4\sqrt{3} = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}$.
20. $24 \div 3\sqrt{3} = \frac{8}{\sqrt{3}} = \frac{8}{3}\sqrt{3}$.
21. $\sqrt{5} \div \sqrt[3]{3} = \sqrt[6]{\frac{5^3}{3^2}} = \frac{1}{3}\sqrt[6]{10,125}$.
22. $\sqrt{a} \div \sqrt[4]{2} = \sqrt[4]{\frac{a^2}{2}} = \frac{1}{2}\sqrt[4]{8a^2}$.
23. $\sqrt[4]{8} \div \sqrt{2} = \sqrt[4]{\frac{8}{4}} = \sqrt[4]{2}$.
24. $\sqrt[3]{a} \div \sqrt[6]{a} = \sqrt[6]{a^2 \div a} = \sqrt[6]{a}$.
25. $\sqrt[3]{\frac{1}{4}} \div \sqrt{\frac{1}{2}} = \sqrt[6]{\frac{1}{16} \div \frac{1}{8}} = \sqrt[6]{\frac{1}{2}} = \frac{1}{2}\sqrt[6]{32}$.
26. $\frac{\sqrt{3}}{\sqrt{3}-2} = \frac{\sqrt{3}(\sqrt{3}+2)}{(\sqrt{3}-2)(\sqrt{3}+2)} = \frac{3+2\sqrt{3}}{3-4} = -3-2\sqrt{3}$.
27. $\frac{\sqrt{5}}{\sqrt{5}+\sqrt{2}} = \frac{\sqrt{5}(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{5-\sqrt{10}}{5-2} = \frac{5-\sqrt{10}}{3}$.
28. $\frac{2\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}} = \frac{(2\sqrt{3}+\sqrt{5})(\sqrt{3}+\sqrt{5})}{(\sqrt{3}-\sqrt{5})(\sqrt{3}+\sqrt{5})} = \frac{11+3\sqrt{15}}{3-5} = -\frac{11+3\sqrt{15}}{2}$.
29. $\frac{\sqrt{3}}{2\sqrt{6}-\sqrt{3}} = \frac{\sqrt{3}(2\sqrt{6}+\sqrt{3})}{(2\sqrt{6}-\sqrt{3})(2\sqrt{6}+\sqrt{3})} = \frac{6\sqrt{2}+3}{21} = \frac{2\sqrt{2}+1}{7}$.
30. $\frac{2\sqrt{5}+3\sqrt{7}}{3\sqrt{5}-2\sqrt{7}} = \frac{(2\sqrt{5}+3\sqrt{7})(3\sqrt{5}+2\sqrt{7})}{(3\sqrt{5}-2\sqrt{7})(3\sqrt{5}+2\sqrt{7})} = \frac{72+13\sqrt{35}}{17}$.
31. $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{(\sqrt{a}+\sqrt{b})^2}{a-b} = \frac{a+b+2\sqrt{ab}}{a-b}$.
32. $\frac{\sqrt{x-3}+\sqrt{3}}{\sqrt{x-3}-\sqrt{3}} = \frac{(\sqrt{x-3}+\sqrt{3})^2}{x-3-3} = \frac{x+2\sqrt{3x-9}}{x-6}$.

$$34. \frac{\sqrt{a+x}}{\sqrt{5}-\sqrt{a+x}} = \frac{\sqrt{a+x}(\sqrt{5}+\sqrt{a+x})}{5-a-x} = \frac{\sqrt{5a+5x+a+x}}{5-a-x}.$$

$$35. \frac{4}{\sqrt{2}-\sqrt{2}} = \frac{4\sqrt{2+\sqrt{2}}}{\sqrt{(2-\sqrt{2})(2+\sqrt{2})}} = \frac{4\sqrt{2+\sqrt{2}}}{\sqrt{4-2}} = \frac{4\sqrt{4+2\sqrt{2}}}{2} \\ = 2\sqrt{4+2\sqrt{2}}.$$

$$36. \frac{\sqrt{10}-\sqrt{5}}{\sqrt{10}+\sqrt{5}} = \frac{(\sqrt{10}-\sqrt{5})^2}{10-5} = \frac{15-2\sqrt{50}}{5} = 3-2\sqrt{2}.$$

$$37. \frac{2-\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(2-\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} \\ = -5-2\sqrt{2}+2\sqrt{3}+2\sqrt{6}.$$

$$38. \frac{\sqrt{5}+\sqrt{7}-\sqrt{2}}{\sqrt{5}-\sqrt{7}} = \frac{(\sqrt{5}+\sqrt{7}-\sqrt{2})(\sqrt{5}+\sqrt{7})}{(\sqrt{5}-\sqrt{7})(\sqrt{5}+\sqrt{7})} \\ = \frac{-12-2\sqrt{35}+\sqrt{10}+\sqrt{14}}{2}.$$

39. No.

$$40. (3-\sqrt{7})^2 - 6(3-\sqrt{7}) + 2 = 16 - 6\sqrt{7} - 18 + 6\sqrt{7} + 2 = 0. \text{ Yes.}$$

$$41. 2\left(\frac{7-\sqrt{3}}{2}\right)^2 - 75\left(\frac{7-\sqrt{3}}{2}\right) + 161 \\ = \frac{52-14\sqrt{3}}{2} - \frac{525-75\sqrt{3}}{2} + 161, \text{ which does not equal zero. No.}$$

$$42. 3\left(\frac{5\pm\sqrt{109}}{6}\right)^2 - 5\left(\frac{5\pm\sqrt{109}}{6}\right) - 7 \\ = \frac{134\pm 10\sqrt{109}}{12} - \frac{50\pm 10\sqrt{109}}{12} - 7 = \frac{84}{12} - 7 = 0. \text{ Yes.}$$

Page 354

$$1. \sqrt{8-2\sqrt{15}} = \sqrt{3-2\sqrt{15}+5} = \sqrt{5}-\sqrt{3}.$$

$$2. \sqrt{5-2\sqrt{6}} = \sqrt{3-2\sqrt{6}+2} = \sqrt{3}-\sqrt{2}.$$

$$3. \sqrt{13+\sqrt{48}} = \sqrt{13+2\sqrt{12}} = \sqrt{12+2\sqrt{12}+1} = \sqrt{12}+1 = 2\sqrt{3}+1.$$

$$4. \sqrt{7-\sqrt{40}} = \sqrt{7-2\sqrt{10}} = \sqrt{5-2\sqrt{10}+2} = \sqrt{5}-\sqrt{2}.$$

$$5. \sqrt{16-6\sqrt{7}} = \sqrt{16-2\sqrt{63}} = \sqrt{7-2\sqrt{63}+9} = 3-\sqrt{7}.$$

$$6. \sqrt{17+12\sqrt{2}} = \sqrt{17+2\sqrt{72}} = \sqrt{9+2\sqrt{72}+8} = 2\sqrt{2}+3.$$

$$7. \sqrt{11-3\sqrt{8}} = \sqrt{11-2\sqrt{18}} = \sqrt{9-2\sqrt{18}+2} = 3-\sqrt{2}.$$

8. $\sqrt{11 - \sqrt{120}} = \sqrt{11 - 2\sqrt{30}} = \sqrt{6 - 2\sqrt{30} + 5} = \sqrt{6} - \sqrt{5}.$
9. $\sqrt{65x - 20\sqrt{3}x^2} = \sqrt{65x - 2\sqrt{300}x^2}$
 $= \sqrt{5x - 2\sqrt{300}x^2 + 60x}$
 $= 2\sqrt{15x} - \sqrt{5x}.$
10. $\sqrt{126a - 10a\sqrt{5}} = \sqrt{126a - 2\sqrt{125}a^2}$
 $= \sqrt{125a - 2\sqrt{125}a^2 + a}$
 $= 5\sqrt{5a} - \sqrt{a}.$
11. $\sqrt{\frac{13a}{4} - \sqrt{3}a^2} = \sqrt{\frac{13a}{4} - 2\sqrt{\frac{3a^2}{4}}} = \sqrt{3a - 2\sqrt{\frac{3a^2}{4}} + \frac{a}{4}}$
 $= \sqrt{3a} - \frac{1}{2}\sqrt{a}.$
12. $\sqrt{2x + 2\sqrt{x^2 - 49}} = \sqrt{x + 7 + 2\sqrt{x^2 - 49} + x - 7} = \sqrt{x + 7} + \sqrt{x - 7}.$
13. $\sqrt{a + \sqrt{a^2 - 1}} = \sqrt{a + 2\sqrt{\frac{a^2 - 1}{4}}} = \sqrt{\frac{a - 1}{2} + 2\sqrt{\frac{a^2 - 1}{4}} + \frac{a + 1}{2}}$
 $= \sqrt{\frac{a - 1}{2}} + \sqrt{\frac{a + 1}{2}} = \frac{1}{2}\sqrt{2a + 2} + \frac{1}{2}\sqrt{2a - 2}.$
14. $\sqrt{9 + 3\sqrt{8}} = \sqrt{9 + 2\sqrt{18}} = \sqrt{6 + 2\sqrt{18} + 3} = \sqrt{6} + \sqrt{3}.$
15. $\sqrt{15 - 5\sqrt{8}} = \sqrt{15 - 2\sqrt{50}} = \sqrt{5 - 2\sqrt{50} + 10} = \sqrt{10} - \sqrt{5}.$
16. $\sqrt{a + \sqrt{a^2 - 4b^2}} = \sqrt{a + 2\sqrt{\frac{a^2 - 4b^2}{4}}} = \sqrt{\frac{a - 2b}{2} + 2\sqrt{\frac{a^2 - 4b^2}{4}} + \frac{a + 2b}{2}}$
 $= \sqrt{\frac{a - 2b}{2}} + \sqrt{\frac{a + 2b}{2}} = \frac{1}{2}\sqrt{2a - 4b} + \frac{1}{2}\sqrt{2a + 4b}.$
17. $\sqrt{m^2 + m + 2n + 2m\sqrt{m + 2n}} = \sqrt{m^2 + m + 2n + 2\sqrt{m^2(m + 2n)}}$
 $= \sqrt{m + 2n + 2\sqrt{m^2(m + 2n)}} + m^2 = \sqrt{m + 2n} + m.$

Page 355

1. $x^2 - 11 = (x - \sqrt{11})(x + \sqrt{11}).$
2. $3x^2 - 8 = (x\sqrt{3} - 2\sqrt{2})(x\sqrt{3} + 2\sqrt{2}).$
3. $x^3 + 3 = (x + \sqrt[3]{3})(x^2 - x\sqrt[3]{3} + \sqrt[3]{9}).$
4. $x^3 - 12 = (x - \sqrt[3]{12})(x^2 + x\sqrt[3]{12} + 2\sqrt[3]{18}).$
5. $3x^3 - 27 = 3(x - \sqrt[3]{9})(x^2 + x\sqrt[3]{9} + 3\sqrt[3]{3}).$
6. $5x^3 + 125 = 5(x + \sqrt[3]{25})(x^2 - x\sqrt[3]{25} + 5\sqrt[3]{5}).$

$$7. \frac{2\sqrt{b}}{a-b} + \frac{2}{\sqrt{a} + \sqrt{b}} = \frac{2\sqrt{b}}{a-b} + \frac{2\sqrt{a} - 2\sqrt{b}}{a-b} = \frac{2\sqrt{a}}{a-b}.$$

$$\begin{aligned} 8. \frac{x+c}{\sqrt{x}-\sqrt{c}} - \frac{x^{\frac{3}{2}} + c^{\frac{3}{2}}}{x-c} &= \frac{(x+c)(\sqrt{x} + \sqrt{c}) - x^{\frac{3}{2}} - c^{\frac{3}{2}}}{x-c} \\ &= \frac{x^{\frac{3}{2}} + x\sqrt{c} + c\sqrt{x} + c^{\frac{3}{2}} - x^{\frac{3}{2}} - c^{\frac{3}{2}}}{x-c} \\ &= \frac{x\sqrt{c} + c\sqrt{x}}{x-c}. \end{aligned}$$

$$\begin{aligned} 9. \quad & x^2 - 5 = 0. \\ & (x - \sqrt{5})(x + \sqrt{5}) = 0. \\ & x = \sqrt{5}, -\sqrt{5}. \end{aligned}$$

$$\begin{aligned} 10. \quad & 2x^2 - 3 = 0. \\ & (x\sqrt{2} - \sqrt{3})(x\sqrt{2} + \sqrt{3}) = 0. \\ & x\sqrt{2} - \sqrt{3} = 0. \\ & x = \frac{\sqrt{3}}{\sqrt{2}} = \frac{1}{2}\sqrt{6}. \\ & x\sqrt{2} + \sqrt{3} = 0. \\ & x = -\frac{\sqrt{3}}{\sqrt{2}} = -\frac{1}{2}\sqrt{6}. \end{aligned}$$

$$\begin{aligned} 11. \quad & x^4 - 26x^2 + 144 = 0. \\ & (x^2 - 8)(x^2 - 18) = 0. \\ & (x - \sqrt{8})(x + \sqrt{8})(x - \sqrt{18})(x + \sqrt{18}) = 0. \\ & (x - 2\sqrt{2})(x + 2\sqrt{2})(x - 3\sqrt{2})(x + 3\sqrt{2}) = 0. \\ & x = 2\sqrt{2}, -2\sqrt{2}, 3\sqrt{2}, -3\sqrt{2}. \end{aligned}$$

$$\begin{aligned} 12. \quad & 4x^4 + c = x^2 + 4cx^2. \\ & 4x^4 - 4cx^2 - x^2 + c = 0. \\ & 4x^2(x^2 - c) - 1(x^2 - c) = 0. \\ & (4x^2 - 1)(x^2 - c) = 0. \\ & (2x - 1)(2x + 1)(x + \sqrt{c})(x - \sqrt{c}) = 0. \\ & x = \frac{1}{2}, -\frac{1}{2}, -\sqrt{c}, \sqrt{c}. \end{aligned}$$

Page 356

$$1. \quad \text{Altitude} = \sqrt{8^2 - 4^2} \\ = 4\sqrt{3}.$$

$$2. \quad \text{Altitude} = \sqrt{s^2 - \left(\frac{s}{2}\right)^2} \\ = \frac{s}{2} \sqrt{3}. \\ \text{Area} = \frac{1}{2} \left(s \cdot \frac{s}{2} \sqrt{3} \right) \\ = \frac{s^2}{4} \sqrt{3}.$$

3. Let s represent the side.

$$\text{Then } s^2 - \frac{s^2}{4} = 576.$$

$$s = 16\sqrt{3}.$$

$$\text{Area} = \frac{24 \cdot 16\sqrt{3}}{2} \\ = 192\sqrt{3}.$$

$$4. \quad s^2 - \frac{s^2}{4} = a^2. \\ s = \frac{2a}{3} \sqrt{3}.$$

$$5. \quad x^2 + y^2 = 121. \quad (1)$$

$$x^2 + (20 - y)^2 = 169. \quad (2)$$

$$(2) - (1), \\ 400 - 40y = 48. \quad (3)$$

$$\text{From (3), } y = 8.8. \quad (4)$$

$$\text{From (4) and (1),} \\ x = 6.6. \quad (5)$$

$$\text{Area} = \frac{20 \cdot 6.6}{2} \\ = 66.$$

6. Referring to the Hint to Exercise 5 of text,

$$x^2 + y^2 = 100. \quad (1)$$

$$x^2 + (16 - y)^2 = 144. \quad (2)$$

$$(2) - (1), 256 - 32y = 44. \quad (3)$$

$$\text{From (3), } y = \frac{53}{8}. \quad (4)$$

$$\text{From (4) and (1),} \\ x = \frac{3}{8} \sqrt{399} \\ = 7.49, \text{ or } 7.5 \text{ nearly.}$$

7. (a) Referring to the figure on page 356 of text,

$$OK = \sqrt{18^2 - 9^2} \\ = 9\sqrt{3}.$$

$$\text{Area of } \triangle ABO = \frac{18 \cdot 9\sqrt{3}}{2} \\ = 81\sqrt{3}.$$

$$\text{Area of the hexagon} \\ = 6 \cdot 81\sqrt{3} \\ = 486\sqrt{3}.$$

(b) Referring to the figure on page 356 of text,

$$OK = \sqrt{s^2 - \left(\frac{s}{2}\right)^2} \\ = \frac{s}{2} \sqrt{3}.$$

$$\text{Area of } \triangle ABO = \frac{s^2}{4} \sqrt{3}.$$

$$\text{Area of the hexagon} \\ = 6 \cdot \frac{s^2}{4} \sqrt{3} \\ = \frac{3s^2}{2} \sqrt{3}.$$

8. (a) Referring to the figure on page 356 of text,

$$s^2 - \left(\frac{s}{2}\right)^2 = 30^2.$$

$$s = 20\sqrt{3}.$$

$$\text{Area of } \triangle ABO = \frac{30 \cdot 20\sqrt{3}}{2} \\ = 300\sqrt{3}.$$

$$\text{Area of the hexagon} \\ = 6 \cdot 300\sqrt{3} \\ = 1800\sqrt{3}.$$

(b) Referring to the figure on page 356 of text,

$$s^2 - \left(\frac{s}{2}\right)^2 = h^2.$$

$$s = \frac{2h}{3} \sqrt{3}.$$

$$\text{Area of } \triangle ABO = \frac{1}{2} h \cdot \frac{2h}{3} \sqrt{3}$$

$$= \frac{h^2}{3} \sqrt{3}.$$

$$\text{Area of the hexagon}$$

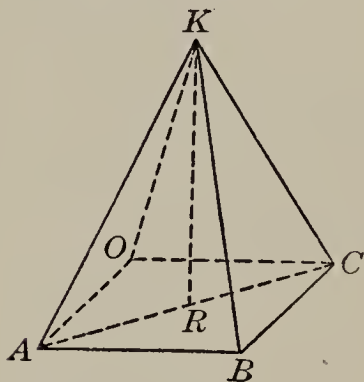
$$= 6 \cdot \frac{h^2}{3} \sqrt{3}$$

$$= 2h^2 \sqrt{3}.$$

$$9. \quad AC = \sqrt{10^2 + 10^2}$$

$$= 10\sqrt{2}.$$

$$AR = 5\sqrt{2}.$$



$$\text{Altitude} = KR$$

$$= \sqrt{(20)^2 - (5\sqrt{2})^2}$$

$$= \sqrt{350} = 5\sqrt{14}.$$

$$\text{Volume} = \frac{1}{3} \cdot 5\sqrt{14} \cdot 10^2$$

$$= \frac{500}{3} \sqrt{14}.$$

$$10. \text{ Altitude} = \sqrt{36^2 - 18^2}$$

$$= 18\sqrt{3}.$$

$$\text{One part is } \frac{2}{3} \cdot 18\sqrt{3}.$$

$$\frac{2}{3} \cdot 18\sqrt{3} = 12\sqrt{3}.$$

$$\text{The other part is } \frac{1}{3} \cdot 18\sqrt{3}.$$

$$\frac{1}{3} \cdot 18\sqrt{3} = 6\sqrt{3}.$$

11. Referring to the figure on page 357 of text,

$$CR = \sqrt{24^2 - 12^2}$$

$$= 12\sqrt{3}.$$

$$CK = \frac{2}{3} \cdot 12\sqrt{3} = 8\sqrt{3}.$$

$$KD = \sqrt{24^2 - (8\sqrt{3})^2}$$

$$= 8\sqrt{6}.$$

12. Referring to the figure on page 357 of text,

$$CR = 15\sqrt{3}.$$

$$CK = \frac{2}{3} \cdot 15\sqrt{3}$$

$$= 10\sqrt{3}.$$

$$KD = \sqrt{30^2 - (10\sqrt{3})^2}$$

$$= 10\sqrt{6}.$$

$$\text{Area of } \triangle ABC = \frac{15\sqrt{3} \cdot 30}{2}$$

$$= 225\sqrt{3}.$$

$$\text{Volume} = \frac{10\sqrt{6} \cdot 225\sqrt{3}}{3}$$

$$= 2250\sqrt{2}.$$

13. Referring to the figure on page 357 of text,

$$CR = \sqrt{e^2 - \left(\frac{e}{2}\right)^2}$$

$$= \frac{e}{2} \sqrt{3}.$$

$$CK = \frac{2}{3} \cdot \frac{e}{2} \sqrt{3}$$

$$= \frac{e}{3} \sqrt{3}.$$

$$KD = \sqrt{e^2 - \left(\frac{e}{3} \sqrt{3}\right)^2}$$

$$= \frac{e}{3} \sqrt{6}.$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot \frac{e}{2} \sqrt{3} \cdot e$$

$$= \frac{e^2}{4} \sqrt{3}.$$

$$\text{Volume} = \frac{1}{3} \left(\frac{e}{3} \sqrt{6} \cdot \frac{e^2}{4} \sqrt{3} \right)$$

$$= \frac{e^3}{12} \sqrt{2}.$$

Page 357

$$1. \frac{3\sqrt{5}}{\sqrt{3}} = \frac{3\sqrt{5}\sqrt{3}}{\sqrt{3}\sqrt{3}} = \sqrt{15}.$$

$$2. \frac{2}{3\sqrt{7}} = \frac{2\sqrt{7}}{3(\sqrt{7})^2} = \frac{2}{21}\sqrt{7}.$$

$$3. \frac{2}{\sqrt{3}-\sqrt{2}} = \frac{2(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} = 2\sqrt{3}+2\sqrt{2}.$$

$$4. \frac{2\sqrt{3}+3\sqrt{5}}{2\sqrt{3}-3\sqrt{5}} = \frac{(2\sqrt{3}+3\sqrt{5})^2}{(2\sqrt{3}-3\sqrt{5})(2\sqrt{3}+3\sqrt{5})} = -\frac{57+12\sqrt{15}}{33}$$

$$= -\frac{19+4\sqrt{15}}{11}.$$

$$5. \frac{a\sqrt{m}-b\sqrt{c}}{a\sqrt{m}+b\sqrt{c}} = \frac{(a\sqrt{m}-b\sqrt{c})^2}{a^2m-b^2c} = \frac{a^2m+b^2c-2ab\sqrt{cm}}{a^2m-b^2c}.$$

$$6. \frac{a-\sqrt{a^2-1}}{a+\sqrt{a^2-1}} = \frac{(a-\sqrt{a^2-1})^2}{a^2-a^2+1} = \frac{a^2-2a\sqrt{a^2-1}+a^2-1}{1}$$

$$= 2a^2-2a\sqrt{a^2-1}-1.$$

$$7. \sqrt{28+10\sqrt{3}} = \sqrt{28+2\sqrt{75}} = \sqrt{25+2\sqrt{75}+3} = 5+\sqrt{3}.$$

$$8. \sqrt{57-12\sqrt{15}} = \sqrt{57-2\sqrt{540}} = \sqrt{45-2\sqrt{540}+12} = \sqrt{45}-\sqrt{12}$$

$$= 3\sqrt{5}-2\sqrt{3}.$$

$$9. \sqrt{c^4+c+2c^2\sqrt{c}} = \sqrt{c^4+c+2\sqrt{c^5}} = \sqrt{c^4+2\sqrt{c^5}+c}$$

$$= \sqrt{c^4}+\sqrt{c} = c^2+\sqrt{c}.$$

$$10. \left(\frac{7+3\sqrt{5}}{2}\right)^2 - 7\left(\frac{7+3\sqrt{5}}{2}\right) + 1 = \frac{49+42\sqrt{5}+45}{4} - \frac{49+21\sqrt{5}}{2} + 1$$

$$= \frac{94}{4} - \frac{49}{2} + 1 = 0.$$

$$11. 3\left(\frac{-3\pm 2\sqrt{3}}{3}\right)^2 + 6\left(\frac{-3\pm 2\sqrt{3}}{3}\right) = \frac{9\mp 12\sqrt{3}+12}{3} + \frac{-18\pm 12\sqrt{3}}{3}$$

$$= \frac{21}{3} - \frac{18}{3} = 1.$$

$$\begin{aligned}
 12. \quad & 3a \left(\frac{-ab + \sqrt{60ab + a^2b^2}}{6a} \right)^2 + ab \left(\frac{-ab + \sqrt{60ab + a^2b^2}}{6a} \right) \\
 &= \frac{a^2b^2 - 2ab\sqrt{60ab + a^2b^2} + 60ab + a^2b^2}{12a} + \frac{-a^2b^2 + ab\sqrt{60ab + a^2b^2}}{6a} \\
 &= \frac{2a^2b^2 + 60ab}{12a} - \frac{a^2b^2}{6a} = \frac{60ab}{12a} = 5b.
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & 3\sqrt{\frac{2}{5}} + 2\sqrt{\frac{1}{10}} - 4\sqrt{\frac{5}{2}} + \sqrt{640} = \frac{3}{5}\sqrt{10} + \frac{1}{5}\sqrt{10} - 2\sqrt{10} + 8\sqrt{10} \\
 &= \frac{3+1-2+8}{5}\sqrt{10}.
 \end{aligned}$$

$$14. \quad \frac{2 + \sqrt{3}}{\sqrt{6}} = \frac{2\sqrt{6} + 3\sqrt{2}}{6}.$$

$$15. \quad \frac{5\sqrt{7} - \sqrt{2}}{\sqrt{2} + \sqrt{7}} = \frac{(5\sqrt{7} - \sqrt{2})(\sqrt{2} - \sqrt{7})}{(\sqrt{2} + \sqrt{7})(\sqrt{2} - \sqrt{7})} = -\frac{6\sqrt{14} - 37}{5} = \frac{37 - 6\sqrt{14}}{5}.$$

$$\begin{aligned}
 16. \quad & 3\sqrt{\frac{1}{6}} + 5\sqrt{24} + \frac{1}{3}\sqrt{96} - \sqrt{66\frac{2}{3}} = \frac{1}{2}\sqrt{6} + 10\sqrt{6} + \frac{4}{3}\sqrt{6} - \frac{10}{3}\sqrt{6} \\
 &= \frac{1+7}{2}\sqrt{6}.
 \end{aligned}$$

$$17. \quad \frac{\sqrt{x}}{\sqrt{x} - \sqrt{a}} - \frac{\sqrt{a}}{\sqrt{x} + \sqrt{a}} = \frac{x + \sqrt{ax} - \sqrt{ax} + a}{x - a} = \frac{x + a}{x - a}.$$

$$\begin{aligned}
 18. \quad & \sqrt{40} - \frac{1}{\sqrt{10}} + \sqrt{\frac{2}{5}} + 7\sqrt{\frac{5}{2}} = 2\sqrt{10} - \frac{1}{10}\sqrt{10} + \frac{1}{5}\sqrt{10} + \frac{7}{2}\sqrt{10} \\
 &= \frac{2+8}{5}\sqrt{10}.
 \end{aligned}$$

$$19. \quad \sqrt{(10)^2 - (2\sqrt{2})^2} = \sqrt{100 - 8} = \sqrt{92} = 2\sqrt{23}.$$

$$\begin{aligned}
 20. \quad & \sqrt{1 - \left(\frac{\sqrt{2} - 2}{2}\right)^2} = \sqrt{\frac{4 - (2 - 4\sqrt{2} + 4)}{4}} = \sqrt{\frac{-2 + 4\sqrt{2}}{4}} \\
 &= \frac{1}{2}\sqrt{4\sqrt{2} - 2}.
 \end{aligned}$$

$$21. \quad \sqrt{R^2 - \left(\frac{R}{3}\sqrt{2}\right)^2} = \sqrt{\frac{9R^2 - 2R^2}{9}} = \sqrt{\frac{7R^2}{9}} = \frac{R}{3}\sqrt{7}.$$

$$\begin{aligned}
 22. \quad \sqrt{(2R)^2 - (R - R\sqrt{2})^2} &= \sqrt{4R^2 - (R^2 - 2\sqrt{2}R^2 + 2R^2)} \\
 &= \sqrt{R^2 + 2\sqrt{2}R^2} = R\sqrt{1 + 2\sqrt{2}}.
 \end{aligned}$$

$$\begin{aligned}
 23. \quad \sqrt{(R - R\sqrt{3})^2 - (2R - R\sqrt{2})^2} \\
 &= \sqrt{R^2 - 2\sqrt{3}R^2 + 3R^2 - (4R^2 - 4\sqrt{2}R^2 + 2R^2)} \\
 &= \sqrt{-2R^2 - 2\sqrt{3}R^2 + 4\sqrt{2}R^2} \\
 &= R\sqrt{-2(1 + \sqrt{3} - 2\sqrt{2})}.
 \end{aligned}$$

$$\begin{aligned}
 24. \quad \sqrt{R^2 - \left(\frac{R}{2}\sqrt{5} - \frac{R}{2}\right)^2} &= \sqrt{R^2 - \left(\frac{5R^2}{4} - \frac{2R^2}{4}\sqrt{5} + \frac{R^2}{4}\right)} \\
 &= \sqrt{\frac{-2R^2}{4} + \frac{2R^2}{4}\sqrt{5}} \\
 &= \frac{R}{2}\sqrt{2(\sqrt{5} - 1)}.
 \end{aligned}$$

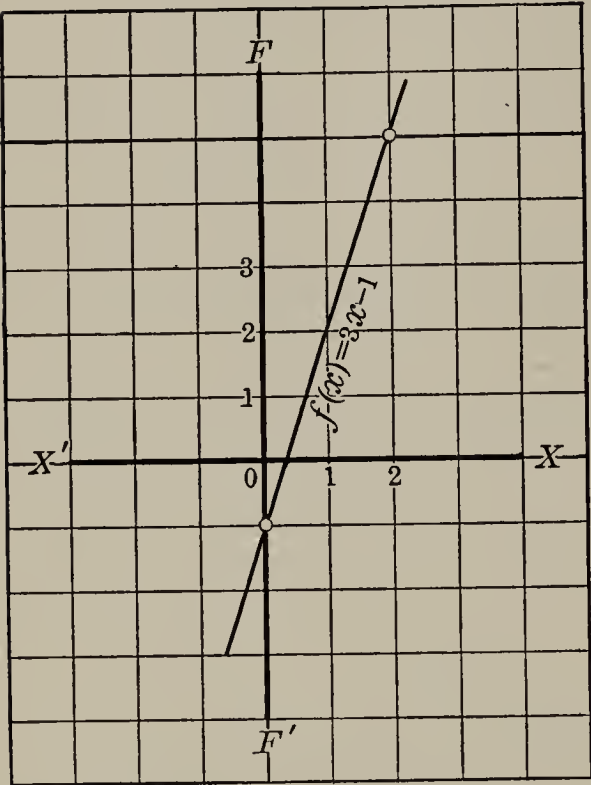
$$\begin{aligned}
 25. \quad \sqrt{\left(e^x + \frac{2}{e^x}\right)^2 - (e^x - 2e^{-x})^2 + e^{2x} + e^{-2x} - 6} \\
 &= \sqrt{e^{2x} + 4 + 4e^{-2x} - (e^{2x} - 4 + 4e^{-2x}) + e^{2x} + e^{-2x} - 6} \\
 &= \sqrt{e^{2x} + 2 + e^{-2x}} = e^x + e^{-x}.
 \end{aligned}$$

$$\begin{aligned}
 26. \quad \frac{b\sqrt{ax - x^2} - bx(ax - x^2)^{-\frac{1}{2}}(a - 2x)}{(\sqrt{ax - x^2})^2} &\div \frac{bx}{\sqrt{ax - x^2}} \\
 &= \frac{b\sqrt{ax - x^2} - \frac{bx}{\sqrt{ax - x^2}}(a - 2x)}{bx(\sqrt{ax - x^2})} \\
 &= \frac{\sqrt{ax - x^2} - \frac{x(a - 2x)}{\sqrt{ax - x^2}}}{x(\sqrt{ax - x^2})} = \frac{1}{x} - \frac{a - 2x}{ax - x^2} \\
 &= \frac{a - x - (a - 2x)}{ax - x^2} = \frac{x}{ax - x^2} = \frac{1}{a - x}.
 \end{aligned}$$

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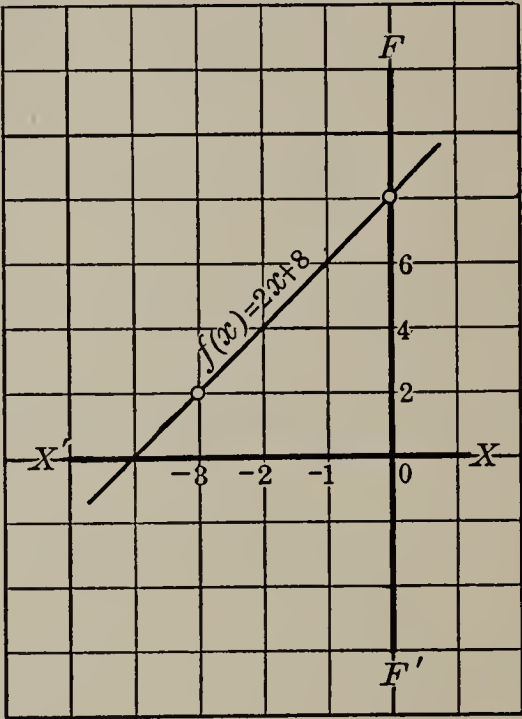
1. $f(x) = 3x - 1$.

If $x =$	0	2
then $f(x) =$	-1	5



2. $f(x) = 2x + 8$.

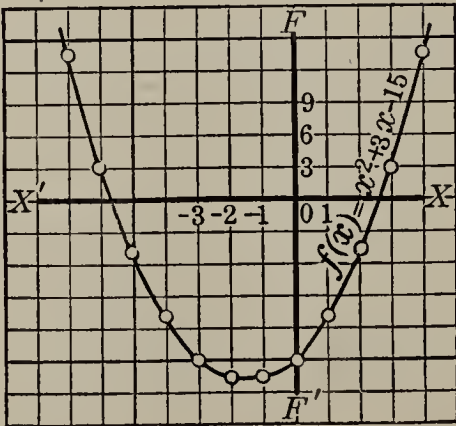
If $x =$	0	-3
then $f(x) =$	8	2



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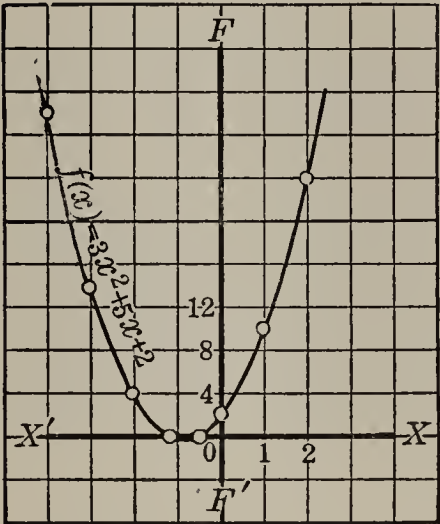
1. $f(x) = x^2 + 3x - 15$.

If $x =$	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
then $f(x) =$	13	3	-5	-11	-15	-17	-17	-15	-11	-5	3	13



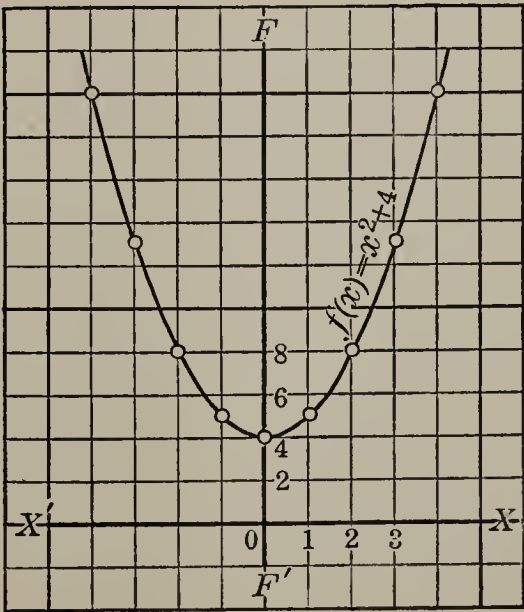
2. $f(x) = 3x^2 + 5x + 2$.

If $x =$	-4	-3	-2	-1	$-\frac{2}{3}$	0	1	2
then $f(x) =$	30	14	4	0	0	2	10	24



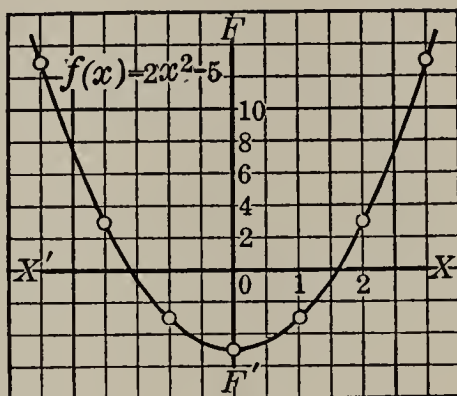
3. $f(x) = x^2 + 4$.

If $x =$	-4	-3	-2	-1	0	1	2	3	4
then $f(x) =$	20	13	8	5	4	5	8	13	20



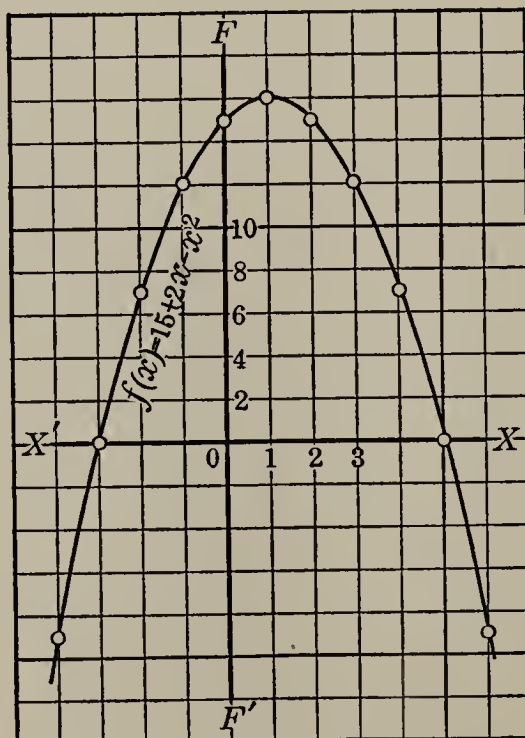
4. $f(x) = 2x^2 - 5.$

If $x =$	- 3	- 2	- 1	0	1	2	3
then $f(x) =$	13	3	- 3	- 5	- 3	3	13



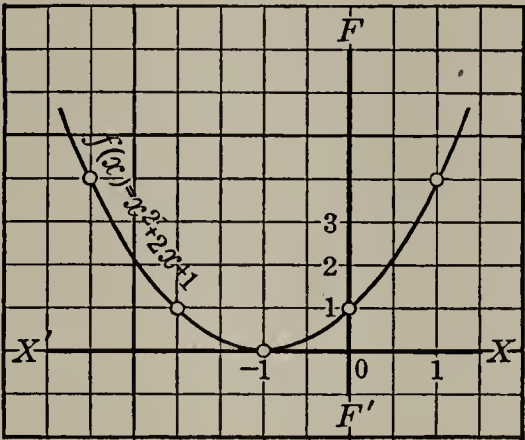
5. $f(x) = 15 + 2x - x^2.$

If $x =$	- 4	- 3	- 2	- 1	0	1	2	3	4	5	6
then $f(x) =$	- 9	0	7	12	15	16	15	12	7	0	- 9



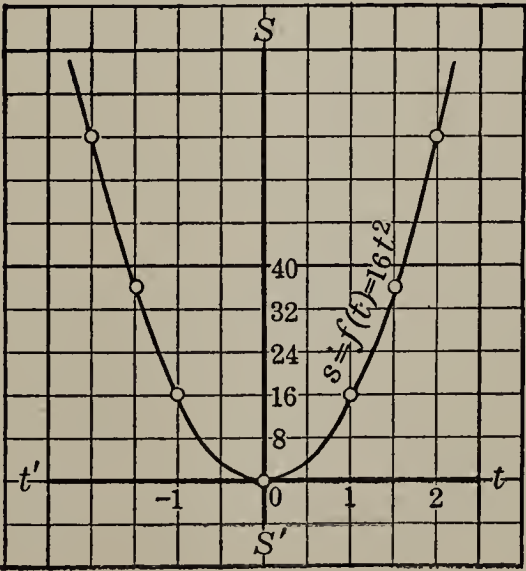
6. $f(x) = x^2 + 2x + 1$.

If $x =$	- 3	- 2	- 1	0	1
then $f(x) =$	4	1	0	1	4



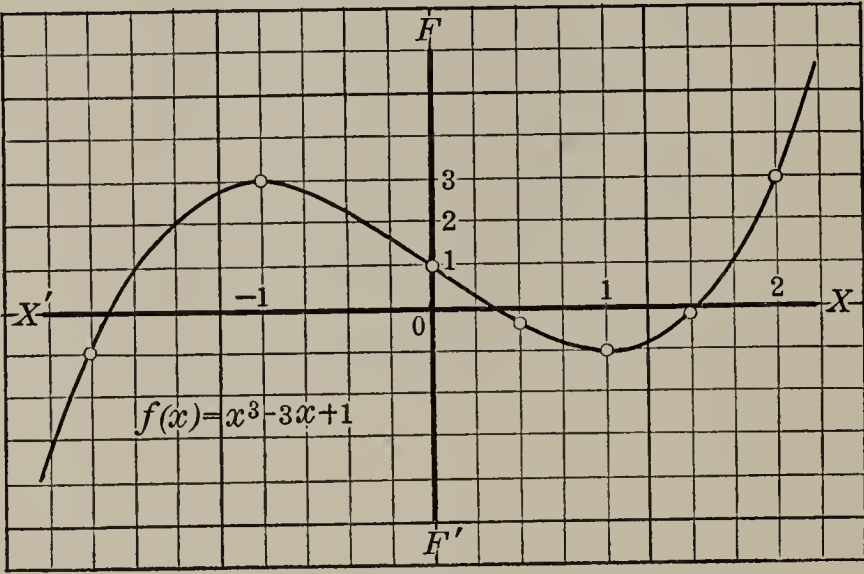
7. $s = f(t) = 16t^2$.

If $t =$	- 2	- 1.5	- 1	0	1	1.5	2
then $f(t) =$	64	36	16	0	16	36	64



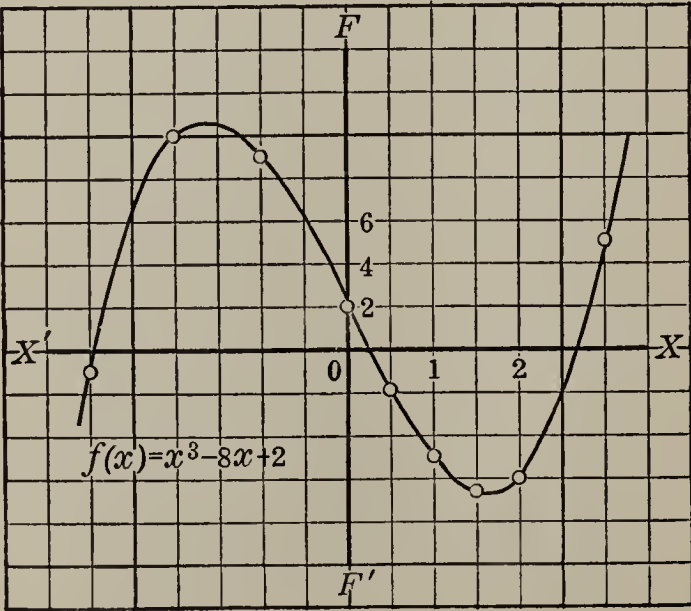
1. $f(x) = x^3 - 3x + 1$.

If $x =$	- 3	- 2	- 1	0	.5	1	1.5	2
then $f(x) =$	- 17	- 1	3	1	-.375	- 1	-.125	3



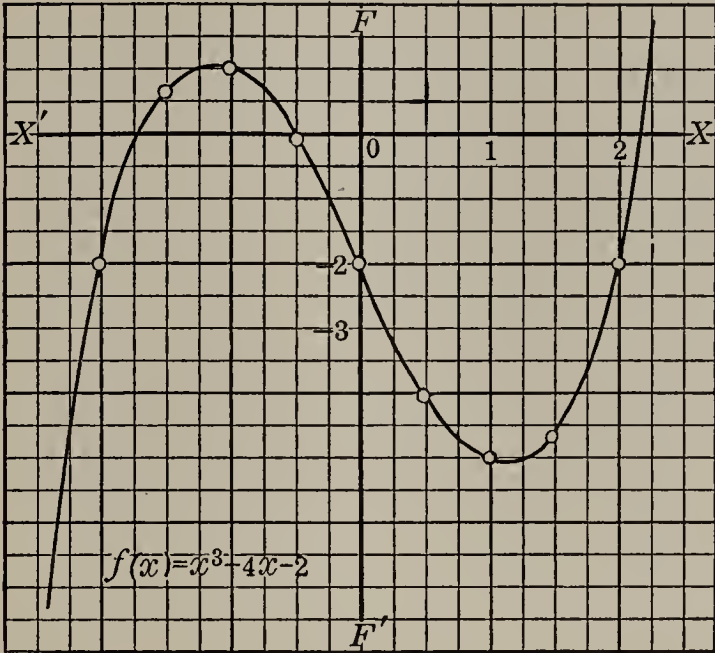
2. $f(x) = x^3 - 8x + 2$.

If $x =$	- 4	- 3	- 2	- 1	0	.5	1	2	3	4
then $f(x) =$	- 30	- 1	10	9	2	- 1.875	- 5	- 6	5	34



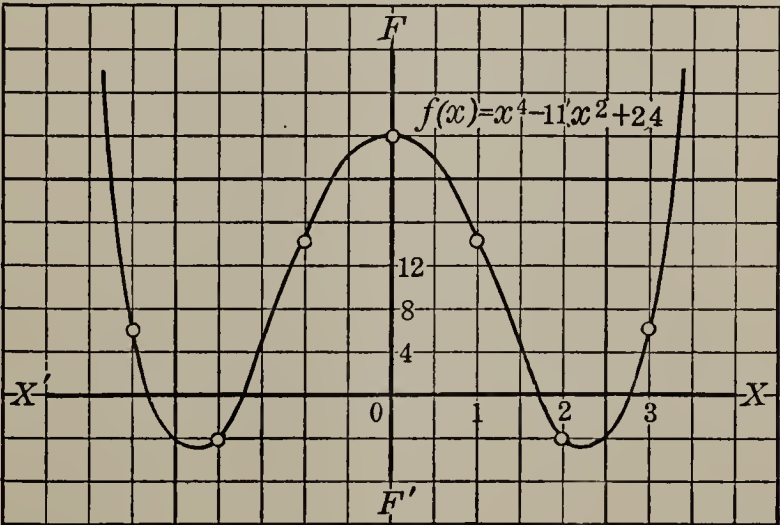
3. $f(x) = x^3 - 4x - 2$.

If $x =$	- 2	- 1.5	- 1	- .5	0	.5	1	1.5	2
then $f(x) =$	- 2	.6	1	- .1	- 2	- 3.9	- 5	- 4.6	- 2



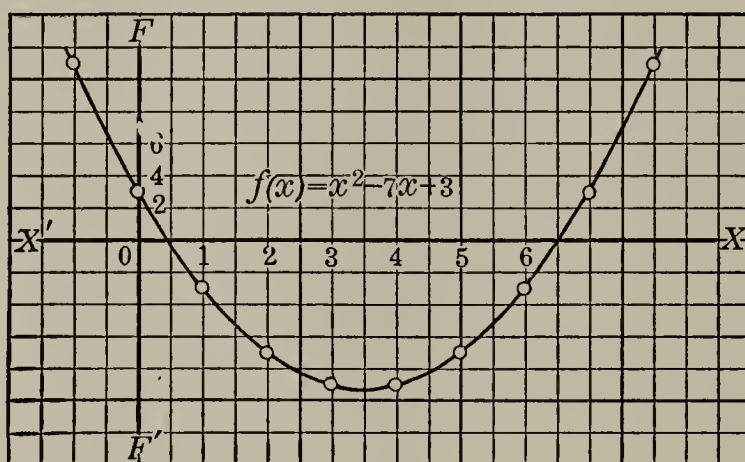
4. $f(x) = x^4 - 11x^2 + 24$.

If $x =$	- 4	- 3	- 2	- 1	0	1	2	3	4
then $f(x) =$	104	6	- 4	14	24	14	- 4	6	104



1. $f(x) = x^2 - 7x + 3$.

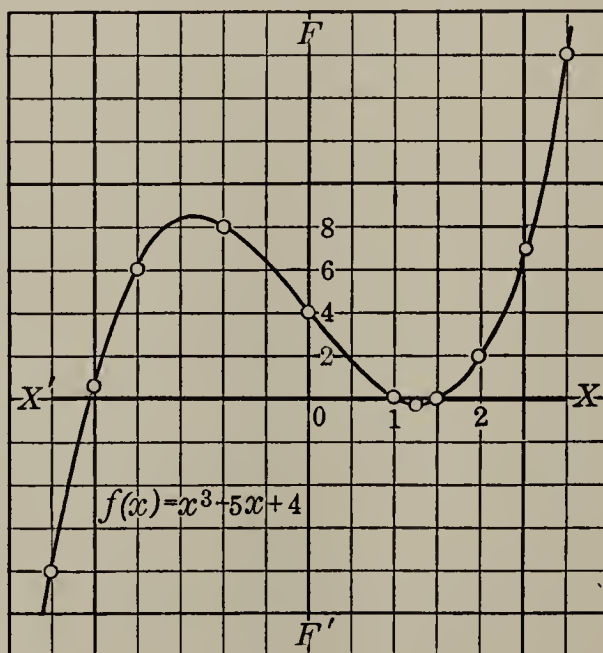
If $x =$	-1	0	1	2	3	4	5	6	7	8
then $f(x) =$	11	3	-3	-7	-9	-9	-7	-3	3	11



From the graph $x = .45, 6.5$.

2. $f(x) = x^3 - 5x + 4$.

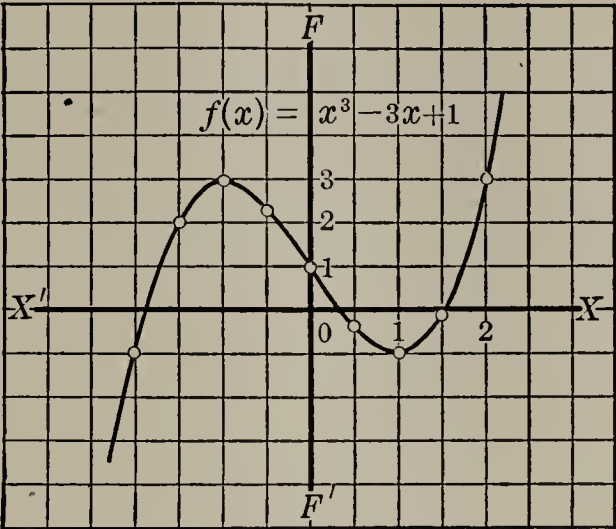
If $x =$	-3	-2.5	-2	-1	0	1	1.25	1.5	2	2.5	3
then $f(x) =$	-8	.875	6	8	4	0	-.296	-.125	2	7.125	16



From the graph $x = 1, 1.6, -2.6$.

3. $f(x) = x^3 - 3x + 1.$

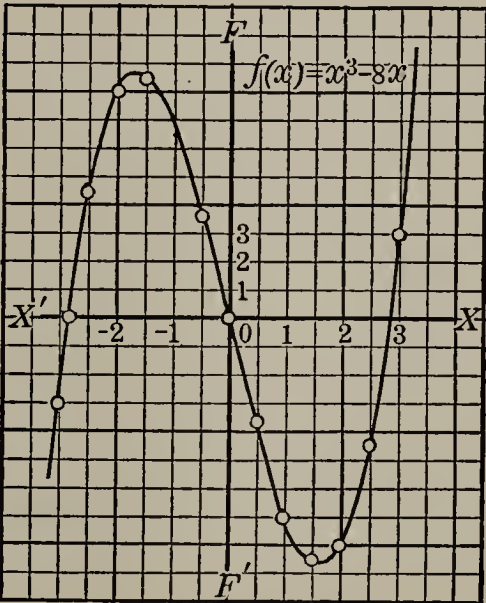
If $x =$	- 2	- 1.5	- 1	- .5	0	.5	1	1.5	2	3
then $f(x) =$	- 1	2.125	3	2.375	1	-.375	- 1	-.125	3	19



From the graph $x = .3, 1.6, -1.8.$

4. $f(x) = x^3 - 8x.$

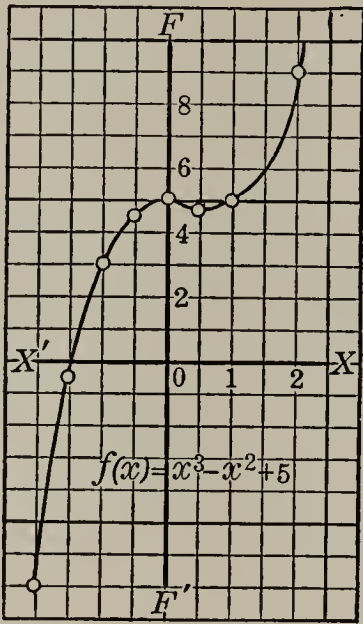
If $x =$	- 3	- 2.5	- 2	- 1.5	- 1	- .5	0	.5	1	1.5	2	2.5	3
then $f(x) =$	- 3	4.3	8	8.6	7	3.8	0	- 3.8	- 7	- 8.6	- 8	- 4.3	3



From the graph $x = 2.8, 0,$ and $- 2.8.$
CR

5. $f(x) = x^3 - x^2 + 5.$

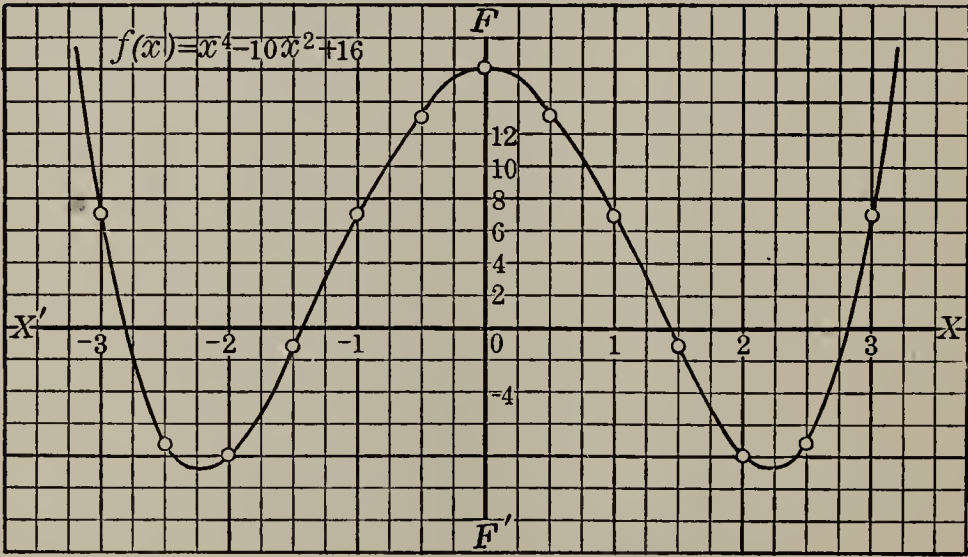
If $x =$	- 2	- 1.5	- 1	- .5	0	.5	1	2
then $f(x) =$	- 7	-.625	3	4.625	5	4.875	5	9



From the graph $x = -1.4.$

6. $f(x) = x^4 - 10x^2 + 16.$

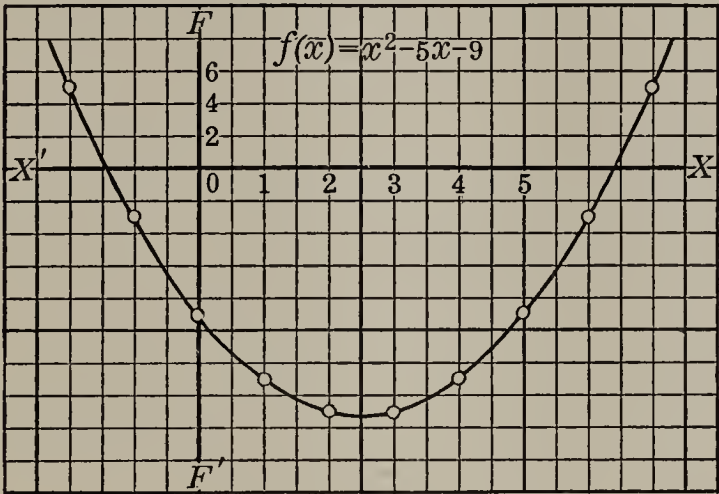
If $x =$	-3	-2.5	-2	-1.5	-1	-.5	0	.5	1	1.5	2	2.5	3
then $f(x) =$	7	-7.4	-8	-1.4	7	13.5	16	13.5	7	-1.4	-8	-7.4	7



From the graph $x = 2.8, 1.4, -1.4,$ and $-2.8.$

1. $f(x) = x^2 - 5x - 9$.

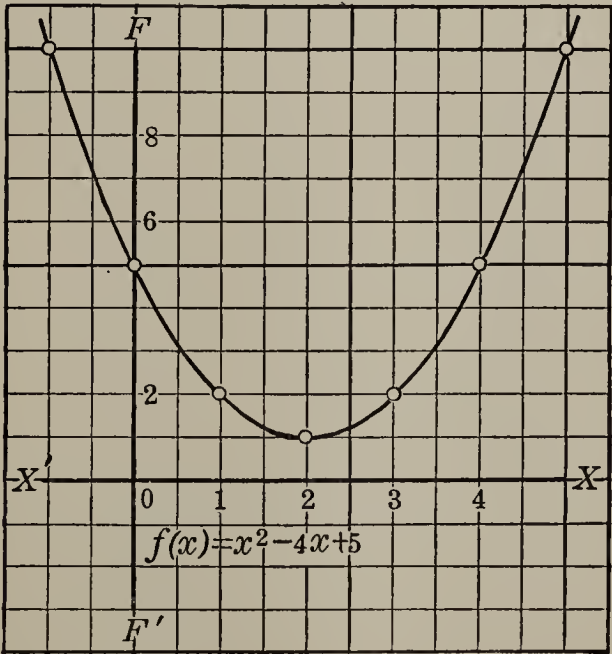
If $x =$	-3	-2	-1	0	1	2	3	4	5	6	7	8
then $f(x) =$	15	5	-3	-9	-13	-15	-15	-13	-9	-3	5	15



From the graph $x = 6.4, -1.4$.

2. $f(x) = x^2 - 4x + 5$.

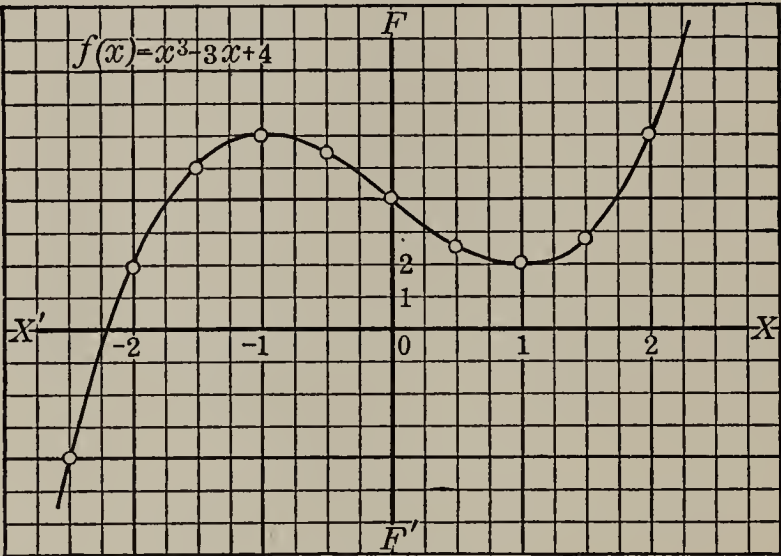
If $x =$	-1	0	1	2	3	4	5
then $f(x) =$	10	5	2	1	2	5	10



There are no real roots.

3. $f(x) = x^3 - 3x + 4$.

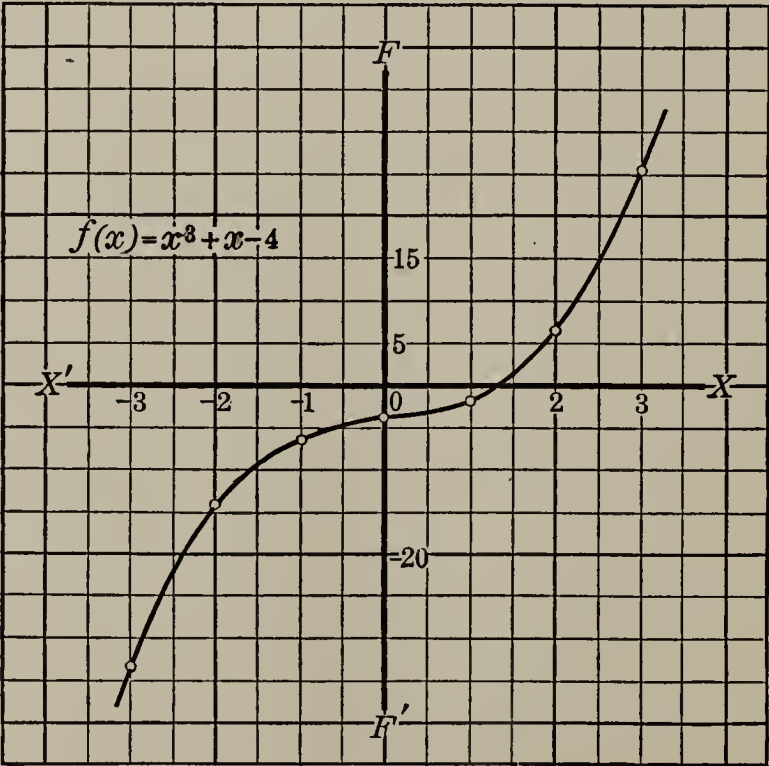
If $x =$	- 2.5	- 2	- 1.5	- 1	- .5	0	.5	1	1.5	2
then $f(x) =$	- 4.1	2	5.1	6	5.3	4	2.6	2	2.8	6



From the graph $x = - 2.2$, approximately.

4. $f(x) = x^3 + x - 4$.

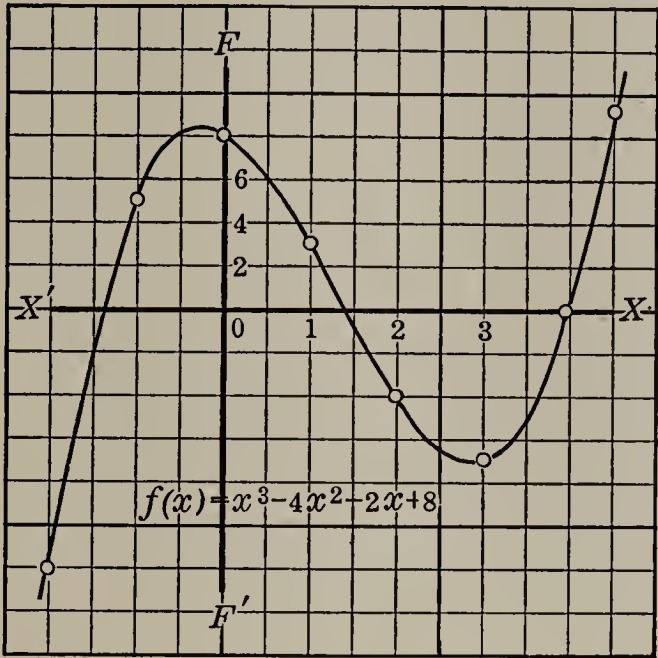
If $x =$	- 3	- 2	- 1	0	1	2	3
then $f(x) =$	- 34	- 14	- 6	- 4	- 2	6	26



From the graph $x = 1.3$, approximately.

5. $f(x) = x^3 - 4x^2 - 2x + 8$.

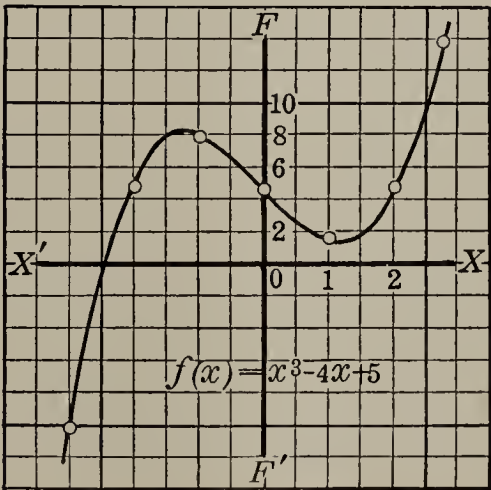
If $x =$	- 2	- 1	0	1	2	3	4	4.5
then $f(x) =$	- 12	5	8	3	- 4	- 7	0	9.125



From the graph $x = 4, 1.4$, and $- 1.4$.

6. $f(x) = x^3 - 4x + 5$.

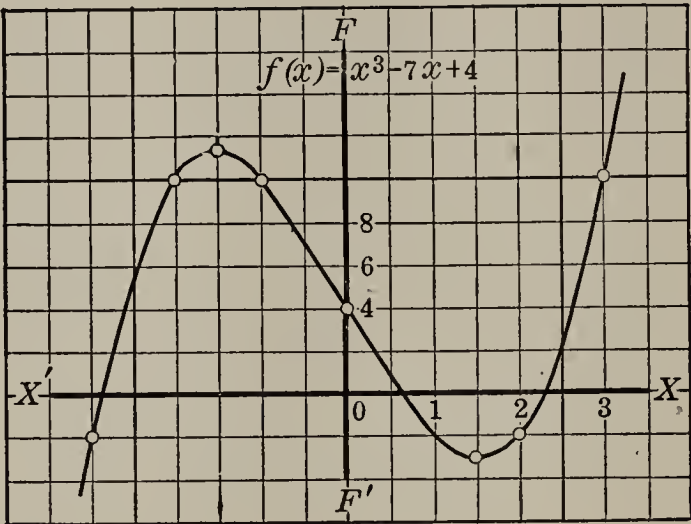
If $x =$	- 3	- 2	- 1	0	1	2	3
then $f(x) =$	- 10	5	8	5	2	5	20



From the graph $x = - 2.4$.

7. $f(x) = x^3 - 7x + 4$.

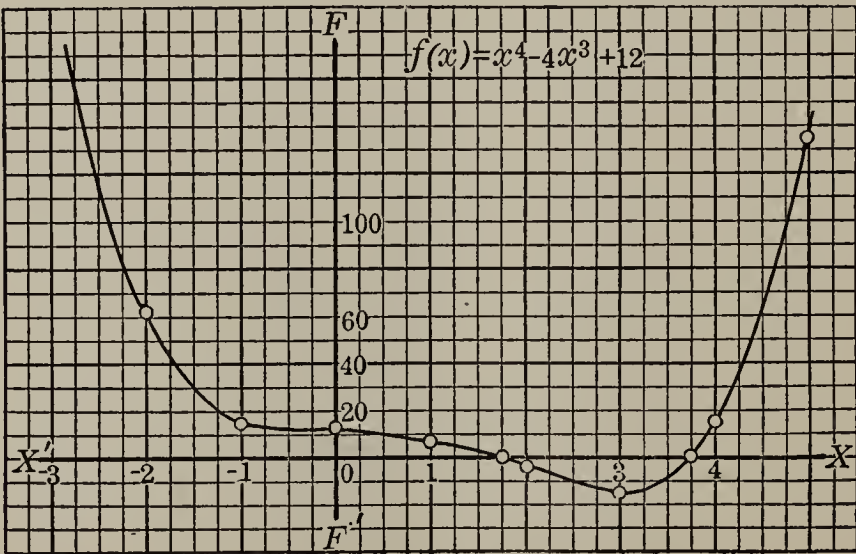
If $x =$	- 4	- 3	- 2	- 1.5	- 1	0	1	1.5	2	3
then $f(x) =$	- 32	- 2	10	11.125	10	4	- 2	- 3.125	- 2	10



From the graph $x = .7, 2.3$, and $- 2.8$.

8. $f(x) = x^4 - 4x^3 + 12$.

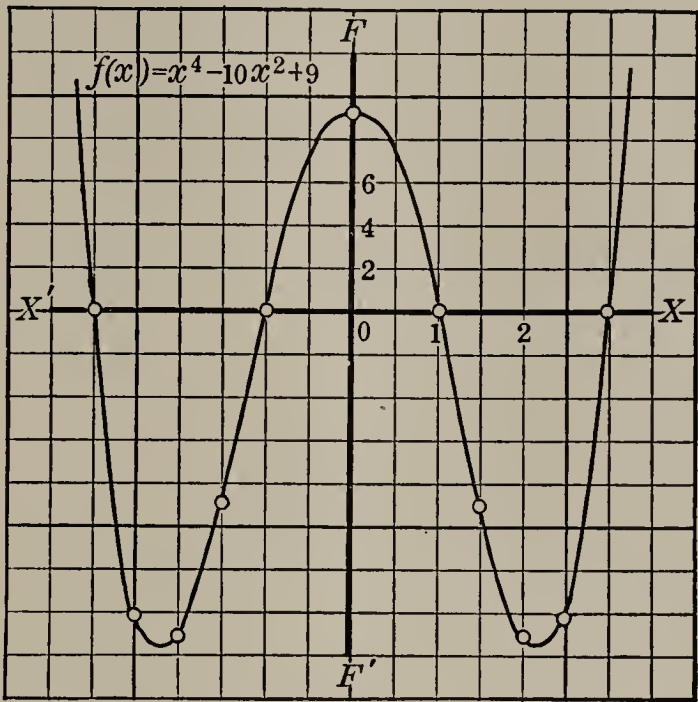
If $x =$	- 2	- 1	0	1	2	3	4	5
then $f(x) =$	60	17	12	9	- 4	- 15	12	137



From the graph $x = 1.7$ and 3.8 .

9. $f(x) = x^4 - 10x^2 + 9$.

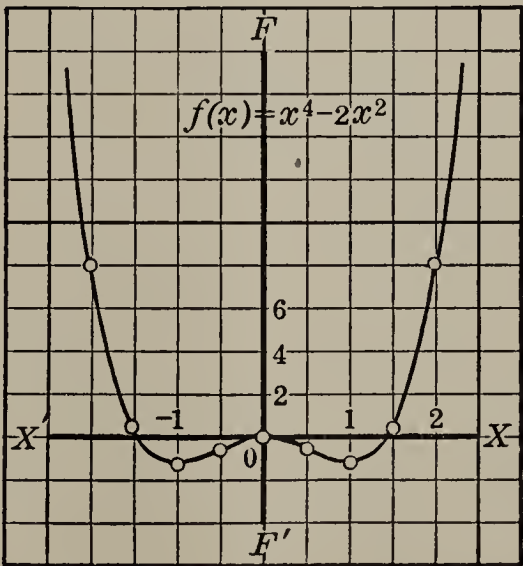
If $x =$	-3.5	-3	-2.5	-2	-1.5	-1	0	1	1.5	2	2.5	3	3.5
then $f(x) =$	36.5	0	-14.4	-15	-8.4	0	9	0	-8.4	-15	-14.4	0	36.5



From the graph $x = 1, -1, 3, -3$.

10. $f(x) = x^4 - 2x^2$.

If $x =$	-2.5	-2	-1.5	-1	-.5	0	.5	1	1.5	2	2.5
then $f(x) =$	26.5	8	.5	-1	-.4	0	-.4	-1	.5	8	26.5



From the graph $x = 0, 0, 1.4, -1.4$.

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1. $x^2 + 4x + 3 = 0.$

$$x^2 + 4x = -3.$$

$$x^2 + 4x + 4 = 1.$$

$$(x + 2)^2 = 1.$$

$$x + 2 = \pm 1.$$

$$x = -1, -3.$$

2. $s^2 - s - 2 = 0.$

$$s^2 - s = 2.$$

$$s^2 - s + \frac{1}{4} = 2\frac{1}{4}.$$

$$(s - \frac{1}{2})^2 = \frac{9}{4}.$$

$$s - \frac{1}{2} = \pm \frac{3}{2}.$$

$$s = 2, -1.$$

3. $x^2 + 2 = -3x.$

$$x^2 + 3x = -2.$$

$$x^2 + 3x + \frac{9}{4} = \frac{1}{4}.$$

$$(x + \frac{3}{2})^2 = \frac{1}{4}.$$

$$x + \frac{3}{2} = \pm \frac{1}{2}.$$

$$x = -1, -2.$$

4. $2x^2 + 5x + 3 = 0.$

$$2x^2 + 5x = -3.$$

$$x^2 + \frac{5}{2}x = -\frac{3}{2}.$$

$$x^2 + \frac{5}{2}x + \frac{25}{16} = \frac{1}{16}.$$

$$(x + \frac{5}{4})^2 = \frac{1}{16}.$$

$$x + \frac{5}{4} = \pm \frac{1}{4}.$$

$$x = -1, -\frac{3}{2}.$$

5. $3x^2 + 7x - 6 = 0.$

$$x^2 + \frac{7}{3}x = 2.$$

$$x^2 + \frac{7}{3}x + \frac{49}{36} = 2 + \frac{49}{36} = \frac{121}{36}.$$

$$(x + \frac{7}{6})^2 = \frac{121}{36}.$$

$$x + \frac{7}{6} = \pm \frac{11}{6}.$$

$$x = \frac{2}{3}, -3.$$

6. $2x^2 - 3x - 5 = 0.$

$$x^2 - \frac{3}{2}x = \frac{5}{2}.$$

$$(x - \frac{3}{4})^2 = \frac{5}{2} + \frac{9}{16} = \frac{49}{16}.$$

$$x - \frac{3}{4} = \pm \frac{7}{4}.$$

$$x = \frac{5}{2}, -1.$$

7. $5x^2 - 7x - 6 = 0.$

$$x^2 - \frac{7}{5}x = \frac{6}{5}.$$

$$(x - \frac{7}{10})^2 = \frac{6}{5} + \frac{49}{100} = \frac{169}{100}.$$

$$x - \frac{7}{10} = \pm \frac{13}{10}.$$

$$x = 2, -\frac{3}{5}.$$

8. $x - 4 + x^2 = 6 - 2x^2 + 8x.$

$$3x^2 - 7x = 10.$$

$$x^2 - \frac{7}{3}x = \frac{10}{3}.$$

$$(x - \frac{7}{6})^2 = \frac{10}{3} + \frac{49}{36} = \frac{169}{36}.$$

$$x - \frac{7}{6} = \pm \frac{13}{6}.$$

$$x = \frac{10}{3}, -1.$$

10. $n^2 + 2 = 5n.$

$$n^2 - 5n = -2.$$

$$(n - \frac{5}{2})^2 = -2 + \frac{25}{4} = \frac{17}{4}.$$

$$n - \frac{5}{2} = \pm \frac{1}{2}\sqrt{17}$$

$$= \pm 2.062.$$

$$n = 2.5 \pm 2.062$$

$$= 4.562, .438.$$

11. $3x^2 - 12x + 9 = 2.$

$$x^2 - 4x = -\frac{7}{3}.$$

$$(x - 2)^2 = -\frac{7}{3} + 4 = \frac{5}{3}.$$

$$x - 2 = \pm \sqrt{\frac{5}{3}}$$

$$= \pm \frac{1}{3}\sqrt{15}$$

$$= \pm 1.291.$$

$$x = 2 \pm 1.291$$

$$= 3.291, .709.$$

12. $5x^2 + 8x + 2 = 0.$

$$x^2 + \frac{8}{5}x = -\frac{2}{5}.$$

$$(x + \frac{4}{5})^2 = -\frac{2}{5} + \frac{16}{25} = \frac{6}{25}.$$

$$x + \frac{4}{5} = \pm \frac{1}{5}\sqrt{6}$$

$$= \pm .489.$$

$$x = -.8 \pm .489$$

$$= -.311, -1.289.$$

$$\begin{aligned}
 13. \quad & (y+1)(y+2) = 3y(y-4). \\
 & y^2 + 3y + 2 = 3y^2 - 12y. \\
 & 2y^2 - 15y = 2. \\
 & y^2 - \frac{15}{2}y = 1. \\
 & (y - \frac{15}{4})^2 = 1 + \frac{225}{16} = \frac{241}{16}. \\
 & y - \frac{15}{4} = \pm \frac{1}{4}\sqrt{241} \\
 & \quad = \pm 3.881. \\
 & y = 3.750 \pm 3.881 \\
 & \quad = 7.631, -.131.
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & x^4 - 5x^2 + 4 = 0. \\
 & x^4 - 5x^2 = -4. \\
 & (x^2 - \frac{5}{2})^2 = -4 + \frac{25}{4} = \frac{9}{4}. \\
 & x^2 - \frac{5}{2} = \pm \frac{3}{2}. \\
 & x^2 = 4, 1. \\
 & x = \pm 2, \pm 1.
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & x^4 - 13x^2 + 36 = 0. \\
 & (x^2 - \frac{13}{2})^2 = -36 + \frac{169}{4} \\
 & \quad = \frac{25}{4}. \\
 & x^2 - \frac{13}{2} = \pm \frac{5}{2}. \\
 & x^2 = 9, 4. \\
 & x = \pm 3, \pm 2.
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & x^4 - 4x^2 + 3 = 0. \\
 & x^4 - 4x^2 = -3. \\
 & (x^2 - 2)^2 = -3 + 4 = 1. \\
 & x^2 - 2 = \pm 1. \\
 & x^2 = 3, 1. \\
 & x = \pm \sqrt{3}, \pm 1.
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & 6x^4 - 11x^2 + 3 = 0. \\
 & x^4 - \frac{11}{6}x^2 = -\frac{1}{2}. \\
 & (x^2 - \frac{11}{12})^2 = -\frac{1}{2} + \frac{121}{144} = \frac{49}{144}. \\
 & x^2 - \frac{11}{12} = \pm \frac{7}{12}. \\
 & x^2 = \frac{3}{2}, \frac{1}{3}. \\
 & x = \pm \sqrt{\frac{3}{2}}, \pm \sqrt{\frac{1}{3}} \\
 & \quad = \pm \frac{1}{2}\sqrt{6}, \pm \frac{1}{3}\sqrt{3}.
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & x^{-2} + 16x^{-1} - 17 = 0. \\
 & (x^{-1} + 17)(x^{-1} - 1) = 0. \\
 & x = -\frac{1}{17}, 1.
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & y^{-4} - 10y^{-2} + 9 = 0. \\
 & (y^{-2} - 9)(y^{-2} - 1) = 0. \\
 & (y^{-1} + 3)(y^{-1} - 3)(y^{-1} + 1)(y^{-1} - 1) = 0. \\
 & y = -\frac{1}{3}, \frac{1}{3}, -1, 1.
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & x^{-1} - 13x^{-\frac{1}{2}} + 36 = 0. \\
 & (x^{-\frac{1}{2}} - 9)(x^{-\frac{1}{2}} - 4) = 0. \\
 & (x^{-\frac{1}{4}} + 3)(x^{-\frac{1}{4}} - 3)(x^{-\frac{1}{4}} + 2)(x^{-\frac{1}{4}} - 2) = 0. \\
 & x = \frac{1}{81}, \frac{1}{16}.
 \end{aligned}$$

23. If

$$\begin{aligned}
 y &= x - 1, \\
 (x - 1)^2 + 4(x - 1) &= 5 \text{ becomes} \\
 y^2 + 4y &= 5. \\
 (y + 2)^2 &= 5 + 4 = 9. \\
 y + 2 &= \pm 3. \\
 y &= x - 1 = 1, -5. \\
 x &= 2, -4.
 \end{aligned}$$

24. If

$$\begin{aligned}
 y &= x^2 - 4x, \\
 (x^2 - 4x)^2 - 5(x^2 - 4x) - 24 &= 0 \text{ becomes} \\
 y^2 - 5y &= 24. \\
 (y - \frac{5}{2})^2 &= 24 + \frac{25}{4} = 1\frac{21}{4}. \\
 y - \frac{5}{2} &= \pm 1\frac{1}{2}. \\
 y &= 8, -3.
 \end{aligned}$$

Then

$$\left. \begin{aligned}
 x^2 - 4x &= 8, \\
 (x - 2)^2 &= 8 + 4 = 12, \\
 x - 2 &= \pm \sqrt{12} = \pm 2\sqrt{3}, \\
 x &= 2 \pm 2\sqrt{3},
 \end{aligned} \right\} \text{ and } \left\{ \begin{aligned}
 x^2 - 4x &= -3, \\
 (x - 2)^2 &= -3 + 4 = 1, \\
 x - 2 &= \pm 1, \\
 x &= 3, 1.
 \end{aligned} \right.$$

25. If

$$\begin{aligned}
 y &= x - \frac{6}{x}, \\
 \left(x - \frac{6}{x}\right)^2 + 4\left(x - \frac{6}{x}\right) - 5 &= 0 \text{ becomes} \\
 y^2 + 4y &= 5. \\
 (y + 2)^2 &= 5 + 4 = 9. \\
 y + 2 &= \pm 3. \\
 y &= 1, -5.
 \end{aligned}$$

Then

$$\left. \begin{aligned}
 x - \frac{6}{x} &= 1, \\
 x^2 - 6 &= x, \\
 x^2 - x &= 6, \\
 (x - \frac{1}{2})^2 &= 6 + \frac{1}{4} = 2\frac{5}{4}, \\
 x - \frac{1}{2} &= \pm \frac{5}{2}, \\
 x &= 3, -2,
 \end{aligned} \right\} \text{ and } \left\{ \begin{aligned}
 x - \frac{6}{x} &= -5, \\
 x^2 - 6 &= -5x, \\
 x^2 + 5x &= 6, \\
 (x + \frac{5}{2})^2 &= 6 + 2\frac{5}{4} = 4\frac{9}{4}, \\
 x + \frac{5}{2} &= \pm \frac{7}{2}, \\
 x &= 1, -6.
 \end{aligned} \right.$$

27.

$$\begin{aligned}
 x^2 + 3ax + 2a^2 &= 0. \\
 \left(x + \frac{3a}{2}\right)^2 &= -2a^2 + \frac{9a^2}{4} = \frac{a^2}{4}. \\
 x + \frac{3a}{2} &= \pm \frac{a}{2}. \\
 x &= -a, -2a.
 \end{aligned}$$

28.

$$2y^2 + by = 6b^2.$$

$$y^2 + \frac{b}{2}y = 3b^2.$$

$$\left(y + \frac{b}{4}\right)^2 = 3b^2 + \frac{b^2}{16} = \frac{49b^2}{16}.$$

$$y + \frac{b}{4} = \pm \frac{7b}{4}.$$

$$y = \frac{3}{2}b, -2b.$$

29.

$$bx^2 + x = 1 + bx.$$

$$bx^2 + (1 - b)x = 1.$$

$$x^2 + \left(\frac{1 - b}{b}\right)x = \frac{1}{b}.$$

$$x^2 + \left(\frac{1 - b}{b}\right)x + \left(\frac{1 - b}{2b}\right)^2 = \frac{1}{b} + \frac{1 - 2b + b^2}{4b^2} = \frac{1 + 2b + b^2}{4b^2}.$$

$$\left(x + \frac{1 - b}{2b}\right)^2 = \left(\frac{1 + b}{2b}\right)^2.$$

$$x + \frac{1 - b}{2b} = \pm \frac{1 + b}{2b}.$$

$$x = \frac{b - 1}{2b} \pm \frac{1 + b}{2b}$$

$$= 1, -\frac{1}{b}.$$

30.

$$\frac{x - c}{c} - \frac{c}{x - c} = \frac{3}{2}.$$

$$2x^2 - 4cx + 2c^2 - 2c^2 = 3cx - 3c^2.$$

$$2x^2 - 7cx + 3c^2 = 0.$$

$$x = 3c, \frac{c}{2}.$$

31.

$$bx^2 - a(b + 1)x + a^2 = 0.$$

$$x^2 - \frac{a}{b}(b + 1)x = -\frac{a^2}{b}.$$

$$x^2 - \frac{a}{b}(b + 1)x + \frac{a^2}{4b^2}(b + 1)^2 = -\frac{a^2}{b} + \frac{a^2(b + 1)^2}{4b^2}$$

$$\left[x - \frac{a}{2b}(b + 1)\right]^2 = \frac{a^2(b - 1)^2}{4b^2}.$$

$$x - \frac{a}{2b}(b + 1) = \pm \frac{a}{2b}(b - 1).$$

$$x = \frac{a}{2b}(b + 1) \pm \frac{a}{2b}(b - 1)$$

$$= a, \frac{a}{b}.$$

$$32. \quad \frac{b}{2a}x^2 = \frac{b}{2a} + 1 - x.$$

$$\frac{b}{2a}x^2 + x = \frac{b}{2a} + 1.$$

$$x^2 + \frac{2a}{b}x = \frac{b+2a}{2a} \cdot \frac{2a}{b} = \frac{b+2a}{b}.$$

$$\left(x + \frac{a}{b}\right)^2 = \frac{b+2a}{b} + \frac{a^2}{b^2} = \frac{b^2 + 2ab + a^2}{b^2}.$$

$$x + \frac{a}{b} = \pm \frac{a+b}{b}.$$

$$x = -\frac{a}{b} \pm \frac{a+b}{b}$$

$$= 1, -\frac{2a+b}{b}.$$

$$33. \quad cx^2 - (c^2 + d)x + cd = 0.$$

$$x^2 - \frac{c^2 + d}{c}x = -d.$$

$$\left(x - \frac{c^2 + d}{2c}\right)^2 = -d + \frac{c^4 + 2c^2d + d^2}{4c^2}$$

$$= \frac{c^4 - 2c^2d + d^2}{4c^2}.$$

$$x - \frac{c^2 + d}{2c} = \pm \frac{c^2 - d}{2c}.$$

$$x = \frac{c^2 + d \pm (c^2 - d)}{2c}$$

$$= c, \frac{d}{c}.$$

$$34. \quad abx^2 - x(a+b)\sqrt{ab} + ab = 0.$$

$$x^2 - \frac{a+b}{\sqrt{ab}}x = -1.$$

$$\left(x - \frac{a+b}{2\sqrt{ab}}\right)^2 = -1 + \frac{a^2 + 2ab + b^2}{4ab}$$

$$= \frac{a^2 - 2ab + b^2}{4ab}.$$

$$x - \frac{a+b}{2\sqrt{ab}} = \pm \frac{a-b}{2\sqrt{ab}}.$$

$$x = \frac{a+b \pm (a-b)}{2\sqrt{ab}}$$

$$= \frac{a}{\sqrt{ab}}, \frac{b}{\sqrt{ab}}$$

$$= \frac{1}{b}\sqrt{ab}, \frac{1}{a}\sqrt{ab}.$$

35. $6a^2y^2 - 7ary + 2r^2 = 0.$

$$y^2 - \frac{7r}{6a}y = -\frac{r^2}{3a^2}.$$

$$\left(y - \frac{7r}{12a}\right)^2 = -\frac{r^2}{3a^2} + \frac{49r^2}{144a^2} = \frac{r^2}{144a^2}.$$

$$y - \frac{7r}{12a} = \pm \frac{r}{12a}.$$

$$y = \frac{2r}{3a}, \frac{r}{2a}.$$

36. If $y = ax + b,$
 $(ax + b)^2 + 3(ax + b) + 2 = 0$ becomes

$$y^2 + 3y + 2 = 0.$$

$$\left(y + \frac{3}{2}\right)^2 = -2 + \frac{9}{4} = \frac{1}{4}.$$

$$y + \frac{3}{2} = \pm \frac{1}{2}.$$

$$y = ax + b = -2, -1.$$

Then

$$x = -\frac{2+b}{a}, -\frac{1+b}{a}.$$

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2. $x^2 - 6x - 16 = 0.$

$$x = \frac{6 \pm \sqrt{36 + 64}}{2} = \frac{6 \pm 10}{2} = 8, -2.$$

3. $x^2 - x - 1 = 0.$

$$x = \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1 \pm \sqrt{5}}{2}.$$

4. $3x^2 + 7x - 3 = 0.$

$$x = \frac{-7 \pm \sqrt{49 + 36}}{6} = \frac{-7 \pm \sqrt{85}}{6}.$$

5. $2x + 4 = x^2.$

$$x^2 - 2x - 4 = 0.$$

$$x = \frac{2 \pm \sqrt{4 + 16}}{2} = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}.$$

6. $x = 1 - x^2.$

$$x^2 + x - 1 = 0.$$

$$x = \frac{-1 \pm \sqrt{1 + 4}}{2} = \frac{-1 \pm \sqrt{5}}{2}.$$

7. $3x + 4 = x^2.$

$$x^2 - 3x - 4 = 0.$$

$$x = \frac{3 \pm \sqrt{9 + 16}}{2} = \frac{3 \pm 5}{2} = 4, -1.$$

$$8. \quad 11x^2 - 15x = 26.$$

$$11x^2 - 15x - 26 = 0.$$

$$\begin{aligned} x &= \frac{15 \pm \sqrt{225 + 1144}}{22} \\ &= \frac{15 \pm \sqrt{1369}}{22} \\ &= \frac{15 \pm 37}{22} \\ &= \frac{26}{11}, -1. \end{aligned}$$

$$9. \quad \frac{3}{2}x - \frac{1}{2} = x^2.$$

$$x^2 - \frac{3}{2}x + \frac{1}{2} = 0.$$

The equation could be solved in the above form, but clearing of fractions makes the solution easier.

$$2x^2 - 3x + 1 = 0.$$

$$x = \frac{3 \pm \sqrt{9 - 8}}{4} = \frac{3 \pm 1}{4} = 1, \frac{1}{2}.$$

$$11. \quad x^2 - 2kx - 3k^2 = 0.$$

$$\begin{aligned} x &= \frac{2k \pm \sqrt{4k^2 + 12k^2}}{2} \\ &= \frac{2k \pm 4k}{2} \\ &= 3k, -k. \end{aligned}$$

$$12. \quad 3a^2 = 7ax + 6x^2.$$

$$6x^2 + 7ax - 3a^2 = 0.$$

$$\begin{aligned} x &= \frac{-7a \pm \sqrt{49a^2 + 72a^2}}{12} \\ &= \frac{-7a \pm 11a}{12} \\ &= \frac{a}{3}, -\frac{3a}{2}. \end{aligned}$$

$$13. \quad 4x^2 + kx - 14k^2 = 0.$$

$$\begin{aligned} x &= \frac{-k \pm \sqrt{k^2 + 224k^2}}{8} \\ &= \frac{-k \pm 15k}{8} \\ &= \frac{7k}{4}, -2k. \end{aligned}$$

14.

$$bx = 12x^2 - b^2.$$

$$12x^2 - bx - b^2 = 0.$$

$$\begin{aligned} x &= \frac{b \pm \sqrt{b^2 + 48b^2}}{24} = \frac{b \pm 7b}{24} \\ &= \frac{b}{3}, -\frac{b}{4}. \end{aligned}$$

15.

$$a^2x^2 + 4abx + 3b^2 = 0.$$

$$\begin{aligned} x &= \frac{-4ab \pm \sqrt{16a^2b^2 - 12a^2b^2}}{2a^2} \\ &= \frac{-4ab \pm 2ab}{2a^2} \\ &= -\frac{3b}{a}, -\frac{b}{a}. \end{aligned}$$

16.

$$12p^2x^2 - 4prx - r^2 = 0.$$

$$\begin{aligned} x &= \frac{4pr \pm \sqrt{16p^2r^2 + 48p^2r^2}}{24p^2} \\ &= \frac{4pr \pm 8pr}{24p^2} \\ &= \frac{r}{2p}, -\frac{r}{6p}. \end{aligned}$$

17.

$$3a^2x^2 + 8abx + 4b^2 = 0.$$

$$\begin{aligned} x &= \frac{-8ab \pm \sqrt{64a^2b^2 - 48a^2b^2}}{6a^2} \\ &= \frac{-8ab \pm 4ab}{6a^2} \\ &= -\frac{2b}{a}, -\frac{2b}{3a}. \end{aligned}$$

18.

$$x - 3\sqrt{c} - \frac{4c}{x} = 0.$$

$$x^2 - 3\sqrt{c} \cdot x - 4c = 0.$$

$$\begin{aligned} x &= \frac{3\sqrt{c} \pm \sqrt{9c + 16c}}{2} \\ &= \frac{3\sqrt{c} \pm 5\sqrt{c}}{2} \\ &= 4\sqrt{c}, -\sqrt{c}. \end{aligned}$$

19. $6h^2 + \frac{13hn}{x} = \frac{8n^2}{x^2}.$

$$6h^2x^2 + 13hnx - 8n^2 = 0.$$

$$\begin{aligned} x &= \frac{-13hn \pm \sqrt{169h^2n^2 + 192h^2n^2}}{12h^2} \\ &= \frac{-13hn \pm \sqrt{361h^2n^2}}{12h^2} \\ &= \frac{-13hn \pm 19hn}{12h^2} \\ &= \frac{n}{2h}, \frac{-8n}{3h}. \end{aligned}$$

20. $13a^2b^2x^2 = 9b^4 + 4a^4x^4.$

$$4a^4x^4 - 13a^2b^2x^2 + 9b^4 = 0.$$

This is a quadratic in x^2 (cf. text, p. 370, Ex. 14).

$$\begin{aligned} x^2 &= \frac{13a^2b^2 \pm \sqrt{169a^4b^4 - 144a^4b^4}}{8a^4} \\ &= \frac{13a^2b^2 \pm 5a^2b^2}{8a^4}. \\ x^2 &= \frac{9b^2}{4a^2}, \frac{b^2}{a^2}. \\ x &= \pm \frac{3b}{2a}, \pm \frac{b}{a}. \end{aligned}$$

22. $2x^2 + (a + 2b)x - a^2 - ab = 0.$

$$\begin{aligned} x &= \frac{-a - 2b \pm \sqrt{a^2 + 4ab + 4b^2 + 8a^2 + 8ab}}{4} \\ &= \frac{-a - 2b \pm (3a + 2b)}{4} \\ &= \frac{a}{2} \text{ and } -a - b. \end{aligned}$$

23. $ax^2 - 2ax - 3a = bx - \frac{b}{3}x^2.$

$$3ax^2 - 6ax - 9a = 3bx - bx^2.$$

$$(3a + b)x^2 - 3(2a + b)x - 9a = 0.$$

$$\begin{aligned} x &= \frac{+3(2a + b) \pm \sqrt{36a^2 + 36ab + 9b^2 + 108a^2 + 36ab}}{6a + 2b} \\ &= \frac{6a + 3b \pm (12a + 3b)}{6a + 2b} \\ &= 3 \text{ or } \frac{-3a}{3a + b}. \end{aligned}$$

24. $p^2 + x^2 = 2px + 2x - 2p.$

$x^2 - (2p + 2)x + (p^2 + 2p) = 0.$

$$\begin{aligned} x &= \frac{2p + 2 \pm \sqrt{4p^2 + 8p + 4 - 4p^2 - 8p}}{2} \\ &= \frac{2p + 2 \pm 2}{2} \\ &= p + 2, p. \end{aligned}$$

25. $x^2 - cx = \frac{1}{2}(ax - ac).$

$2x^2 - 2cx = ax - ac.$

$2x^2 - (a + 2c)x + ac = 0.$

$$\begin{aligned} x &= \frac{a + 2c \pm \sqrt{a^2 + 4ac + 4c^2 - 8ac}}{4} \\ &= \frac{a + 2c \pm (a - 2c)}{4} \\ &= \frac{a}{2}, c. \end{aligned}$$

26. $4a^2x^4 + 4c^2 = 16a^2x^2 + c^2x^2.$

$4a^2x^4 - (16a^2 + c^2)x^2 + 4c^2 = 0.$

$$\begin{aligned} x^2 &= \frac{16a^2 + c^2 \pm \sqrt{256a^4 + 32a^2c^2 + c^4 - 64a^2c^2}}{8a^2} \\ &= \frac{16a^2 + c^2 \pm (16a^2 - c^2)}{8a^2} \\ &= 4, \frac{c^2}{4a^2}. \\ x &= \pm 2, \pm \frac{c}{2a}. \end{aligned}$$

27. $x^6 - a^3x^3 = 8b^3x^3 - 8a^3b^3.$

$x^6 - (a^3 + 8b^3)x^3 + 8a^3b^3 = 0.$

$$\begin{aligned} x^3 &= \frac{a^3 + 8b^3 \pm \sqrt{a^6 + 16a^3b^3 + 64b^6 - 32a^3b^3}}{2} \\ &= \frac{a^3 + 8b^3 \pm (a^3 - 8b^3)}{2} \\ &= a^3, 8b^3. \\ x &= a, 2b. \end{aligned}$$

28. $a^2 + x^2 = 2ax + b^2.$

$x^2 - 2ax + (a^2 - b^2) = 0.$

$$x = \frac{2a \pm \sqrt{4a^2 - 4a^2 + 4b^2}}{2} = \frac{2a \pm 2b}{2} = a \pm b.$$

$$29. \quad 12pq = x^2 + 4qx - 3px.$$

$$x^2 + (4q - 3p)x - 12pq = 0.$$

$$\begin{aligned} x &= \frac{3p - 4q \pm \sqrt{16q^2 - 24qp + 9p^2 + 48qp}}{2} \\ &= \frac{3p - 4q \pm (4q + 3p)}{2} \\ &= 3p, -4q. \end{aligned}$$

$$30. \quad x^4 - a^2x^2 + a^2b^4 = b^4x^2.$$

$$x^4 - (a^2 + b^4)x^2 + a^2b^4 = 0.$$

$$\begin{aligned} x^2 &= \frac{a^2 + b^4 \pm \sqrt{a^4 + 2a^2b^4 + b^8 - 4a^2b^4}}{2} \\ &= \frac{a^2 + b^4 \pm (a^2 - b^4)}{2} \\ &= a^2, b^4. \\ x &= \pm a, \pm b^2. \end{aligned}$$

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$$1. \quad 3x^2 - 7x - 10 = 0.$$

$$\begin{aligned} x &= \frac{7 \pm \sqrt{49 + 120}}{6} = \frac{7 \pm 13}{6} \\ &= \frac{10}{3}, -1. \end{aligned}$$

$$2. \quad 5x^2 + 14x + 8 = 0.$$

$$\begin{aligned} x &= \frac{-14 \pm \sqrt{196 - 160}}{10} \\ &= \frac{-14 \pm 6}{10} \\ &= -\frac{4}{5}, -2. \end{aligned}$$

(Note that even when an expression of the type in Exs. 1 and 2 can be factored, it is often easier to solve by formula than by factoring.)

$$3. \quad 4x^2 + 3x - 7 = 0.$$

$$\begin{aligned} x &= \frac{-3 \pm \sqrt{9 + 112}}{8} = \frac{-3 \pm 11}{8} \\ &= 1, -\frac{7}{4}. \end{aligned}$$

$$4. \quad x^2 - 1.7x - .84 = 0.$$

$$(x - 2.1)(x + .4) = 0.$$

$$x = 2.1, - .4.$$

$$\begin{aligned}
 5. \quad & 8x^2 + 2\sqrt{5}x - 15 = 0. \\
 & x = \frac{-2\sqrt{5} \pm \sqrt{20 + 480}}{16} \\
 & = \frac{-2\sqrt{5} \pm 10\sqrt{5}}{16} \\
 & = \frac{1}{2}\sqrt{5}, -\frac{3}{4}\sqrt{5}.
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & x^4 - 7x^2 + 12 = 0. \\
 & (x^2 - 4)(x^2 - 3) = 0. \\
 & x^2 = 4, 3. \\
 & x = \pm 2, \pm \sqrt{3}.
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & x^3 - x = 0. \\
 & x(x - 1)(x + 1) = 0. \\
 & x = 0, \pm 1.
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & x^4 = x^2. \\
 & x^2(x^2 - 1) = 0. \\
 & x^2(x - 1)(x + 1) = 0. \\
 & x = 0, 0, \pm 1.
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & x^4 - 64x^2 = 0. \\
 & x^2(x^2 - 64) = 0. \\
 & x = 0, 0, \pm 8.
 \end{aligned}$$

$$\begin{aligned}
 10. \text{ If } & y = x^2 - 3x, \\
 & (x^2 - 3x)^2 - 2(x^2 - 3x) = 8 \text{ becomes} \\
 & y^2 - 2y - 8 = 0. \\
 & (y - 4)(y + 2) = 0.
 \end{aligned}$$

$$\text{Then } \left. \begin{aligned} x^2 - 3x - 4 &= 0, \\ (x - 4)(x + 1) &= 0, \\ x &= 4, -1, \end{aligned} \right\} \text{ and } \left\{ \begin{aligned} x^2 - 3x + 2 &= 0, \\ (x - 2)(x - 1) &= 0, \\ x &= 1, 2. \end{aligned} \right.$$

$$\begin{aligned}
 11. \quad & 5x^2 - 9x + 3 = 0. \\
 & x = \frac{9 \pm \sqrt{81 - 60}}{10} = \frac{9 \pm \sqrt{21}}{10}.
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & .03x^2 + .01x = .1. \\
 & 3x^2 + x - 10 = 0. \\
 & x = \frac{-1 \pm \sqrt{1 + 120}}{6} \\
 & = \frac{-1 \pm 11}{6} \\
 & = \frac{5}{3}, -2.
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & x^2 + x\sqrt{2} - \sqrt{6} = x\sqrt{3}. \\
 & x^2 + x(\sqrt{2} - \sqrt{3}) - \sqrt{6} = 0. \\
 & (x - \sqrt{3})(x + \sqrt{2}) = 0. \\
 & x = \sqrt{3}, -\sqrt{2}.
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & \frac{x-1}{x} = \frac{x+1}{6}. \\
 & 6x - 6 = x^2 + x. \\
 & x^2 - 5x + 6 = 0. \\
 & (x-2)(x-3) = 0. \\
 & x = 2, 3.
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \frac{x-3}{x+4} = \frac{x-4}{2x+6}. \\
 & 2(x^2 - 9) = x^2 - 16. \\
 & x^2 = 2. \\
 & x = \pm \sqrt{2}.
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{x+1}{x+2} + \frac{x+3}{x+4} = \frac{22}{15}. \\
 & 15(x+1)(x+4) + 15(x+2)(x+3) = 22(x+2)(x+4). \\
 & 8x^2 + 18x - 26 = 0. \\
 & 4x^2 + 9x - 13 = 0. \\
 & x = \frac{-9 \pm \sqrt{81 + 208}}{8} \\
 & = \frac{-9 \pm 17}{8} \\
 & = 1, -\frac{13}{4}.
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{2x-1}{x-1} + \frac{1-x}{x-2} = \frac{x+1}{x+2}. \\
 & (2x-1)(x^2-4) - (x-1)^2(x+2) = (x^2-1)(x-2). \\
 & x^2 - 4x = 0. \\
 & x = 0, 4.
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & pqx^2 - r qx + psx = rs. \\
 & pqx^2 - r qx + psx - rs = 0. \\
 & (qx + s)(px - r) = 0. \\
 & x = \frac{r}{p}, -\frac{s}{q}.
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & x^2 - 2ax + a^2 - b^2 = 0. \\
 & (x - a - b)(x - a + b) = 0. \\
 & x = a + b, a - b.
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & \frac{m}{m-x} - \frac{m-x}{m} = 2. \\
 & m^2 - m^2 + 2mx - x^2 = 2m^2 - 2mx. \\
 & x^2 - 4mx + 2m^2 = 0. \\
 & x = \frac{4m \pm \sqrt{16m^2 - 8m^2}}{2} \\
 & = 2m \pm m\sqrt{2}.
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & 3x^2 + 2a = x + 6ax. \\
 & 3x^2 - (1 + 6a)x + 2a = 0. \\
 & x = \frac{1 + 6a \pm \sqrt{(1 + 6a)^2 - 24a}}{6} \\
 & = \frac{1 + 6a \pm (1 - 6a)}{6} \\
 & x = \frac{1}{3}, 2a.
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & 5x^2 + 3m = 5mx + 3x. \\
 & 5x^2 - (5m + 3)x + 3m = 0. \\
 & x = \frac{5m + 3 \pm \sqrt{(5m + 3)^2 - 60m}}{10} \\
 & = \frac{5m + 3 \pm (5m - 3)}{10} \\
 & x = m, \frac{3}{5}.
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & ax^2 + hm = mx - ahx. \\
 & ax^2 - (m - ah)x + hm = 0. \\
 & x = \frac{m - ah \pm \sqrt{(m - ah)^2 + 4ahm}}{2a} \\
 & = \frac{m - ah \pm (m + ah)}{2a} \\
 & = \frac{m}{a}, -h.
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & cx^2 + kx = 2mk + 2cmx. \\
 & cx^2 + (k - 2cm)x - 2mk = 0. \\
 & x = \frac{-k + 2cm \pm \sqrt{(k - 2cm)^2 + 8cmk}}{2c} \\
 & = \frac{-k + 2cm \pm (k + 2cm)}{2c}. \\
 & x = 2m, -\frac{k}{c}.
 \end{aligned}$$

25.

$$\begin{aligned}
 x^2 + 2s + s^2 &= 2sx + 2x. \\
 x^2 - (2s + 2)x + 2s + s^2 &= 0. \\
 x &= \frac{2s + 2 \pm \sqrt{4(s + 1)^2 - 8s - 4s^2}}{2} \\
 &= \frac{2s + 2 \pm 2}{2}. \\
 x &= s, s + 2.
 \end{aligned}$$

26.

$$\begin{aligned}
 c^2x^2 + a^2 &= m^2 + 2acx. \\
 c^2x^2 - 2acx + a^2 - m^2 &= 0. \\
 x &= \frac{2ac \pm \sqrt{4a^2c^2 - 4c^2(a^2 - m^2)}}{2c^2} \\
 &= \frac{2ac \pm 2cm}{2c^2}. \\
 x &= \frac{a - m}{c}, \frac{a + m}{c}.
 \end{aligned}$$

27.

$$\begin{aligned}
 cmx^2 + m^2x + a^2 &= am + acx + amx. \\
 cmx^2 + (m^2 - ac - am)x + a^2 - am &= 0. \\
 x &= \frac{-(m^2 - ac - am) \pm \sqrt{(m^2 - ac - am)^2 - 4a^2cm + 4acm^2}}{2cm} \\
 &= \frac{-(m^2 - ac - am) \pm (m^2 + ac - am)}{2cm} \\
 &= \frac{a}{m}, \frac{a - m}{c}.
 \end{aligned}$$

28.

$$\begin{aligned}
 ax^2 - 4a &= bx^2 + 2bx. \\
 (a - b)x^2 - 2bx - 4a &= 0. \\
 x &= \frac{2b \pm \sqrt{4b^2 + 16a(a - b)}}{2(a - b)} \\
 &= \frac{2b \pm (4a - 2b)}{2(a - b)} \\
 &= \frac{2a}{a - b}, -2.
 \end{aligned}$$

29.

$$\begin{aligned}
 2mx^2 + 3ax &= 18m + ax^2. \\
 (2m - a)x^2 + 3ax - 18m &= 0. \\
 x &= \frac{-3a \pm \sqrt{9a^2 + 72m(2m - a)}}{2(2m - a)} \\
 &= \frac{-3a \pm (3a - 12m)}{2(2m - a)} \\
 x &= -\frac{6m}{2m - a}, 3.
 \end{aligned}$$

$$30. \quad (s-x)^2 + (t-x)^2 = (s-t)^2.$$

$$2x^2 - 2(s+t)x + 2st = 0.$$

$$(x-s)(x-t) = 0.$$

$$x = s, t.$$

$$31. \quad 3cx^2 + 50a^2 = 15acx + 2x^2.$$

$$(3c-2)x^2 - 15acx + 50a^2 = 0.$$

$$x = \frac{15ac \pm \sqrt{215a^2c^2 - 200a^2(3c-2)}}{2(3c-2)}$$

$$= \frac{15ac \pm (15ac - 20a)}{2(3c-2)}.$$

$$x = 5a, \frac{10a}{3c-2}.$$

$$32. \quad 2x^2 + 5x = cx^2 + 3cx + 3.$$

$$(2-c)x^2 + (5-3c)x - 3 = 0.$$

$$x = \frac{-(5-3c) \pm \sqrt{(5-3c)^2 + 12(2-c)}}{2(2-c)}$$

$$= \frac{-5+3c \pm (7-3c)}{2(2-c)}.$$

$$x = \frac{1}{2-c}, -3.$$

$$33. \quad x^2 + 2x + 1 = mx^2 + mx.$$

$$(x+1)^2 = mx(x+1).$$

$$(x+1)^2 - mx(x+1) = 0.$$

$$(x+1)(x+1-mx) = 0.$$

$$x = -1, \frac{1}{m-1}.$$

$$34. \quad ax^2 + 3x = 2x^2 + 2ax - 2.$$

$$(a-2)x^2 + (3-2a)x + 2 = 0.$$

$$x = \frac{-(3-2a) \pm \sqrt{(3-2a)^2 - 8(a-2)}}{2(a-2)}$$

$$= \frac{-3+2a \pm (2a-5)}{2(a-2)}.$$

$$x = 2, \frac{1}{a-2}.$$

$$35. \quad 2x^2 + 5x = hx^2 + 3hx + 3.$$

$$(2-h)x^2 + (5-3h)x - 3 = 0.$$

$$\begin{aligned} x &= \frac{-(5-3h) \pm \sqrt{(5-3h)^2 + 12(2-h)}}{2(2-h)} \\ &= \frac{-5+3h \pm (7-3h)}{2(2-h)}. \\ x &= \frac{1}{2-h}, \quad -3. \end{aligned}$$

$$36. \quad 2x^2 + (a+2b)x = a^2 + ab.$$

$$2x^2 + (a+2b)x - a^2 - ab = 0.$$

$$\begin{aligned} x &= \frac{-(a+2b) \pm \sqrt{(a+2b)^2 + 8(a^2+ab)}}{4} \\ &= \frac{-a-2b \pm (3a+2b)}{4}. \\ x &= \frac{1}{2}a, \quad -a-b. \end{aligned}$$

$$37. \quad (3a+b)x^2 - (2a+b)x = a.$$

$$(3a+b)x^2 - (2a+b)x - a = 0.$$

$$\begin{aligned} x &= \frac{2a+b \pm \sqrt{(2a+b)^2 + 4a(3a+b)}}{2(3a+b)} \\ &= \frac{2a+b \pm (4a+b)}{2(3a+b)}. \\ x &= 1, \quad -\frac{a}{3a+b}. \end{aligned}$$

$$38.$$

$$\frac{t^2}{x^2} = \frac{t+1}{x+1}.$$

$$t^2x + t^2 = tx^2 + x^2.$$

$$(t+1)x^2 - t^2x - t^2 = 0.$$

$$\begin{aligned} x &= \frac{t^2 \pm \sqrt{t^4 + 4t^2(t+1)}}{2(t+1)} \\ &= \frac{t^2 \pm t(t+2)}{2(t+1)}. \\ x &= t, \quad -\frac{t}{t+1}. \end{aligned}$$

39.

$$\frac{x - 4a + c}{c - a} = \frac{9c + x}{x}.$$

$$x^2 - 4ax + cx = 9c^2 + cx - 9ac - ax.$$

$$x^2 - 3ax - 9c^2 + 9ac = 0.$$

$$x = \frac{3a \pm \sqrt{9a^2 + 36c^2 - 36ac}}{2}$$

$$= \frac{3a \pm (3a - 6c)}{2}.$$

$$x = 3a - 3c, 3c.$$

40.

$$\frac{1}{mx + 4} + \frac{mx - 4}{16} = 1.$$

$$16 + m^2x^2 - 16 = 16mx + 64.$$

$$m^2x^2 - 16mx = 64.$$

$$m^2x^2 - 16mx + 64 = 128.$$

$$mx - 8 = \pm 8\sqrt{2}.$$

$$x = \frac{8}{m}(1 \pm \sqrt{2}).$$

41.

$$\frac{2a + x}{2a - x} + \frac{a - 2x}{a + 2x} = \frac{8}{3}.$$

$$6a^2 + 15ax + 6x^2 + 6a^2 - 15ax + 6x^2 = 16a^2 + 24ax - 16x^2.$$

$$28x^2 - 24ax - 4a^2 = 0.$$

$$7x^2 - 6ax - a^2 = 0.$$

$$(7x + a)(x - a) = 0.$$

$$x = -\frac{a}{7}, a.$$

42.

$$\frac{x^2 + 1}{x} = \frac{a + m}{t} + \frac{t}{a + m}.$$

Let

$$\frac{a + m}{t} = b.$$

Then

$$\frac{x^2 + 1}{x} = b + \frac{1}{b}.$$

$$bx^2 - (b^2 + 1)x + b = 0.$$

$$x = \frac{b^2 + 1 \pm \sqrt{(b^2 + 1)^2 - 4b^2}}{2b}$$

$$= \frac{b^2 + 1 \pm (b^2 - 1)}{2b}.$$

$$x = b, \frac{1}{b}.$$

Replacing $\frac{a + m}{t}$ for b ,

$$x = \frac{a + m}{t}, \frac{t}{a + m}.$$

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$$43. \quad \frac{2x-r}{s} - \frac{4r}{2x-s} + 3 = 0.$$

$$4x^2 - 2sx - 2rx + rs - 4rs + 3s(2x-s) = 0.$$

$$4x^2 + (4s-2r)x - 3rs - 3s^2 = 0.$$

$$x = \frac{-(4s-2r) \pm \sqrt{(4s-2r)^2 + 16(3rs+3s^2)}}{8}$$

$$= \frac{-4s+2r \pm (8s+2r)}{8}$$

$$x = \frac{r+s}{2}, -\frac{3s}{2}.$$

44.

$$\frac{1}{m+c+x} = \frac{1}{m} + \frac{1}{c} + \frac{1}{x}.$$

$$mcx = cx(m+c+x) + mx(m+c+x) + mc(m+c+x).$$

$$(c+m)x^2 + (c^2+2cm+m^2)x + mc(c+m) = 0.$$

$$x^2 + (c+m)x + mc = 0.$$

$$(x+m)(x+c) = 0.$$

$$x = -m, -c.$$

45.

$$\frac{x+a}{x-t} + \frac{x-a}{x+t} = 1 + \frac{a^2+2t^2}{x^2-t^2}.$$

$$x^2+ax+tx+at+x^2-ax-tx+at = x^2-t^2+a^2+2t^2.$$

$$x^2+2at-a^2-t^2 = 0.$$

$$x-(a-t)^2 = 0.$$

$$x = a-t, t-a.$$

46.

$$\frac{x^2+2ax}{a-b} - \frac{x}{2b} = \frac{a}{b}.$$

$$2bx^2+4abx-(a-b)x = 2a(a-b).$$

$$2bx^2+(4ab-(a-b))x-2a(a-b) = 0.$$

$$(x+2a)(2bx-(a-b)) = 0.$$

$$x = -2a, \frac{a-b}{2b}.$$

47. If

$$y = x^2 + 5x + 2,$$

$$(x^2 + 5x + 2)^2 - 6(x^2 + 5x + 2) = 16 \text{ becomes}$$

$$y^2 - 6y - 16 = 0.$$

$$(y-8)(y+2) = 0.$$

Then

$$x^2 + 5x + 2 - 8 = 0,$$

$$x^2 + 5x - 6 = 0,$$

$$(x+6)(x-1) = 0,$$

$$x = -6, 1,$$

$$\left. \begin{array}{l} x^2 + 5x + 2 - 8 = 0, \\ x^2 + 5x - 6 = 0, \\ (x+6)(x-1) = 0, \\ x = -6, 1, \end{array} \right\} \text{ and } \left\{ \begin{array}{l} x^2 + 5x + 2 + 2 = 0, \\ x^2 + 5x + 4 = 0, \\ (x+4)(x+1) = 0, \\ x = -4, -1. \end{array} \right.$$

$$48. \quad \frac{3x-1}{5x+1} - \frac{x+1}{x-1} = \frac{48}{(5x+1)(1-x)}.$$

$$(3x-1)(x-1) - (x+1)(5x+1) + 48 = 0.$$

$$x^2 + 5x - 24 = 0.$$

$$(x-3)(x+8) = 0.$$

$$x = 3, -8.$$

$$49. \quad x-3 = \frac{x^3 - 10x^2 + 1}{(x-3)^2}.$$

$$(x-3)^3 = x^3 - 10x^2 + 1.$$

$$x^2 + 27x - 28 = 0.$$

$$(x-1)(x+28) = 0.$$

$$x = 1, -28.$$

$$50. \quad 15x^2 - 1.95x + .054 = 0.$$

$$x^2 - .13x + .0036 = 0.$$

$$(x-.04)(x-.09) = 0.$$

$$x = .04, .09.$$

$$51. \quad 10 - 9x = 7x^2.$$

$$7x^2 + 9x - 10 = 0.$$

$$x = \frac{-9 \pm \sqrt{81 + 280}}{14}$$

$$= \frac{-9 \pm 19}{14}$$

$$= \frac{5}{7}, -2.$$

$$52. \quad x^2 + 961a^2 = 62ax.$$

$$x^2 - 62ax + 961a^2 = 0.$$

$$(x-31a)^2 = 0.$$

$$x = 31a, 31a.$$

$$53. \quad (2x+3)(x-4) = (3x-8)(4x-1).$$

$$2x^2 - 5x - 12 = 12x^2 - 35x + 8.$$

$$10x^2 - 30x + 20 = 0.$$

$$x^2 - 3x + 2 = 0.$$

$$(x-2)(x-1) = 0.$$

$$x = 2, 1.$$

$$54. \quad (3x-2)(x-5) = (4x-3)(x+1).$$

$$3x^2 - 17x + 10 = 4x^2 + x - 3.$$

$$x^2 + 18x - 13 = 0.$$

$$x = \frac{-18 \pm \sqrt{324 + 52}}{2}$$

$$= \frac{-18 \pm \sqrt{376}}{2}$$

$$= -9 \pm \sqrt{94}.$$

$$\begin{aligned}
 55. \quad & \frac{1}{x+7} = \frac{2}{5} - \frac{1}{3-x}. \\
 & 5(3-x) = 2(x+7)(3-x) - 5(x+7). \\
 & 15 - 5x = 42 - 8x - 2x^2 - 5x - 35. \\
 & 2x^2 + 8x + 8 = 0. \\
 & x^2 + 4x + 4 = 0. \\
 & (x+2)^2 = 0. \\
 & x = -2, -2.
 \end{aligned}$$

$$\begin{aligned}
 56. \quad & \frac{x}{x^2 + 3x - 4} = \frac{x+2}{x+4}. \\
 & \frac{x}{(x+4)(x-1)} = \frac{x+2}{x+4}. \\
 & x = x^2 + x - 2. \\
 & x^2 - 2 = 0. \\
 & x = \pm \sqrt{2}.
 \end{aligned}$$

$$\begin{aligned}
 57. \quad & \frac{x+3}{x^2 - 3x + 2} = \frac{x-1}{x-2} - \frac{x-2}{x-1}. \\
 & \frac{x+3}{(x-1)(x-2)} = \frac{(x-1)^2 - (x-2)^2}{(x-1)(x-2)}. \\
 & x+3 = 2x-3. \\
 & x = 6.
 \end{aligned}$$

$$\begin{aligned}
 58. \quad & \frac{5x-11}{x-1} + \frac{1-3x}{x+2} = \frac{2x-1}{x+1}. \\
 & (5x-11)(x^2+3x+2) + (1-3x)(x^2-1) = (2x-1)(x^2+x-2). \\
 & \text{Expanding and collecting,}
 \end{aligned}$$

$$\begin{aligned}
 & 4x^2 - 15x - 25 = 0. \\
 & x = \frac{15 \pm \sqrt{225 + 400}}{8} \\
 & = \frac{15 \pm 25}{8} \\
 & = 5, -\frac{5}{4}.
 \end{aligned}$$

$$\begin{aligned}
 59. \quad & 17x = 6x^2 - 10. \\
 & 6x^2 - 17x - 10 = 0. \\
 & x = \frac{17 \pm \sqrt{289 + 240}}{12} \\
 & = \frac{17 \pm 23}{12} \\
 & = \frac{10}{3}, -\frac{1}{2}.
 \end{aligned}$$

$$60. \quad 2k^2x^2 + 8kx = 5(3kx - 1).$$

$$2k^2x^2 - 7kx + 5 = 0.$$

$$(2kx - 5)(kx - 1) = 0.$$

$$x = \frac{5}{2k}, \frac{1}{k}.$$

$$61. \quad 37pqx = 210q^2 - p^2x^2.$$

$$p^2x^2 + 37pqx - 210q^2 = 0.$$

$$(px + 42q)(px - 5q) = 0.$$

$$x = \frac{5q}{p}, -\frac{42q}{p}.$$

$$62. \quad \frac{3x-2}{x-1} + \frac{2}{3} = 9\frac{1}{6} - \frac{5(x+1)}{4x-1}.$$

$$\frac{3x-2}{x-1} = \frac{51}{6} - \frac{5(x+1)}{4x-1}.$$

$$6(12x^2 - 11x + 2) = 51(4x^2 - 5x + 1) - 30(x^2 - 1).$$

$$102x^2 - 189x + 69 = 0.$$

$$34x^2 - 63x + 23 = 0.$$

$$x = \frac{63 \pm \sqrt{3969 - 3128}}{68} = \frac{63 \pm \sqrt{841}}{68} = \frac{63 \pm 29}{68}$$

$$= \frac{2}{1}\frac{3}{7}, \frac{1}{2}.$$

$$63. \quad 3 + \frac{4k}{c-2x} = \frac{k-2x}{c}.$$

$$3c^2 - 6cx + 4ck = ck - 2cx - 2kx + 4x^2.$$

$$4x^2 + (4c - 2k)x - (3c^2 + 3ck) = 0.$$

$$x = \frac{2k - 4c \pm \sqrt{16c^2 - 16ck + 4k^2 + 48c^2 + 48ck}}{8}$$

$$= \frac{2k - 4c \pm \sqrt{64c^2 + 32ck + 4k^2}}{8}$$

$$= \frac{2k - 4c \pm (8c + 2k)}{8}$$

$$= \frac{k+c}{2}, -\frac{3c}{2}.$$

$$64. \quad 3x^2 + .7x = .2.$$

$$30x^2 + 7x - 2 = 0.$$

$$x = \frac{-7 \pm \sqrt{49 + 240}}{60}$$

$$= \frac{-7 \pm 17}{60}$$

$$= \frac{1}{6}, -\frac{2}{5}.$$

$$\begin{aligned}
 65. \quad & \frac{1}{x^2 - 3x + 2} + \frac{1}{x^2 + x - 6} = \frac{1}{3 - 2x - x^2} \\
 & \frac{1}{(x-2)(x-1)} + \frac{1}{(x-2)(x+3)} + \frac{1}{(x+3)(x-1)} = 0. \\
 & (x+3) + (x-1) + (x-2) = 0. \\
 & 3x = 0. \\
 & x = 0.
 \end{aligned}$$

$$\begin{aligned}
 66. \text{ If } & y = x^2 - 2, \\
 & \frac{2(x^2 - 2) - 1}{x^2 - 1} = \frac{x^2 + 2}{(x^2 - 2) - 4} \text{ becomes} \\
 & \frac{2y - 1}{y + 1} = \frac{y + 4}{y - 4}. \\
 & 2y^2 - 9y + 4 = y^2 + 5y + 4. \\
 & y^2 - 14y = 0.
 \end{aligned}$$

$$\text{Then } \left. \begin{aligned} x^2 - 2 &= 0, \\ x^2 &= 2, \\ x &= \pm \sqrt{2}, \end{aligned} \right\} \text{ and } \left\{ \begin{aligned} x^2 - 2 &= 14, \\ x^2 &= 16, \\ x &= \pm 4. \end{aligned} \right.$$

$$67. \quad (x-2)^2 - (1-2x)(3x+5) = 5 - (1-2x)(3x+2).$$

Combining the terms in $1-2x$,

$$\begin{aligned}
 (x-2)^2 - 3(1-2x) &= 5. \\
 x^2 + 2x - 4 &= 0. \\
 x &= \frac{-2 \pm \sqrt{4+16}}{2} \\
 &= -1 \pm \sqrt{5}.
 \end{aligned}$$

$$\begin{aligned}
 68. \text{ If } & y = x^2 - 2x - 2, \\
 & (x^2 - 2x - 2)^2 + 2(x^2 - 2x - 2) - 3 = 0 \text{ becomes} \\
 & y^2 + 2y - 3 = 0. \\
 & (y+3)(y-1) = 0. \\
 \text{Then } & \left. \begin{aligned} x^2 - 2x - 2 + 3 &= 0, \\ x^2 - 2x + 1 &= 0, \\ (x-1)^2 &= 0, \\ x &= 1, 1, \end{aligned} \right\} \text{ and } \left\{ \begin{aligned} x^2 - 2x - 2 - 1 &= 0, \\ x^2 - 2x - 3 &= 0, \\ (x-3)(x+1) &= 0, \\ x &= 3, -1 \end{aligned} \right.
 \end{aligned}$$

$$\begin{aligned}
 69. \text{ If } & y = x^2 - 4x + 1, \\
 & (x^2 - 4x + 1)^2 - 4(x^2 - 4x) - 16 = 0 \text{ becomes} \\
 & y^2 - 4y - 12 = 0. \\
 & (y-6)(y+2) = 0.
 \end{aligned}$$

$$\text{Then } \left. \begin{aligned} x^2 - 4x + 1 - 6 &= 0, \\ (x-5)(x+1) &= 0, \\ x &= 5, -1, \end{aligned} \right\} \text{ and } \left\{ \begin{aligned} x^2 - 4x + 1 + 2 &= 0, \\ (x-3)(x-1) &= 0, \\ x &= 3, 1 \end{aligned} \right.$$

70. If

$$y = 2x - b,$$

$$(2x - b)^2 = a(2x - b) + 2a^2 \text{ becomes}$$

$$y^2 - ay - 2a^2 = 0.$$

$$(y - 2a)(y + a) = 0.$$

Then

$$2x - b - 2a = 0,$$

$$x = \frac{b + 2a}{2},$$

and

$$\left\{ \begin{array}{l} 2x - b + a = 0, \\ x = \frac{b - a}{2}. \end{array} \right.$$

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1. Let x = the second part.Then $42 - x$ = the first part,and $42 - x = x^2$.Whence $x = 6$ or -7 ,and $42 - x = 36$ or 49 .2. Let x = the first number.Then $x + 2$ = the second,and $x(x + 2) = 143$.Whence $x = 11$ or -13 ,and $x + 2 = 13$ or -11 .3. Let x and $x + 1$ = the numbers, respectively.Then $\frac{1}{x} + \frac{1}{x + 1} = \frac{13}{42}$.

$$42(x + 1) + 42x = 13x(x + 1).$$

Whence $x = 6$ or $-\frac{7}{13}$,and $x + 1 = 7$ or $\frac{6}{13}$.

But the term *consecutive numbers* implies integers; therefore we must reject the fractional solutions.

4. Let x = the rate in miles per hour.Then $x + 6$ = the time in hours,and $x(x + 6) = 216$, or the distance in miles.Whence $x = 12$, (Root -18 rejected.)and $x + 6 = 18$.5. Let x = the base in feet.Then $x - 6$ = the altitude in feet,and $\frac{x(x - 6)}{2} = 56$.Whence $x = 14$, (Root -8 rejected.)and $x - 6 = 8$.6. Let x and $x - 9$ = the legs, respectively, in feet,and $\sqrt{x^2 + (x - 9)^2}$ = the hypotenuse.Then $\frac{x(x - 9)}{2} = 45$.Whence $x = 15$, (Root -6 rejected.)and $x - 9 = 6$.

$$\sqrt{x^2 + (x - 9)^2} = \sqrt{15^2 + 6^2} = \sqrt{261} = 3\sqrt{29}.$$

7. Let $x =$ the altitude.
 Then $x + 4$ and $3x =$ the bases, respectively,
 and $\left(\frac{x + 4 + 3x}{2}\right)x = 180$, or the area.
 Whence $x = 9$, (Root $- 10$ rejected.)
 and $x + 4 = 13$,
 and $3x = 27$.
8. Let $x =$ the shorter side in inches.
 Then $\frac{2}{3}x - x =$ the longer side in inches,
 and $x(\frac{2}{3}x - x) = 27$, or the area.
 Whence $x = 4\frac{1}{2}, 6$,
 or $\frac{2}{3}x - x = 6, 4\frac{1}{2}$, which duplicates the first answer
 and may therefore be ignored.
9. Let $n =$ the number of sides in a polygon with
 119 diagonals.
 Then $\frac{1}{2}n(n - 3) = 119$.
 Whence $n = 17$. (Root $- 14$ rejected.)
10. Let $BC = x$.
 Then $6^2 = x(x + 10)$.
 Whence $x = -5 + \sqrt{61}$. (Root $- 5 - \sqrt{61}$ rejected.)
 $= 2.8$.
11. Let $x =$ the number of days for B .
 Then $x + 4 =$ the number of days for A ,
 and $\frac{1}{x} + \frac{1}{x + 4} = 1 \div 2\frac{2}{3} = \frac{3}{8}$.
 Whence $x = 4$. (Root $-\frac{8}{3}$ rejected.)
 $x + 4 = 8$.
12. Let $x =$ the original cost in dollars.
 Then $20 - x =$ the profit in dollars,
 and $\frac{20 - x}{x} =$ per cent of profit *expressed as a decimal*.
 Therefore $\frac{20 - x}{x} = \frac{x + 9}{100}$.
 Whence $x = 16$, (Root $- 125$ rejected.)
 and $\frac{20 - x}{x} = \frac{1}{4} = 25\%$.
13. $(x - 6)(x - 6)(3) = 12$, or the volume of the box.
 Whence $x = 4$ and 8 .
 But if $x = 4$, $x - 6 = -2$, which is absurd.
 Therefore $x = 8$ inches is the only solution.

14. Let $x =$ the number of letters.
 Then $x - 20 =$ the number of lines,
 and $(x - 16)(x - 20 + 15) = x(x - 20)$.
 Whence $x = 80$,
 and $x - 20 = 60$.
15. $(x - 5)(x + 5) - 1 = x + 5 - (x - 5)$.
 $x^2 - 26 = 10$.
 $x = 6$. (Root $- 6$ rejected.)
 Or $(x - 5)(x + 5) - 1 = x - 5 - (x + 5) = -10$.
 Solving, $x = 4$. (Root $- 4$ rejected.)
16. $(4x - 5)(x - 2) = x + 4$.
 $4x^2 - 13x + 10 = x + 4$.
 Whence $x = 3, \frac{1}{2}$.
17. Let $x =$ rate per cent of interest.
 Then $4000 \cdot \frac{x}{100}$, or $40x =$ interest in dollars for first year,
 and $4000 + 40x + 400$, or $4400 + 40x =$ principal for second year. (1)
 Therefore $\frac{x}{100}(4400 + 40x)$, or $\frac{x}{5}(220 + 2x) =$ interest for second year. (2)
 Now $400 + (1) + (2) = 5230$, principal for third year.
 Then $(4400 + 40x) + \frac{x}{5}(220 + 2x) + 400 = 5230$.
 Whence $x = 5$. (Root $- 215$ rejected.)
 Therefore the rate is 5%.
18. Let $x =$ the number in the party.
 Then $\frac{60}{x} =$ the share of each,
 and $\frac{60}{x} - 1 = \frac{60}{x + 2}$.
 Whence $x = 10$. (Root $- 12$ rejected.)
19. Let $x, x + 1$, and $x + 2 =$ the numbers, respectively.
 Then $2x(x + 1) + 2x(x + 2) + 2(x + 1)(x + 2) = 214$.
 $6x^2 + 12x - 210 = 0$.
 Whence $x = 5$, (Root $- 7$ rejected.)
 and $x + 1 = 6, x + 2 = 7$.
20. Let $x =$ the radius of the reduced circle
 in inches,
 and $28 - x =$ the amount in inches the radius
 of the given circle must be
 shortened.

Then $27^2 x^2 = 27^2 \cdot 28^2 - 1078 = 2464 - 1078 = 1386$.
 Whence $x = 21$, (Root $- 21$ rejected.)
 and $28 - x = 28 - 21 = 7$.

21. Let A be the point from which the body A starts, and let B be the point from which the body B starts.

Let x = the number of seconds after which the distance $AB = 75$.

In x seconds A travels $10x$ feet away from the vertex, while B travels $5x$ feet toward it.

After x seconds A will be $5 + 10x$ feet from the vertex, and B will be $35 - 5x$ feet from the vertex.

Then $(5 + 10x)^2 + (35 - 5x)^2 = 75^2 = 5625$.

Whence $x^2 - 2x - 35 = 0$,
 and $x = 7, -5$.

7 is positive and gives a point of time in the future at which A and B will be 75 feet apart.

-5 is negative and gives a point of time in the past at which A and B were 75 feet apart.

Therefore the times are 5 seconds ago and 7 seconds hence.

22. C represents the base and B the top of the mountain. A is the point 40 miles distant (from B , not from C). AB is the line of sight, tangent to the earth's surface. DB here passes through O , the earth's center, therefore $DC = 8000$ miles. Let $BC = x$, and we have

$$\begin{aligned}\overline{AB}^2 &= BC \cdot BD. \\ 1600 &= x(x + 8000). \\ x^2 + 8000x - 1600 &= 0. \\ x &= \frac{-8000 \pm \sqrt{64,006,400}}{2} \\ &= -4000 \pm \sqrt{16,001,600} \\ &= -4000 \pm 4000.2 \text{ (approximately).}\end{aligned}$$

Rejecting the negative root, $x = .2$ miles.

23. $s = \frac{1}{2} \cdot 32 \cdot 8^2 = 16 \cdot 64 = 1024$ feet.

24. $100t - 16t^2 = 136$.

Whence $t = 2, 4\frac{1}{4}$.

The stone goes up beyond a height of 136 feet and passes that height twice — once going up (after 2 seconds) and once going down (after $4\frac{1}{4}$ seconds).

25. $100t - 16t^2 = 0$.

$t = 0, 6\frac{1}{4}$.

The stone was at the starting point to begin with (that is, after 0 seconds) and returns there after $6\frac{1}{4}$ seconds.

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1. $\sqrt{x+1} = 5.$
 $x+1 = 25.$
 $x = 24.$
2. $\sqrt{x+1} = \sqrt{3x-5}.$
 $x+1 = 3x-5.$
 $x = 3.$
3. $3\sqrt{2x-8}-7=17.$
 $3\sqrt{2x-8}=24.$
 $\sqrt{2x-8}=8.$
 $2x-8=64.$
 $x=\frac{72}{2}=36.$
4. $3\sqrt{2x+6}=\sqrt{6x^2-6}.$
 $9(2x+6)=6x^2-6.$
 $6x^2-18x-60=0.$
 $x^2-3x-10=0.$
 $(x-5)(x+2)=0.$
 $x=5, -2.$
5. $\sqrt[3]{2x+3}=3.$
 $2x+3=27.$
 $x=\frac{24}{2}=12.$
6. $7+2\sqrt[3]{2x-1}=13.$
 $2\sqrt[3]{2x-1}=6.$
 $\sqrt[3]{2x-1}=3.$
 $2x-1=27.$
 $x=\frac{28}{2}=14.$
7. $\sqrt[3]{4x+3}-\sqrt[3]{4-3x}=0.$
 $\sqrt[3]{4x+3}=\sqrt[3]{4-3x}.$
 $4x+3=4-3x.$
 $7x=1.$
 $x=\frac{1}{7}.$
8. $\sqrt[3]{2x+2}=\sqrt{2x-2}.$
 $(\sqrt[3]{2x+2})^6=(\sqrt{2x-2})^6.$
 $(2x+2)^2=(2x-2)^3.$
 $4x^2+8x+4=8x^3-24x^2+24x-8.$
 $2x^3-7x^2+4x-3=0.$
 $(x-3)(2x^2-x+1)=0.$
 $x=3, \frac{1 \pm \sqrt{-7}}{4}.$
9. $\sqrt[4]{x+1}=\sqrt{x-1}.$
 $x+1=(x-1)^2.$
 $x^2-3x=x(x-3)=0.$
 $x=0, 3.$ (Root 0 rejected.)
11. $\sqrt{9x}-4=\sqrt{x}.$
 $\sqrt{9x}=\sqrt{x}+4.$
 $9x=x+16+8\sqrt{x}.$
 $8x-16=8\sqrt{x}.$
 $x-2=\sqrt{x}.$
 $x^2-4x+4=x.$
 $x^2-5x+4=0.$
 $(x-1)(x-4)=0.$
 $x=1, 4.$ (Root 1 rejected.)

$$\begin{aligned}
 12. \quad & \sqrt{3x-2} + 5 = \sqrt{x+35}. \\
 & 3x-2+25+10\sqrt{3x-2} = x+35. \\
 & 2x-12 = -10\sqrt{3x-2}. \\
 & x-6 = -5\sqrt{3x-2}. \\
 & x^2-12x+36 = 25(3x-2). \\
 & x^2-87x+86 = 0. \\
 & (x-86)(x-1) = 0. \\
 & x = 86, 1. \qquad (\text{Root } 86 \text{ rejected.})
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & \sqrt{3x+4} - \sqrt{x+5} = \sqrt{5-x}. \\
 & 3x+4+x+5-2\sqrt{3x^2+19x+20} = 5-x. \\
 & 5x+4 = 2\sqrt{3x^2+19x+20}. \\
 & 25x^2+40x+16 = 12x^2+76x+80. \\
 & 13x^2-36x-64 = 0. \\
 & (13x+16)(x-4) = 0. \\
 & x = -\frac{16}{13}, 4. \qquad (\text{Root } -\frac{16}{13} \text{ rejected.})
 \end{aligned}$$

$$\begin{aligned}
 14. \quad & \sqrt{x+2} = \sqrt{x} + \sqrt{2}. \\
 & x+2 = x+2+2\sqrt{2x}. \\
 & 2\sqrt{2x} = 0. \\
 & 8x = 0. \\
 & x = 0.
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & x+1 = \sqrt{2x^2+3x-1}. \\
 & x^2+2x+1 = 2x^2+3x-1. \\
 & x^2+x-2 = 0. \\
 & (x+2)(x-1) = 0. \\
 & x = -2, 1. \qquad (\text{Root } -2 \text{ rejected.})
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \sqrt{x+2} + \sqrt{x-1} - \sqrt{3x+3} = 0. \\
 & \sqrt{x+2} + \sqrt{x-1} = \sqrt{3x+3}. \\
 & 2x+1+2\sqrt{x^2+x-2} = 3x+3. \\
 & 2\sqrt{x^2+x-2} = x+2. \\
 & 4x^2+4x-8 = x^2+4x+4. \\
 & 3x^2-12 = 3(x^2-4) = 0. \\
 & x = \pm 2.
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & x^3 + 5x^{\frac{3}{2}} - 14 = 0. \\
 & x^3 + 5x^{\frac{3}{2}} + \left(\frac{5}{2}\right)^2 = 14 + \frac{25}{4} = \frac{81}{4}. \\
 & x^{\frac{3}{2}} + \frac{5}{2} = \pm \frac{9}{2}. \\
 & x^{\frac{3}{2}} = 2, -7. \\
 & x = \sqrt[3]{4}, \sqrt[3]{49}. \qquad (\text{Root } \sqrt[3]{49} \text{ rejected.})
 \end{aligned}$$

19. $6 = x^{\frac{1}{2}} + x.$
 $x + x^{\frac{1}{2}} + \frac{1}{4} = \frac{25}{4}.$
 $x^{\frac{1}{2}} + \frac{1}{2} = \pm \frac{5}{2}.$
 $x^{\frac{1}{2}} = 2, -3.$
 $x = 4, 9.$ (Root 9 rejected.)

20. $x^{-4} - 17x^{-2} + 52 = 0.$
 $(x^{-2} - 13)(x^{-2} - 4) = 0.$
 $x^{-2} = 4, 13.$
 $x = \pm \frac{1}{2}, \pm \frac{1}{\sqrt{13}} \sqrt{13}.$

21. $2x^{\frac{2}{3}} - 7\sqrt[3]{x} + 6 = 0.$
 $2x^{\frac{2}{3}} - 7x^{\frac{1}{3}} + 6 = 0.$
 $(2x^{\frac{1}{3}} - 3)(x^{\frac{1}{3}} - 2) = 0.$
 $x^{\frac{1}{3}} = \frac{3}{2}, 2.$
 $x = \frac{27}{8}, 8.$

22. $x^{-\frac{3}{2}} = 4x^{-\frac{3}{4}} + 32.$
 $x^{-\frac{3}{2}} - 4x^{-\frac{3}{4}} - 32 = 0.$
 $(x^{-\frac{3}{4}} - 8)(x^{-\frac{3}{4}} + 4) = 0.$
 $x^{-\frac{3}{4}} = 8, -4.$

Whence
and $x^{\frac{3}{4}} = \frac{1}{8}, -\frac{1}{4},$
 $x = \frac{1}{64}, \sqrt[3]{\frac{1}{2 \cdot \frac{1}{8} \cdot 6}}. \text{ (Root } \sqrt[3]{\frac{1}{2 \cdot \frac{1}{8} \cdot 6}} \text{ rejected.)}$

23. $x^{\frac{1}{3}} - x^{\frac{1}{6}} - 6 = 0.$
 $(x^{\frac{1}{6}} - 3)(x^{\frac{1}{6}} + 2) = 0.$
 $x^{\frac{1}{6}} = 3, -2.$
 $x = 3^6, (-2)^6$
 $= 729, 64.$ (Root 64 rejected.)

24. $x^{-1} - \frac{25}{36x^{\frac{1}{2}}} + \frac{1}{9} = 0.$
 $36x^{-1} - 25x^{-\frac{1}{2}} + 4 = 0.$
 $(4x^{-\frac{1}{2}} - 1)(9x^{-\frac{1}{2}} - 4) = 0.$
 $x^{-\frac{1}{2}} = \frac{1}{4}, \frac{4}{9}.$
 $x = 16, \frac{81}{16}.$

25. If

$$y = x^2 + 5x,$$

$$x^2 + 5x + 3\sqrt{x^2 + 5x} - 54 = 0 \text{ becomes}$$

$$y + 3y^{\frac{1}{2}} - 54 = 0.$$

$$(y^{\frac{1}{2}} + 9)(y^{\frac{1}{2}} - 6) = 0.$$

$$y^{\frac{1}{2}} = 6, -9.$$

$$y = 36, 81.$$

But $y = 81$ does not satisfy $y + 3y^{\frac{1}{2}} - 54 = 0$.

Therefore

$$x^2 + 5x = 36 \text{ only.}$$

$$x^2 + 5x - 36 = (x + 9)(x - 4) = 0.$$

$$x = 4, -9.$$

26. If

$$y = 3x^2 - 4x,$$

$$3x^2 - 4x - 11\sqrt{3x^2 - 4x} + 28 = 0 \text{ becomes}$$

$$y - 11y^{\frac{1}{2}} + 28 = 0.$$

$$(y^{\frac{1}{2}} - 7)(y^{\frac{1}{2}} - 4) = 0.$$

$$y = 49, 16.$$

Then

$$3x^2 - 4x - 49 = 0.$$

$$x = \frac{4 \pm \sqrt{16 + 588}}{6} = \frac{2 \pm \sqrt{151}}{3}.$$

Or

$$3x^2 - 4x - 16 = 0.$$

$$x = \frac{4 \pm \sqrt{16 + 192}}{6} = \frac{2 \pm 2\sqrt{13}}{3}.$$

27. If

$$y = 2x^2 - 3x - 1,$$

$$2x^2 - 3x - 4 - \sqrt{2x^2 - 3x - 1} + 1 = 0, \text{ or}$$

$$2x^2 - 3x - 1 - \sqrt{2x^2 - 3x - 1} - 2 = 0, \text{ becomes}$$

$$y - y^{\frac{1}{2}} - 2 = 0.$$

$$(y^{\frac{1}{2}} - 2)(y^{\frac{1}{2}} + 1) = 0.$$

$$y = 4, 1.$$

But $y = 1$ does not satisfy $y - y^{\frac{1}{2}} - 2 = 0$.

Therefore

$$2x^2 - 3x - 1 = 4 \text{ only.}$$

$$2x^2 - 3x - 5 = 0.$$

$$x = \frac{3 \pm \sqrt{9 + 40}}{4}$$

$$= \frac{5}{2}, -1.$$

28. If

$$y = x^2 - 2x - 4,$$

$$x^2 - 2x - 5\sqrt{x^2 - 2x - 4} + 2 = 0, \text{ or}$$

$$x^2 - 2x - 4 - 5\sqrt{x^2 - 2x - 4} + 6 = 0, \text{ becomes}$$

$$y - 5y^{\frac{1}{2}} + 6 = 0.$$

$$(y^{\frac{1}{2}} - 2)(y^{\frac{1}{2}} - 3) = 0.$$

$$y = 4, 9.$$

$$\text{Then } \left. \begin{aligned} x^2 - 2x - 4 &= 4, \\ x^2 - 2x - 8 &= 0, \\ (x-4)(x+2) &= 0, \\ x &= 4, -2, \end{aligned} \right\} \text{ and } \left\{ \begin{aligned} x^2 - 2x - 4 &= 9, \\ x^2 - 2x - 13 &= 0, \\ x &= \frac{2 \pm \sqrt{56}}{2} \\ &= 1 \pm \sqrt{14}. \end{aligned} \right.$$

29. $12x^{\frac{4}{3}} - 27x^{\frac{2}{3}} = 20x^{\frac{2}{3}} - 45.$
 $12x^{\frac{4}{3}} - 47x^{\frac{2}{3}} + 45 = 0.$

$$x^{\frac{2}{3}} = \frac{47 \pm \sqrt{2209 - 2160}}{24}$$

$$= \frac{47 \pm 7}{24}$$

$$= \frac{9}{4}, \frac{5}{3}.$$

$$x = \pm \frac{27}{8}, \pm \frac{5}{9} \sqrt{15}.$$

30. If $y = x^2 + 9,$
 $15 - 2\sqrt{x^2 + 9} = x^2 + 9,$ or
 $x^2 + 9 + 2\sqrt{x^2 + 9} - 15 = 0,$ becomes
 $y + 2y^{\frac{1}{2}} - 15 = 0.$
 $(y^{\frac{1}{2}} + 5)(y^{\frac{1}{2}} - 3) = 0.$
 $y^{\frac{1}{2}} = 3, -5.$

But $y = 25$ does not satisfy $y + 2y^{\frac{1}{2}} - 15 = 0.$

Therefore $x^2 + 9 = 9$ only,

and $x = 0.$

31.
$$\frac{6\sqrt{x} - 8}{\sqrt{2x} - \sqrt{8}} = \sqrt{2}.$$

 $6\sqrt{x} - 8 = 2\sqrt{x} - 4.$
 $4\sqrt{x} = 4.$
 $\sqrt{x} = 1.$
 $x = 1.$

32.
$$\frac{2\sqrt{3}}{\sqrt{2x} - 3} = \frac{\sqrt{2x + 12}}{3\sqrt{3}}.$$

$$18 = \sqrt{4x^2 + 18x - 36}.$$

$$324 = 4x^2 + 18x - 36.$$

$$4x^2 + 18x - 360 = 0.$$

$$2x^2 + 9x - 180 = (2x - 15)(x + 12) = 0.$$

$$x = \frac{15}{2}, -12. \quad (\text{Root } -12 \text{ rejected.})$$

33. If

$$y = x - 5,$$

$$4(x - 5) - 2(x - 5)^{\frac{1}{2}} - 2 = 0, \text{ or}$$

$$2(x - 5) - (x - 5)^{\frac{1}{2}} - 1 = 0, \text{ becomes}$$

$$2y - y^{\frac{1}{2}} - 1 = 0.$$

$$(2y^{\frac{1}{2}} + 1)(y^{\frac{1}{2}} - 1) = 0.$$

$$y^{\frac{1}{2}} = 1, -\frac{1}{2}.$$

$$y = 1, \frac{1}{4}.$$

But $y = \frac{1}{4}$ does not satisfy $2y - y^{\frac{1}{2}} - 1 = 0$.

Therefore $x - 5 = 1$ only.

$$x = 6.$$

34.

$$4x\sqrt{x} = (x + 3)\sqrt{9x}.$$

$$4x\sqrt{x} = x\sqrt{9x} + 3\sqrt{9x}.$$

$$4x\sqrt{x} = 3x\sqrt{x} + 9\sqrt{x}.$$

$$(x - 9)\sqrt{x} = 0.$$

$$x = 9, 0.$$

35.

$$\frac{\sqrt{x+2}}{\sqrt{8-x}} - \frac{\sqrt{8-x}}{\sqrt{x+2}} = \frac{8}{3}.$$

$$\frac{x+2}{8-x} - 2 + \frac{8-x}{x+2} = \frac{64}{9}.$$

$$\frac{x+2}{8-x} + \frac{8-x}{x+2} = \frac{82}{9}.$$

$$9(x+2)^2 + 9(8-x)^2 = 82(x+2)(8-x).$$

Expanding and combining,

$$x^2 - 6x - 7 = 0.$$

$$x = 7, -1.$$

(Root -1 rejected.)

36.

$$t = \pi \sqrt{\frac{l}{g}}.$$

$$t^2 = \pi^2 \frac{l}{g}.$$

Whence

$$l = \frac{gt^2}{\pi^2},$$

and then

$$g = \frac{\pi^2 l}{t^2}.$$

37.

and

$$r = \frac{1}{2}x\sqrt{3},$$

$$A = 2\pi r^2.$$

Then

$$A = 2\pi \left[\frac{1}{2}x\sqrt{3} \right]^2 = \frac{3\pi x^2}{2}.$$

38. $a^2 = 2r^2$,
 and $C = 2\pi r$.
 Then $r^2 = \frac{a^2}{2}$,
 and $r = \frac{a}{2}\sqrt{2}$.
 Therefore $C = 2\pi \cdot \frac{a}{2}\sqrt{2} = \pi a\sqrt{2}$.
39. $A = \frac{3r^2}{2}\sqrt{3}$,
 and $x = \frac{1}{2}r\sqrt{3}$.
 Then $r = \frac{2x}{\sqrt{3}} = \frac{2}{3}x\sqrt{3}$.
 Therefore $A = \frac{3\sqrt{3}}{2}\left[\frac{2}{3}x\sqrt{3}\right]^2 = 2x^2\sqrt{3}$.
40. $K = 3r^2$,
 and $a = \frac{1}{2}r\sqrt{3 + \sqrt{2}}$.
 Then $r = \frac{2a}{\sqrt{3 + \sqrt{2}}}$,
 and $r^2 = \frac{4a^2}{3 + \sqrt{2}} = \frac{4a^2(3 - \sqrt{2})}{7}$.
 Therefore $K = \frac{12a^2}{7}(3 - \sqrt{2})$.

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1. $3\sqrt{-1} + 2\sqrt{-1} = 5\sqrt{-1}$.
2. $4\sqrt{-1} + \sqrt{-4} = 4\sqrt{-1} + 2\sqrt{-1} = 6\sqrt{-1}$.
3. $\sqrt{-25} - \sqrt{-16} = 5\sqrt{-1} - 4\sqrt{-1} = \sqrt{-1}$.
4. $\sqrt{-9} + \sqrt{-4} = 3\sqrt{-1} + 2\sqrt{-1} = 5\sqrt{-1}$.
5. $\sqrt{-4} + \sqrt{-16} = 2\sqrt{-1} + 4\sqrt{-1} = 6\sqrt{-1}$.
6. $(-8)^{\frac{1}{2}} + (-32)^{\frac{1}{2}} = \sqrt{-8} + \sqrt{-32}$
 $= 2\sqrt{2} \cdot i + 4\sqrt{2} \cdot i$
 $= 6\sqrt{2} \cdot i$
 $= 6\sqrt{-2}$.
7. $\sqrt{-18} + \sqrt{-8} = 3\sqrt{2} \cdot i + 2\sqrt{2} \cdot i = 5\sqrt{2} \cdot i = 5\sqrt{-2}$.
8. $4\sqrt{-25x^2} - 2\sqrt{-36x^2} = 4 \cdot 5x\sqrt{-1} - 2 \cdot 6x\sqrt{-1}$
 $= (20x - 12x)\sqrt{-1}$
 $= 8x\sqrt{-1}$.

9. $2 + 3\sqrt{-1} + 6 - 5\sqrt{-1} = 8 - 2\sqrt{-1}.$
10. $7\sqrt{-a^2} - 5a + 4\sqrt{-a^2} = 11\sqrt{-a^2} - 5a = 11a\sqrt{-1} - 5a.$
11. $(3a - 6ib) + (a + ib) = 4a - 5ib.$
12. $4 - 8i + 16 - 3\sqrt{-9} = 4 - 8i + 16 - 3 \cdot 3i = 20 - 17i.$
13. $5 + 3\sqrt{-49x^2} - 6\sqrt{-16x^2} + 4 = 9 + 3 \cdot 7x \cdot \sqrt{-1} - 6 \cdot 4x \cdot \sqrt{-1}$
 $= 9 - 3x\sqrt{-1}.$
14. $18 - 3(-1)^{\frac{1}{2}} + 6(-25)^{\frac{1}{2}} + 4 = 22 - 3\sqrt{-1} + 6 \cdot 5 \cdot \sqrt{-1}$
 $= 22 + 27\sqrt{-1}.$
15. $5\sqrt{-3} + 3\sqrt{-2} - \sqrt{-27} + 2\sqrt{-8}$
 $= 5\sqrt{-3} + 3\sqrt{-2} - 3\sqrt{-3} + 4\sqrt{-2}$
 $= 2\sqrt{-3} + 7\sqrt{-2}.$
16. $(8 - 5\sqrt{-16}) - (7 + 3\sqrt{-25}) = 8 - 20\sqrt{-1} - 7 - 15\sqrt{-1}$
 $= 1 - 35\sqrt{-1}.$
17. $4\sqrt{-9a^4} - 6a^2\sqrt{-16} + 3\sqrt{-6} + 5\sqrt{-54}$
 $= 12a^2\sqrt{-1} - 24a^2\sqrt{-1} + 3\sqrt{-6} + 15\sqrt{-6}$
 $= 18\sqrt{-6} - 12a^2\sqrt{-1}.$
18. $(5x - 6iy) - (3x + 2iy) = 2x - 8iy.$

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1. $(-1)^5 = (-1)^4(-1) = -1.$
2. $(-1)^6 = 1.$
3. $(-1)^7 = (-1)^6(-1) = -1.$
4. $(-1)^8 = 1.$
5. $(\sqrt{-1})^{-4} = \frac{1}{(\sqrt{-1})^4} = \frac{1}{1} = 1.$
6. $(\sqrt{-1})^7 = (\sqrt{-1})^4 \cdot (\sqrt{-1})^3 = (1)(-\sqrt{-1}) = -\sqrt{-1}.$
7. $(\sqrt{-2})^3 = -2\sqrt{-2}.$
8. $(\sqrt{-3})^5 = (\sqrt{-3})^4 \cdot \sqrt{-3} = 9\sqrt{-3}.$
9. $(2\sqrt{-5})^7 = (2)^7 \cdot (\sqrt{-5})^6 \cdot (\sqrt{-5})$
 $= (128)(-125)\sqrt{-5}$
 $= -16,000\sqrt{-5}.$
10. $\sqrt{-4} \cdot \sqrt{-25} = 2i \cdot 5i = 10i^2 = -10.$
11. $\sqrt{-4} \cdot (-\sqrt{-9}) = (2i)(-1)(3i) = -6i^2 = 6.$
12. $\sqrt{-7} \cdot (-\sqrt{-6}) = (\sqrt{7}i)(-1)(\sqrt{6}i) = -\sqrt{42}i^2 = \sqrt{42}.$
13. $\sqrt{-16} \cdot \sqrt{-10} = 4i \cdot \sqrt{10}i = 4\sqrt{10}i^2 = -4\sqrt{10}.$
14. $2\sqrt{-3} \cdot 3\sqrt{-2} = 6\sqrt{3}i\sqrt{2}i = 6\sqrt{6}i^2 = -6\sqrt{6}.$
15. $\sqrt{-a} \cdot \sqrt{-b} = i\sqrt{a} \cdot i\sqrt{b} = \sqrt{ab} \cdot i^2 = -\sqrt{ab}.$
16. $3\sqrt{-7}(-2\sqrt{-5}) = -6i\sqrt{7}i\sqrt{5} = -6i^2\sqrt{35} = 6\sqrt{35}.$
17. $\sqrt{m+n} \cdot \sqrt{-m-n} = \sqrt{m+n} \cdot \sqrt{m+n} \cdot i = (m+n)\sqrt{-1}.$
18. $(3 + \sqrt{-1})(3 - \sqrt{-1}) = 3^2 - (\sqrt{-1})^2 = 9 + 1 = 10.$

19. $(5 + \sqrt{-3})(5 - \sqrt{-3}) = 5^2 - (\sqrt{-3})^2 = 25 + 3 = 28.$
20. $(3 - 4\sqrt{2}i)(3 + 2\sqrt{2}i) = 9 + 6\sqrt{2}i - 12\sqrt{2}i + 16 = 25 - 6\sqrt{-2}.$
21. $(4 + \sqrt{-1})(5 - \sqrt{-3}) = 20 + 5\sqrt{-1} - 4\sqrt{-3} + \sqrt{3}$
 $= 20 + \sqrt{3} + 5\sqrt{-1} - 4\sqrt{-3}.$
22. $(5 - 3i)(6 - 5\sqrt{2}i) = 30 - 25\sqrt{2}i - 18i - 15\sqrt{2}$
 $= (30 - 15\sqrt{2}) - (18 + 25\sqrt{2})i.$
23. $(a + ib)(c + id) = ac + adi + bci - bd = (ac - bd) + (ad + bc)i.$
24. $(a + ib)^2 = a^2 + 2abi - b^2 = a^2 - b^2 + 2abi.$
25. $(a + ib)(a - ib) = a^2 + b^2.$
26. $(-\frac{1}{2} + \frac{1}{2}\sqrt{-3})^2 = \frac{1}{4} - \frac{1}{2}\sqrt{-3} - \frac{3}{4} = -\frac{1}{2} - \frac{1}{2}\sqrt{-3}.$
27. $(-\frac{1}{2} - \frac{1}{2}\sqrt{-3})^2 = \frac{1}{4} + \frac{1}{2}\sqrt{-3} - \frac{3}{4} = -\frac{1}{2} + \frac{1}{2}\sqrt{-3}.$
28. $(-2 + 2\sqrt{-3})^3 = -8 + 3 \cdot 8\sqrt{-3} - 3 \cdot 8(-3) + 8(-3\sqrt{-3})$
 $= -8 + 72$
 $= 64.$
29. $(-2 - 2\sqrt{-3})^3 = -8 - 3 \cdot 8\sqrt{-3} - 3 \cdot 8(-3) - 8(-3\sqrt{-3})$
 $= -8 + 72$
 $= 64.$
30. $(x - iy)^3 = x^3 - 3x^2yi + 3xy^2i^2 - y^3i^3 = (x^3 - 3xy^2) + (y^3 - 3x^2y)i.$
31. $(a + ib)^2 - (a - ib)^2 = (a^2 + 2abi + i^2b^2) - (a^2 - 2abi + i^2b^2) = 4abi.$
32. $(2 + 2\sqrt{-3})^3 - (2 - 2\sqrt{-3})^3$
 $= (8 + 24\sqrt{-3} - 72 - 24\sqrt{-3}) - (8 - 24\sqrt{-3} - 72 + 24\sqrt{-3})$
 $= -64 + 64 = 0.$
33. $(a + i\sqrt{1 - b^2})(a - i\sqrt{1 - b^2}) = a^2 - i^2(1 - b^2) = a^2 - b^2 + 1.$
34. $(5 + 6\sqrt{-2})(5 - 6\sqrt{-2}) = 5^2 - (6\sqrt{-2})^2$
 $= 25 + 72$
 $= 97, \text{ a real number.}$
 $(5 + 6\sqrt{-2}) + (5 - 6\sqrt{-2}) = 10, \text{ a real number.}$
35. $(2 + \sqrt{-3})(2 - \sqrt{-3}) = 2^2 - (\sqrt{-3})^2 = 4 + 3 = 7, \text{ a real number.}$
 $(2 + \sqrt{-3}) + (2 - \sqrt{-3}) = 4, \text{ a real number.}$

Page 392 (First set)

1. $\frac{\sqrt{-12}}{\sqrt{2}} = \frac{\sqrt{-12} \cdot \sqrt{2}}{2} = \frac{2\sqrt{-3} \cdot \sqrt{2}}{2} = \sqrt{-6}.$
2. $\frac{\sqrt{6}}{\sqrt{-2}} = \frac{\sqrt{6} \cdot \sqrt{-2}}{-2} = \frac{\sqrt{-12}}{-2} = \frac{2\sqrt{-3}}{-2} = -\sqrt{-3}.$
3. $\frac{2\sqrt{5}}{4\sqrt{-1}} = \frac{2\sqrt{5}\sqrt{-1}}{-4} = -\frac{1}{2}\sqrt{-5}.$

$$4. \frac{\sqrt{-9}}{\sqrt{-1}} = \frac{\sqrt{-9} \cdot \sqrt{-1}}{-1} = \frac{-3}{-1} = 3.$$

$$5. \frac{1}{\sqrt{-3}} = \frac{\sqrt{-3}}{-3} = -\frac{1}{3}\sqrt{-3}.$$

$$6. \frac{4}{\sqrt{-5}} = \frac{4\sqrt{-5}}{-5} = -\frac{4}{5}\sqrt{-5}.$$

$$7. \frac{\sqrt{8}}{\sqrt{-2}} = \frac{2\sqrt{2} \cdot \sqrt{-2}}{-2} = -\sqrt{2} \cdot \sqrt{2} \cdot \sqrt{-1} = -2\sqrt{-1}.$$

$$8. \frac{(-49)^{\frac{1}{2}}}{(-64)^{\frac{1}{2}}} = \frac{\sqrt{49}\sqrt{-1}}{\sqrt{64}\sqrt{-1}} = \frac{7}{8}.$$

$$9. \frac{\sqrt{by}}{\sqrt{-y}} = \frac{\sqrt{by}\sqrt{-y}}{-y} = \frac{y\sqrt{-b}}{-y} = -\sqrt{-b}.$$

$$10. \frac{\sqrt{-m}}{\sqrt{-n}} = \frac{\sqrt{-m}\sqrt{-1}}{\sqrt{-n}\sqrt{-1}} = \frac{-\sqrt{m}}{-\sqrt{n}} = \sqrt{\frac{m}{n}} = \frac{\sqrt{mn}}{n}.$$

$$11. \frac{(-6bx)^{\frac{1}{2}}}{(-5x)^{\frac{1}{2}}} = \frac{\sqrt{-6bx} \cdot \sqrt{-1}}{\sqrt{-5x} \cdot \sqrt{-1}} = \frac{-\sqrt{6bx}}{-\sqrt{5x}} = \sqrt{\frac{6b}{5}} = \frac{1}{5}\sqrt{30b}.$$

$$12. \frac{(-x^6)^{\frac{1}{2}} - (-x^2)^{\frac{1}{2}}}{(-x)^{\frac{1}{2}}} = \frac{(\sqrt{-x^6})(\sqrt{-x}) - (\sqrt{-x^2})(\sqrt{-x})}{\sqrt{-x} \cdot \sqrt{-x}} \\ = \frac{-\sqrt{x^7} + \sqrt{x^3}}{-x} = x^2\sqrt{x} - \sqrt{x} = (x^2 - 1)\sqrt{x}.$$

$$13. \frac{3}{1 + \sqrt{-1}} = \frac{3(1 - \sqrt{-1})}{1 + 1} = \frac{3 - 3\sqrt{-1}}{2}.$$

$$14. \frac{2}{1 - \sqrt{-2}} = \frac{2(1 + \sqrt{-2})}{1 + 2} = \frac{2 + 2\sqrt{-2}}{3}.$$

$$15. \frac{2\sqrt{-5}}{\sqrt{-1} + 6} = \frac{2\sqrt{-5}(\sqrt{-1} - 6)}{-1 - 36} = \frac{2\sqrt{5} + 12\sqrt{-5}}{37}.$$

$$16. \frac{2\sqrt{-3}}{3\sqrt{-2} + 3} = \frac{2\sqrt{-3}(3\sqrt{-2} - 3)}{-18 - 9} = \frac{-6\sqrt{6} - 6\sqrt{-3}}{-27} \\ = \frac{2\sqrt{6} + 2\sqrt{-3}}{9}.$$

$$17. \frac{-1 - \sqrt{-3}}{-1 + \sqrt{-3}} = \frac{1 + 2\sqrt{-3} - 3}{1 + 3} = \frac{-1 + \sqrt{-3}}{2}.$$

$$18. \frac{1 + 2i}{3 - 4i} = \frac{(1 + 2i)(3 + 4i)}{9 + 16} = \frac{10i - 5}{25} = \frac{2i - 1}{5}.$$

$$19. \frac{x}{x+iy} = \frac{x(x-iy)}{x^2+y^2} = \frac{x^2-xyi}{x^2+y^2}.$$

$$20. \frac{a+ib}{c+id} = \frac{(a+ib)(c-id)}{c^2+d^2} = \frac{ac+bci-adi+bd}{c^2+d^2} = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}.$$

$$\begin{aligned} 21. (3+2i)(1-i) \div (3-4i)(1+i) &= (3-i+2) \div (3-i+4) \\ &= \frac{5-i}{7-i} \\ &= \frac{(5-i)(7+i)}{50} \\ &= \frac{36-2i}{50} \\ &= \frac{18-i}{25}. \end{aligned}$$

$$22. (1-\sqrt{-3})^3 = 1-3\sqrt{-3}+(3)(-3)-(-3)(\sqrt{-3}) = -8. \text{ Yes.}$$

$$23. \text{Substituting } 3 \pm \sqrt{-3} \text{ for } x \text{ in } x^2 - 6x + 12 = 0,$$

$$\begin{aligned} (3 \pm \sqrt{-3})^2 - 6(3 \pm \sqrt{-3}) + 12 \\ = 9 \pm 6\sqrt{-3} - 3 - 18 \mp 6\sqrt{-3} + 12 \\ = 0 \text{ with either sign. Yes.} \end{aligned}$$

$$24. \text{Substituting these values in } x^2 - xy - 12y^2 = 8, \text{ we get}$$

$$\left(\frac{6}{5}\right)(-10) - \left(\frac{8}{5}\right)\left(-\frac{3}{5}\right)(-10) - 12\left(\frac{9}{25}\right)(-10) = 8,$$

$$\frac{-128}{5} - \frac{48}{5} + \frac{216}{5} = 8.$$

$$\frac{-176 + 216}{5} = 8.$$

$$\frac{40}{5} = 8, \text{ an identity.}$$

$$\text{Substituting in } x^2 + xy - 10y^2 = 20,$$

$$\left(\frac{6}{5}\right)(-10) + \left(\frac{8}{5}\right)\left(-\frac{3}{5}\right)(-10) - 10\left(\frac{9}{25}\right)(-10) = 20.$$

$$\frac{-128}{5} + \frac{48}{5} + \frac{180}{5} = 20.$$

$$\frac{-128 + 228}{5} = 20.$$

$$\frac{100}{5} = 20, \text{ an identity.}$$

Hence the given pair of values satisfies the given system.

Page 392 (Second set)

$$1. \quad x^2 + 4x + 8 = 0.$$

$$x = \frac{-4 \pm \sqrt{16-32}}{2} = -2 \pm 2\sqrt{-1}.$$

$$2. \quad x^2 - 8x + 24 = 0.$$

$$x = \frac{8 \pm \sqrt{64-96}}{2} = 4 \pm 2\sqrt{-2}.$$

$$3. \quad x^2 + 3x + 9 = 0.$$

$$x = \frac{-3 \pm \sqrt{9 - 36}}{2} = \frac{-3 \pm 3\sqrt{-3}}{2}.$$

$$4. \quad x^2 - 5x + 16 = 0.$$

$$x = \frac{5 \pm \sqrt{25 - 64}}{2} = \frac{5 \pm \sqrt{-39}}{2}.$$

$$5. \quad 3x^2 + 2x + 4 = 0.$$

$$x = \frac{-2 \pm \sqrt{4 - 48}}{6} = \frac{-1 \pm \sqrt{-11}}{3}.$$

$$6. \quad x^2 + x + 1 = 0.$$

$$x = \frac{-1 \pm \sqrt{1 - 4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}.$$

$$7. \quad x^2 - x + 1 = 0.$$

$$x = \frac{1 \pm \sqrt{1 - 4}}{2} = \frac{1 \pm \sqrt{-3}}{2}.$$

$$8. \quad 5x^2 - 6x + 14 = 0.$$

$$x = \frac{6 \pm \sqrt{36 - 280}}{10} = \frac{3 \pm \sqrt{-61}}{5}.$$

$$9. \quad 6x^2 + 10x + 21 = 0.$$

$$x = \frac{-10 \pm \sqrt{100 - 504}}{12} = \frac{-5 \pm \sqrt{-101}}{6}.$$

$$10. \quad 3x^2 + 16x + 21 = 0.$$

$$x = \frac{-16 \pm \sqrt{256 - 252}}{6}$$

$$= \frac{-16 \pm 2}{6}$$

$$= -3, -\frac{7}{3}.$$

$$11. \quad x^3 - 1 = 0.$$

$$(x - 1)(x^2 + x + 1) = 0.$$

$$x = 1, \frac{-1 \pm \sqrt{-3}}{-2}. \quad (\text{See Ex. 6.})$$

$$12. \quad x^3 + 1 = 0.$$

$$(x + 1)(x^2 - x + 1) = 0.$$

$$x = -1, \frac{1 \pm \sqrt{-3}}{2}. \quad (\text{See Ex. 7.})$$

$$13. \quad x^3 = 8.$$

$$x^3 - 8 = 0.$$

$$(x - 2)(x^2 + 2x + 4) = 0.$$

$$x = 2, \frac{-2 \pm \sqrt{4 - 16}}{2}$$

$$= 2, -1 \pm \sqrt{-3}.$$

14. $x^3 = -27$.
 $x^3 + 27 = 0$.
 $(x + 3)(x^2 - 3x + 9) = 0$.
 $x = -3, \frac{3 \pm \sqrt{9 - 36}}{2} = -3, \frac{3 + 3\sqrt{-3}}{2}$.
15. $x^4 = 1$.
 $x^4 - 1 = 0$.
 $(x^2 + 1)(x + 1)(x - 1) = 0$.
 $x = \pm \sqrt{-1}, \pm 1$.
16. $x^4 = 9$.
 $x^4 - 9 = 0$.
 $(x^2 - 3)(x^2 + 3) = 0$.
 $x = \pm \sqrt{3}, \pm \sqrt{-3}$.
17. $x^6 = 1$.
 $x^6 - 1 = 0$.
 $(x + 1)(x^2 - x + 1)(x - 1)(x^2 + x + 1) = 0$.
 $x = \pm 1, \frac{1 \pm \sqrt{-3}}{2}, \frac{-1 \pm \sqrt{-3}}{2}$.
18. $x^6 = 64$.
 $x^6 - 64 = 0$.
 $(x^3 + 8)(x^3 - 8) = 0$.
 $(x + 2)(x - 2)(x^2 - 2x + 4)(x^2 + 2x + 4) = 0$.
 $x = \pm 2, \frac{2 \pm \sqrt{4 - 16}}{2}, \frac{-2 \pm \sqrt{4 - 16}}{2}$
 $= \pm 2, 1 \pm \sqrt{-3}, -1 \pm \sqrt{-3}$.
19. $x^3 = 64$.
 $x^3 - 64 = 0$.
 $(x - 4)(x^2 + 4x + 16) = 0$.
 $x = 4, \frac{-4 \pm \sqrt{16 - 64}}{2}$
 $= 4, -2 \pm 2\sqrt{-3}$.
20. $x^3 = -125$.
 $x^3 + 125 = 0$.
 $(x + 5)(x^2 - 5x + 25) = 0$.
 $x = -5, \frac{5 \pm \sqrt{25 - 100}}{2}$
 $= -5, \frac{5 \pm 5\sqrt{-3}}{2}$.

21. Two ; three ; four ; six.

22. That it has n n th roots.

23. $27x^3 - 8 = 0.$

$$(3x - 2)(9x^2 + 6x + 4) = 0.$$

$$\begin{aligned} x &= \frac{2}{3}, \frac{-6 \pm \sqrt{36 - 144}}{18} \\ &= \frac{2}{3}, \frac{-1 \pm \sqrt{-3}}{3}. \end{aligned}$$

24. $64x^3 + 125 = 0.$

$$(4x + 5)(16x^2 - 20x + 25) = 0.$$

$$\begin{aligned} x &= -\frac{5}{4}, \frac{20 \pm \sqrt{400 - 1600}}{32} \\ &= -\frac{5}{4}, \frac{5 \pm 5\sqrt{-3}}{8}. \end{aligned}$$

25. $x^4 - 2x^2 - 8 = 0.$

$$(x^2 - 4)(x^2 + 2) = 0.$$

$$x = \pm 2, \pm \sqrt{-2}.$$

26. $x^3 + x^2 - 2x - 2 = 0.$

$$(x^2 - 2)(x + 1) = 0.$$

$$x = -1, \pm \sqrt{2}.$$

27. $x^6 + 7x^3 - 8 = 0.$

$$(x^3 + 8)(x^3 - 1) = 0.$$

$$(x + 2)(x - 1)(x^2 - 2x + 4)(x^2 + x + 1) = 0.$$

$$x = -2, 1, 1 \pm \sqrt{-3}, \frac{-1 \pm \sqrt{-3}}{2}.$$

28. $3x^4 + 16x^2 + 21 = 0.$

$$(3x^2 + 7)(x^2 + 3) = 0.$$

$$\begin{aligned} x &= \pm \sqrt{-\frac{7}{3}}, \pm \sqrt{-3} \\ &= \pm \frac{1}{3} \sqrt{-21}, \pm \sqrt{-3}. \end{aligned}$$

29. $27x^4 - 12x^2 - 64 = 0.$

$$x^2 = \frac{12 \pm \sqrt{144 + 6912}}{54} = \frac{12 \pm 84}{54}$$

$$= -\frac{4}{3}, \frac{16}{9}.$$

$$x = \pm \sqrt{-\frac{4}{3}}, \pm \frac{4}{3}$$

$$= \pm \frac{2}{3} \sqrt{-3}, \pm \frac{4}{3}.$$

30. $6x^6 + 21x^3 + 9 = 0.$

$$2x^6 + 7x^3 + 3 = 0.$$

$$(2x^3 + 1)(x^3 + 3) = 0.$$

$$(x\sqrt[3]{2} + 1)(x^2\sqrt[3]{4} - x\sqrt[3]{2} + 1)(x + \sqrt[3]{3})(x^2 - x\sqrt[3]{3} + \sqrt[3]{9}) = 0. \text{ (See § 196.)}$$

From $x\sqrt[3]{2} + 1 = 0$, $x = -\frac{1}{\sqrt[3]{2}} = -\frac{1}{2}\sqrt[3]{4};$

and from $x^2\sqrt[3]{4} - x\sqrt[3]{2} + 1 = 0$,

$$\begin{aligned} x &= \frac{\sqrt[3]{2} \pm \sqrt{\sqrt[3]{4} - 4\sqrt[3]{4}}}{2\sqrt[3]{4}} \\ &= \frac{\sqrt[3]{2} \pm \sqrt{-3\sqrt[3]{4}}}{2\sqrt[3]{4}} \\ &= \frac{\sqrt[3]{4} \pm \sqrt[3]{2} \cdot \sqrt{-3\sqrt[3]{4}}}{4} \\ &= \frac{\sqrt[3]{4} \pm \sqrt[6]{-432}}{4}; \end{aligned}$$

and from $x + \sqrt[3]{3} = 0$, $x = -\sqrt[3]{3};$

and from $x^2 - x\sqrt[3]{3} + \sqrt[3]{9} = 0$,

$$\begin{aligned} x &= \frac{\sqrt[3]{3} \pm \sqrt{\sqrt[3]{9} - 4\sqrt[3]{9}}}{2} \\ &= \frac{\sqrt[3]{3} \pm \sqrt{-3\sqrt[3]{9}}}{2} \\ &= \frac{\sqrt[3]{3} \pm \sqrt[6]{-243}}{2}. \end{aligned}$$

Whence $x = -\frac{1}{2}\sqrt[3]{4}, -\sqrt[3]{3}, \frac{\sqrt[3]{4} \pm \sqrt[6]{-432}}{4}, \frac{\sqrt[3]{3} \pm \sqrt[6]{-243}}{2}.$

31. $25x^4 + 40x^2 + 64 = 0.$

$$x^2 = \frac{-40 \pm \sqrt{1600 - 6400}}{50} = \frac{-4 \pm 4\sqrt{-3}}{5}.$$

$$\begin{aligned} x &= \pm \sqrt{\frac{-4 \pm 4\sqrt{-3}}{5}} = \pm \frac{2}{5} \sqrt{-5 \pm 5\sqrt{-3}} \\ &= \pm \frac{1}{5} (\sqrt{10} \pm \sqrt{-30}). \end{aligned}$$

32. $(x^2 + 4)(x^2 + 3x + 7) = 0.$

$$x = \pm 2\sqrt{-1}, \frac{-3 \pm \sqrt{-19}}{2}.$$

33. If

$$y = x^2 + 2x,$$

$$(x^2 + 2x)^2 + 15(x^2 + 2x) + 54 = 0 \text{ becomes}$$

$$(y + 6)(y + 9) = 0.$$

$$y = -6, -9.$$

$$\left. \begin{aligned} \text{Then } x^2 + 2x &= -6, \\ x^2 + 2x + 6 &= 0, \\ x &= \frac{-2 \pm \sqrt{4 - 24}}{2}, \\ x &= -1 \pm \sqrt{-5}, \end{aligned} \right\} \text{ and } \left\{ \begin{aligned} x^2 + 2x &= -9, \\ x^2 + 2x + 9 &= 0, \\ x &= \frac{-2 \pm \sqrt{4 - 36}}{2}, \\ x &= -1 \pm 2\sqrt{-2}. \end{aligned} \right.$$

34. If $y = x^2 + 5x$,
 $(x^2 + 5x)^2 + 9(x^2 + 5x) - 112 = 0$ becomes
 $(y - 7)(y + 16) = 0.$

$$\text{Then } \left\{ \begin{aligned} x^2 + 5x - 7 &= 0, \\ x &= \frac{-5 \pm \sqrt{53}}{2}, \end{aligned} \right\} \text{ and } \left\{ \begin{aligned} x^2 + 5x + 16 &= 0, \\ x &= \frac{-5 \pm \sqrt{-39}}{2}. \end{aligned} \right.$$

35. $x + y = 4,$ (1)

$x^2 - 3xy - y^2 = -39.$ (2)

From (1), $y = 4 - x.$ (3)

In (2),

$$x^2 - 3x(4 - x) - (4 - x)^2 = -39.$$

$$3x^2 - 4x - 16 = -39.$$

$$3x^2 - 4x + 23 = 0.$$

$$x = \frac{4 \pm \sqrt{16 - 276}}{6} = \frac{2 \pm \sqrt{-65}}{3}.$$

$$y = 4 - x = \frac{12}{3} - \frac{2 \pm \sqrt{-65}}{3} = \frac{10 \mp \sqrt{-65}}{3}.$$

Therefore

$$x = \frac{2 + \sqrt{-65}}{3}, \frac{2 - \sqrt{-65}}{3},$$

and

$$y = \frac{10 - \sqrt{-65}}{3}, \frac{10 + \sqrt{-65}}{3}.$$

36. $x + 2y = 4,$ (1)

$y^2 - x = 0.$ (2)

In (1), $x = 4 - 2y.$

In (2), $x = y^2.$

Hence $y^2 = 4 - 2y.$

$$y^2 + 2y - 4 = 0.$$

$$y = \frac{-2 \pm \sqrt{4 + 16}}{2} = -1 \pm \sqrt{5},$$

and

$$x = 4 - 2y = 4 - (-2 \pm 2\sqrt{5}) = 6 \mp 2\sqrt{5}.$$

Therefore

$$x = 6 - 2\sqrt{5}, 6 + 2\sqrt{5},$$

and

$$y = -1 + \sqrt{5}, -1 - \sqrt{5}.$$

$$37. \quad x^2 + y^2 = 4, \quad (1)$$

$$x - y = 6. \quad (2)$$

$$\text{Squaring (2),} \quad x^2 - 2xy + y^2 = 36. \quad (3)$$

$$(1) - (3), \quad 2xy = -32. \quad (4)$$

$$(1) + (4), \quad (x + y)^2 = -28. \quad (5)$$

$$\text{Then} \quad x + y = \pm \sqrt{-28} = \pm 2\sqrt{-7}. \quad (6)$$

$$x - y = 6. \quad (7)$$

$$\text{From (7) and } x + y = 2\sqrt{-7}, \quad 2x = 6 + 2\sqrt{-7}.$$

$$\text{Then} \quad x = 3 + \sqrt{-7},$$

$$\text{and} \quad y = x - 6 = -3 + \sqrt{-7}.$$

$$\text{From (7) and } x + y = -2\sqrt{-7}, \quad 2x = 6 - 2\sqrt{-7}.$$

$$\text{Then} \quad x = 3 - \sqrt{-7},$$

$$\text{and} \quad y = x - 6 = -3 - \sqrt{-7}.$$

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$$1. \quad (x - 2)(x - 3) = 0.$$

$$x^2 - 5x + 6 = 0.$$

$$2. \quad (x - 3)(x - 7) = 0.$$

$$x^2 - 10x + 21 = 0.$$

$$3. \quad (x - 1)(x + 3) = 0.$$

$$x^2 + 2x - 3 = 0.$$

$$4. \quad (x + 2)(x + 5) = 0.$$

$$x^2 + 7x + 10 = 0.$$

$$5. \quad (x - 2)(x - \frac{2}{5}) = 0.$$

$$(x - 2)(5x - 2) = 0.$$

$$5x^2 - 12x + 4 = 0.$$

$$6. \quad (x - 5)(x - \frac{3}{7}) = 0.$$

$$(x - 5)(7x - 3) = 0.$$

$$7x^2 - 38x + 15 = 0.$$

$$7. \quad (x + \frac{2}{3})(x - \frac{5}{6}) = 0.$$

$$(3x + 2)(6x - 5) = 0.$$

$$18x^2 - 3x - 10 = 0.$$

$$8. \quad (x + \frac{4}{5})(x + \frac{7}{8}) = 0.$$

$$(5x + 4)(8x + 7) = 0.$$

$$40x^2 + 67x + 28 = 0.$$

$$9. \quad (x - 1 - \sqrt{3})(x - 1 + \sqrt{3}) = 0.$$

$$x^2 - 2x - 2 = 0.$$

$$10. \quad (x - 3 - \sqrt{7})(x - 3 + \sqrt{7}) = 0.$$

$$x^2 - 6x + 2 = 0.$$

$$11. \quad (x - 2 - \sqrt{-5})(x - 2 + \sqrt{-5}) = 0.$$

$$x^2 - 4x + 9 = 0.$$

$$12. \quad (x + 7 - \sqrt{-5})(x + 7 + \sqrt{-5}) = 0.$$

$$x^2 + 14x + 54 = 0.$$

$$13. \quad (x - \frac{1}{2} - \sqrt{\frac{3}{2}})(x - \frac{1}{2} + \sqrt{\frac{3}{2}}) = 0.$$

$$x^2 - x - \frac{5}{4} = 0.$$

$$4x^2 - 4x - 5 = 0.$$

14. $(x - \frac{1}{2} - \sqrt{-\frac{3}{2}})(x - \frac{1}{2} + \sqrt{-\frac{3}{2}}) = 0.$
 $x^2 - x + \frac{7}{4} = 0.$
 $4x^2 - 4x + 7 = 0.$
15. $(x - 2)(x - 3)(x - 4) = 0.$
 $x^3 - 9x^2 + 26x - 24 = 0.$
16. $(x - 1)(x - 3)(x - 5) = 0.$
 $x^3 - 9x^2 + 23x - 15 = 0.$
17. $(x - 1)(x + 1)(x - 2) = 0.$
 $x^3 - 2x^2 - x + 2 = 0.$
18. $(x - \frac{1}{2})(x - \frac{1}{3})(x - \frac{1}{4}) = 0.$
 $(2x - 1)(3x - 1)(4x - 1) = 0.$
 $24x^3 - 26x^2 + 9x - 1 = 0.$
19. $(x - \sqrt{-2})(x + \sqrt{-2})(x - 2) = 0.$
 $(x^2 + 2)(x - 2) = 0.$
 $x^3 - 2x^2 + 2x - 4 = 0.$
20. $(x - 1 - \sqrt{2})(x - 1 + \sqrt{2})(x - 3) = 0.$
 $(x^2 - 2x - 1)(x - 3) = 0.$
 $x^3 - 5x^2 + 5x + 3 = 0.$
21. $(x - 1)(x + 1)(x - \sqrt{-1})(x + \sqrt{-1}) = 0.$
 $(x^2 - 1)(x^2 + 1) = 0.$
 $x^4 - 1 = 0.$
22. $(x - \sqrt{2} + \sqrt{3})(x - \sqrt{2} - \sqrt{3})(x + \sqrt{2} + \sqrt{3})(x + \sqrt{2} - \sqrt{3}) = 0.$
 $(x^2 - 2\sqrt{2}x - 1)(x^2 + 2\sqrt{2}x - 1) = 0.$
 $x^4 - 10x^2 + 1 = 0.$

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2. $-(\frac{4}{3} + \frac{1}{2}) = -\frac{11}{6}; (\frac{4}{3})(\frac{1}{2}) = \frac{2}{3}.$
 $x^2 - \frac{11}{6}x + \frac{2}{3} = 0$, or $6x^2 - 11x + 4 = 0.$
3. $-(\frac{1}{4} - \frac{2}{3}) = \frac{5}{12}; (\frac{1}{4})(-\frac{2}{3}) = -\frac{1}{6}.$
 $x^2 + \frac{5}{12}x - \frac{1}{6} = 0$, or $12x^2 + 5x - 2 = 0.$
4. $-(\frac{3}{2} - \frac{16}{5}) = \frac{17}{10}; (\frac{3}{2})(-\frac{16}{5}) = -\frac{24}{5}.$
 $x^2 + \frac{17}{10}x - \frac{24}{5} = 0$, or $10x^2 + 17x - 48 = 0.$
5. $-(4.41 + 1.59) = -6; (4.41)(1.59) = 7.0119.$
 $x^2 - 6x + 7.0119 = 0.$
6. $-(2 + 3\sqrt{3} + 2 - 3\sqrt{3}) = -4; (2 + 3\sqrt{3})(2 - 3\sqrt{3}) = -23.$
 $x^2 - 4x - 23 = 0.$
7. $-(4 + \sqrt{-2} + 4 - \sqrt{-2}) = -8; (4 + \sqrt{-2})(4 - \sqrt{-2}) = 18.$
 $x^2 - 8x + 18 = 0.$

$$8. -\left(\frac{3}{2} + \sqrt{7} + \frac{3}{2} - \sqrt{7}\right) = -3; \left(\frac{3}{2} + \sqrt{7}\right)\left(\frac{3}{2} - \sqrt{7}\right) = -\frac{19}{4}.$$

$$x^2 - 3x - \frac{19}{4} = 0, \text{ or } 4x^2 - 12x - 19 = 0.$$

$$9. -\left(\frac{1}{2} + \frac{1}{2}\sqrt{-3} + \frac{1}{2} - \frac{1}{2}\sqrt{-3}\right) = -1;$$

$$\left(\frac{1}{2} + \frac{1}{2}\sqrt{-3}\right)\left(\frac{1}{2} - \frac{1}{2}\sqrt{-3}\right) = 1.$$

$$x^2 - x + 1 = 0.$$

$$10. -\left(\frac{5}{3} + \sqrt{3} + \frac{5}{3} - \sqrt{3}\right) = -\frac{10}{3}; \left(\frac{5}{3} + \sqrt{3}\right)\left(\frac{5}{3} - \sqrt{3}\right) = -\frac{2}{9}.$$

$$x^2 - \frac{10}{3}x - \frac{2}{9} = 0, \text{ or } 9x^2 - 30x - 2 = 0.$$

$$11. -(1 + \sqrt{-1} + 1 - \sqrt{-1}) = -2; (1 + \sqrt{-1})(1 - \sqrt{-1}) = 2.$$

$$x^2 - 2x + 2 = 0.$$

$$12. -\left(a - \frac{1}{a}\right) = \frac{1 - a^2}{a}; (a)\left(-\frac{1}{a}\right) = -1.$$

$$x^2 + \frac{(1 - a^2)x}{a} - 1 = 0, \text{ or } ax^2 + (1 - a^2)x - a = 0.$$

$$13. -\left(\frac{3a}{2} + \frac{5a}{2}\right) = -4a; \left(\frac{3a}{2}\right)\left(\frac{5a}{2}\right) = \frac{15a^2}{4}.$$

$$x^2 - 4ax + \frac{15a^2}{4} = 0, \text{ or } 4x^2 - 16ax + 15a^2 = 0.$$

$$14. -(1 + a + 1 - a) = -2; (1 + a)(1 - a) = 1 - a^2.$$

$$x^2 - 2x + 1 - a^2 = 0.$$

$$15. -\left(\frac{a+b}{a-b} + \frac{a-b}{a+b}\right) = \frac{-2(a^2 + b^2)}{a^2 - b^2}; \left(\frac{a+b}{a-b}\right)\left(\frac{a-b}{a+b}\right) = 1.$$

$$x^2 + \frac{-2(a^2 + b^2)}{a^2 - b^2}x + 1 = 0,$$

or $(a^2 - b^2)x^2 - 2(a^2 + b^2)x + a^2 - b^2 = 0.$

16. For the roots 6, 8, and $\frac{1}{2}$, the equation is

$$(x - 6)(x - 8)\left(x - \frac{1}{2}\right) = 0.$$

$$(1) \cdot 2, \quad (x - 6)(x - 8)(2x - 1) = 0.$$

$$2x^3 - 29x^2 + 110x - 48 = 0.$$

$$17. \quad (x + 4)(x - 4)\left(x - \frac{1}{4}\right) = 0.$$

$$(x^2 - 16)(4x - 1) = 0.$$

$$4x^3 - x^2 - 64x + 16 = 0.$$

$$18. \quad x^2 - 5x + 6 = 0.$$

$$x = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm 1}{2}$$

$$= 3, 2.$$

$$-(3 + 2) = -5; 3 \cdot 2 = 6.$$

(1)

19.

$$x^2 - x - 3 = 0.$$

$$x = \frac{1 \pm \sqrt{13}}{2}.$$

$$-\left(\frac{1 + \sqrt{13}}{2} + \frac{1 - \sqrt{13}}{2}\right) = -1; \left(\frac{1}{2} + \frac{1}{2}\sqrt{13}\right)\left(\frac{1}{2} - \frac{1}{2}\sqrt{13}\right) = -3.$$

20.

$$x^2 - 2x - 4 = 0.$$

$$x = \frac{2 \pm \sqrt{4 + 16}}{2} = 1 \pm \sqrt{5}.$$

$$-(1 + \sqrt{5} + 1 - \sqrt{5}) = -2; (1 + \sqrt{5})(1 - \sqrt{5}) = -4.$$

21.

$$x^2 - 9x - 10 = 0.$$

$$x = \frac{9 \pm \sqrt{81 + 40}}{2} = \frac{9 \pm 11}{2}$$

$$= 10, -1.$$

$$-(10 - 1) = -9; (10)(-1) = -10.$$

22.

$$x^2 + 2x + 1 = 0.$$

$$x = \frac{-2 \pm \sqrt{4 - 4}}{2}$$

$$= -1, -1.$$

$$-(-1 - 1) = 2; (-1)(-1) = 1.$$

23.

$$x^2 + 8x + 16 = 0.$$

$$x = \frac{-8 \pm \sqrt{64 - 64}}{2}$$

$$= -4, -4.$$

$$-(-4 - 4) = 8; (-4)(-4) = 16.$$

24.

$$x^2 + 5x + 5 = 0.$$

$$x = \frac{-5 \pm \sqrt{25 - 20}}{2} = \frac{-5 \pm \sqrt{5}}{2}.$$

$$-\left(\frac{-5 + \sqrt{5}}{2} + \frac{-5 - \sqrt{5}}{2}\right) = 5;$$

$$\left(\frac{-5 + \sqrt{5}}{2}\right)\left(\frac{-5 - \sqrt{5}}{2}\right) = \frac{25 - 5}{4} = 5.$$

25.

$$2x^2 + 3x - 6 = 0.$$

$$x = \frac{-3 \pm \sqrt{9 + 48}}{4} = \frac{-3 \pm \sqrt{57}}{4}.$$

$$-\left(\frac{-3 + \sqrt{57}}{4} + \frac{-3 - \sqrt{57}}{4}\right) = \frac{3}{2}.$$

$$\left(\frac{-3 + \sqrt{57}}{4}\right)\left(\frac{-3 - \sqrt{57}}{4}\right) = \frac{9 - 57}{16} = -3 = \frac{-6}{2}.$$

26. $3x^2 + 3x - 5 = 0.$

$$x = \frac{-3 \pm \sqrt{9 + 60}}{6} = -\frac{1}{2} \pm \frac{1}{6}\sqrt{69}.$$

$$- \left(-\frac{1}{2} + \frac{1}{6}\sqrt{69} - \frac{1}{2} - \frac{1}{6}\sqrt{69} \right) = 1 = \frac{3}{3};$$

$$\left(-\frac{1}{2} + \frac{1}{6}\sqrt{69} \right) \left(-\frac{1}{2} - \frac{1}{6}\sqrt{69} \right) = \frac{1}{4} - \frac{69}{36} = -\frac{69}{36} = -\frac{5}{3}.$$

27. $5x^2 - 6x + 10 = 0.$

$$x = \frac{6 \pm \sqrt{36 - 200}}{10} = \frac{3 \pm \sqrt{-41}}{5}.$$

$$- \left(\frac{3 + \sqrt{-41}}{5} + \frac{3 - \sqrt{-41}}{5} \right) = -\frac{6}{5};$$

$$\left(\frac{3 + \sqrt{-41}}{5} \right) \left(\frac{3 - \sqrt{-41}}{5} \right) = \frac{9 + 41}{25} = \frac{50}{25} = \frac{10}{5}.$$

28. $x^2 + x + 1 = 0.$

$$x = \frac{-1 \pm \sqrt{-3}}{2}.$$

$$- \left(\frac{-1 + \sqrt{-3}}{2} + \frac{-1 - \sqrt{-3}}{2} \right) = 1;$$

$$\left(\frac{-1 + \sqrt{-3}}{2} \right) \left(\frac{-1 - \sqrt{-3}}{2} \right) = \frac{1 + 3}{4} = 1.$$

29. $\frac{x^2}{3} + \frac{x}{4} + \frac{1}{5} = 0.$

$$20x^2 + 15x + 12 = 0.$$

$$x = \frac{-15 \pm \sqrt{225 - 960}}{40} = \frac{-15 \pm \sqrt{-735}}{40}.$$

$$- \left(\frac{-15 + \sqrt{-735}}{40} + \frac{-15 - \sqrt{-735}}{40} \right) = \frac{30}{40} = \frac{1}{4} \div \frac{1}{3};$$

$$\left(\frac{-15 + \sqrt{-735}}{40} \right) \left(\frac{-15 - \sqrt{-735}}{40} \right) = \frac{225 + 735}{1600} = \frac{960}{1600} = \frac{1}{5} \div \frac{1}{3}.$$

30. $\frac{x^2}{2} + 4x - 7 = 0.$

$$x^2 + 8x - 14 = 0.$$

$$x = \frac{-8 \pm \sqrt{64 + 56}}{2} = -4 \pm \sqrt{30}.$$

$$- (-4 + \sqrt{30} - 4 - \sqrt{30}) = 8; (-4 + \sqrt{30})(-4 - \sqrt{30}) = -14.$$

31.

$$6x^2 - 4x - 3 = 0.$$

$$x = \frac{4 \pm \sqrt{88}}{12} = \frac{2 \pm \sqrt{22}}{6}.$$

$$-\left(\frac{2 + \sqrt{22}}{6} + \frac{2 - \sqrt{22}}{6}\right) = -\frac{4}{6};$$

$$\left(\frac{2 + \sqrt{22}}{6}\right)\left(\frac{2 - \sqrt{22}}{6}\right) = \frac{4 - 22}{36} = -\frac{3}{6}.$$

32. If one root = 2, then $x = 2$ satisfies the equation.Therefore $4 + 4 - c = 0$, or $c = 8$.33. $36 - 6 - c = 0$, and $c = 30$.34. $49 - 7c - 70 = 0$; $7c + 21 = 0$; $c = -3$.35. $16 - 8b + 20 = 0$; $8b - 36 = 0$; $b = \frac{9}{2}$.

36. Let

 r = the lesser root.

Then

 $2r$ = the greater root,

and

$$-(r + 2r) = -3r = -8.$$

Whence

$$r = \frac{8}{3},$$

and

$$2r = \frac{16}{3}.$$

Then

$$c = \frac{8}{3} \cdot \frac{16}{3} = 1\frac{2}{9}8.$$

37. Let

 x = the lesser root.

Then

 $x + 2$ = the greater root.

Then

$$-(x + x + 2) = 7.$$

Whence

$$x = -\frac{9}{2},$$

and

$$x + 2 = -\frac{5}{2}.$$

Then

$$c = \left(-\frac{9}{2}\right)\left(-\frac{5}{2}\right) = 4\frac{5}{4}.$$

38. Let

 x = the greater root.

Then

 $x - 10$ = the lesser root.

Then

$$-(x + x - 10) = 11.$$

Whence

$$x = -\frac{1}{2},$$

and

$$x - 10 = -\frac{21}{2}.$$

Then

$$b = \left(-\frac{1}{2}\right)\left(-\frac{21}{2}\right) = \frac{21}{4}.$$

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2. $b^2 - 4ac = k^2 - 64 = 0$, and $k = \pm 8$.

3. $b^2 - 4ac = 100 - 4k = 0$, and $k = 25$.

4. $b^2 - 4ac = 64 - 8k = 0$, and $k = 8$.

5. $b^2 - 4ac = 9k^2 - 144 = 0$, and $k = \pm 4$.

6. $b^2 - 4ac = 16k^2 - 144 = 0$, and $k = \pm 3$.

7. $b^2 - 4ac = 900 - 36k - 324 = 0$, and $k = 16$.

8. $b^2 - 4ac = 3600 - 400k = 0$, and $k = 9$.

$$\begin{aligned}
 9. \quad & b^2 - 4ac = 84^2 - 4 \cdot 9 \cdot 49k^2 = 0. \\
 & 12^2 - 4 \cdot 9k^2 = 0. \\
 & 12 - 3k^2 = 0, \text{ and } k = \pm 2.
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & b^2 - 4ac = k^2 + 6k + 9 - 784 = 0. \\
 & k^2 + 6k - 775 = 0. \\
 & (k - 25)(k + 31) = 0, \text{ and } k = 25, -31.
 \end{aligned}$$

$$11. \quad b^2 - 4ac = 900 - 100k^2 - 500 = 0, \text{ and } k = \pm 2.$$

$$\begin{aligned}
 12. \quad & y^2 = 2x, & (1) \\
 & y = x + a. & (2)
 \end{aligned}$$

Substituting from (2) in (1), $x^2 + 2ax + a^2 = 2x$.

$$x^2 + (2a - 2)x + a^2 = 0.$$

$$b^2 - 4ac = 4a^2 - 8a + 4 - 4a^2 = 0,$$

and $a = \frac{1}{2}.$

$$\begin{aligned}
 13. \quad & x^2 + y^2 = a^2, & (1) \\
 & y = x + 1. & (2)
 \end{aligned}$$

Substituting from (2) in (1), $x^2 + x^2 + 2x + 1 = a^2$.

$$2x^2 + 2x + 1 - a^2 = 0.$$

$$b^2 - 4ac = 4 - 8 + 8a^2 = 0.$$

$$8a^2 - 4 = 0.$$

$$2a^2 - 1 = 0.$$

$$a = \pm \frac{1}{2}\sqrt{2}.$$

$$\begin{aligned}
 14. \quad & xy = a, & (1) \\
 & x + y = 1. & (2) \\
 \text{From (2),} & y = 1 - x. & (3)
 \end{aligned}$$

Substituting from (3) in (1), $x(1 - x) = a$.

$$x - x^2 = a.$$

$$x^2 - x + a = 0.$$

$$b^2 - 4ac = 1 - 4a = 0,$$

and $a = \frac{1}{4}.$

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$$1. \quad b^2 - 4ac = 9 + 160 = 169 = 13^2.$$

Hence, rational factors.

$$2. \quad b^2 - 4ac = 25 + 56 = 81 = 9^2.$$

Hence, rational factors.

$$3. \quad b^2 - 4ac = 81 - 504 = -423, \text{ not a perfect square.}$$

Hence, no rational factors.

$$4. \quad b^2 - 4ac = 1 + 960 = 961 = 31^2.$$

Hence, rational factors.

5. $b^2 - 4ac = 289 - 288 = 1 = 1^2$.

Hence, rational factors.

6. $b^2 - 4ac = 9 + 400 = 409$, not a perfect square.

Hence, no rational factors.

7. $b^2 - 4ac = 81 - 336 = -255$, not a perfect square.

Hence, no rational factors.

8. $b^2 - 4ac = 233^2 + 24 \cdot 33 = 55,081$, not a perfect square.

Hence, no rational factors.

9. $b^2 - 4ac = 4a^2 - 4a^2 + 4b^2 = 4b^2 = (2b)^2$.

Hence, rational factors.

10. $b^2 - 4ac = b^4 + 2b^2a^2 + a^4 - 4a^2b^2 = (b^2 - a^2)^2$.

Hence, rational factors.

12. Let $x^2 - 7x - 7 = 0$.

Then
$$x = \frac{7 \pm \sqrt{49 + 28}}{2} = \frac{7 \pm \sqrt{77}}{2}.$$

Then
$$\begin{aligned} x^2 - 7x - 7 &= \left(x - \frac{7 + \sqrt{77}}{2}\right)\left(x - \frac{7 - \sqrt{77}}{2}\right) \\ &= \frac{1}{4}(2x - 7 - \sqrt{77})(2x - 7 + \sqrt{77}). \end{aligned}$$

13. Let $x^2 - 4x - 1 = 0$.

Then
$$x = \frac{4 \pm \sqrt{16 + 4}}{2} = 2 \pm \sqrt{5}.$$

Therefore $x^2 - 4x - 1 = (x - 2 - \sqrt{5})(x - 2 + \sqrt{5}).$

14. Let $x^2 + 2x + 2 = 0$.

Then
$$x = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm \sqrt{-1}.$$

Therefore $x^2 + 2x + 2 = (x + 1 - \sqrt{-1})(x + 1 + \sqrt{-1}).$

15. Let $x^2 + 4x - 9 = 0$.

Then
$$x = \frac{-4 \pm \sqrt{16 + 36}}{2} = -2 \pm \sqrt{13}.$$

Therefore $x^2 + 4x - 9 = (x + 2 - \sqrt{13})(x + 2 + \sqrt{13}).$

16. Let $4x^2 - 12x - 9 = 0$.

Then
$$x = \frac{12 \pm \sqrt{144 + 144}}{8} = \frac{3 \pm 3\sqrt{2}}{2}.$$

Therefore
$$\begin{aligned} 4x^2 - 12x - 9 &= 4\left(x - \frac{3 + 3\sqrt{2}}{2}\right)\left(x - \frac{3 - 3\sqrt{2}}{2}\right) \\ &= (2x - 3 - 3\sqrt{2})(2x - 3 + 3\sqrt{2}). \end{aligned}$$

17. Let $25x^2 + 20x + 4 = 0.$

Then
$$x = \frac{-20 \pm \sqrt{400 - 400}}{50}$$
$$= -\frac{2}{5}, -\frac{2}{5}.$$

Therefore $25x^2 + 20x + 40 = (5x + 2)^2.$

18. Let $6x^2 + 14x - 40 = 0.$

Then
$$x = \frac{-14 \pm \sqrt{196 + 960}}{12} = \frac{-14 \pm 34}{12}.$$
$$= -4, \frac{5}{3}.$$

Therefore $6x^2 + 14x - 40 = 2(x + 4)(3x - 5).$

19. Let $10 - 9x - 9x^2 = 0.$

Then
$$x = \frac{9 \pm \sqrt{81 + 360}}{-18} = \frac{9 \pm 21}{-18}$$
$$= -\frac{5}{3}, \frac{2}{3}.$$

Therefore $10 - 9x - 9x^2 = (5 + 3x)(2 - 3x).$

20. Let $10x^2 + 12 - 26x = 0.$

Then
$$x = \frac{26 \pm \sqrt{676 - 480}}{20} = \frac{26 \pm 14}{20}$$
$$= 2, \frac{3}{5}.$$

Therefore $10x^2 + 12 - 26x = 2(x - 2)(5x - 3).$

21. Let $x^2 + 7x + 8 = 0.$

Then
$$x = \frac{-7 \pm \sqrt{17}}{2}.$$

Therefore
$$x^2 + 7x + 8 = \left(x - \frac{-7 + \sqrt{17}}{2}\right)\left(x - \frac{-7 - \sqrt{17}}{2}\right)$$
$$= \frac{1}{4}(2x + 7 - \sqrt{17})(2x + 7 + \sqrt{17}).$$

22. Let $x^2 + x + 1 = 0.$

Then
$$x = \frac{-1 \pm \sqrt{-3}}{2}.$$

And
$$x^2 + x + 1 = \left(x - \frac{-1 + \sqrt{-3}}{2}\right)\left(x - \frac{-1 - \sqrt{-3}}{2}\right)$$
$$= \frac{1}{4}(2x + 1 - \sqrt{-3})(2x + 1 + \sqrt{-3}).$$

23. Let $x^2 + 1 = 0.$

Then
$$x = \pm \sqrt{-1}.$$

Therefore $x^2 + 1 = (x - \sqrt{-1})(x + \sqrt{-1}).$

24. Let $x^2 + 9 = 0.$

Then
$$x = \pm 3\sqrt{-1}.$$

Therefore $x^2 + 9 = (x - 3\sqrt{-1})(x + 3\sqrt{-1}).$

25. Let $x^2 - 2ax + a^2 - b = 0.$

Then
$$x = \frac{2a \pm \sqrt{4a^2 - 4a^2 + 4b}}{2} = a \pm \sqrt{b}.$$

Therefore $x^2 - 2ax + a^2 - b = (x - a - \sqrt{b})(x - a + \sqrt{b}).$

26. Let $x^2 + 6ax + 9a^2 - 4b = 0.$

Then
$$x = \frac{-6a \pm \sqrt{36a^2 - 36a^2 + 16b}}{2}$$

$$= -3a \pm 2\sqrt{b}.$$

Therefore $x^2 + 6ax + 9a^2 - 4b = (x + 3a - 2\sqrt{b})(x + 3a + 2\sqrt{b}).$

27. Let $4x^2 + 4ax + a^2 - 4c = 0.$

Then
$$x = \frac{-4a \pm \sqrt{16a^2 - 16a^2 + 64c}}{8}$$

$$= \frac{-a \pm 2\sqrt{c}}{2}.$$

Therefore $4x^2 + 4ax + a^2 - 4c = (2x + a - 2\sqrt{c})(2x + a + 2\sqrt{c}).$

28. Let $x^2 - 4ax + 4a^2 + c = 0.$

Then
$$x = \frac{4a \pm \sqrt{16a^2 - 16a^2 - 4c}}{2}$$

$$= 2a \pm \sqrt{-c}.$$

Therefore $x^2 - 4ax + 4a^2 + c = (x - 2a - \sqrt{-c})(x - 2a + \sqrt{-c}).$

29. Let $ax^2 + bx + c = 0.$

Then
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Therefore

$$ax^2 + bx + c = a \left(x - \frac{-b + \sqrt{b^2 - 4ac}}{2a} \right) \left(x - \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right)$$

$$= \frac{a}{4a^2} (2ax + b - \sqrt{b^2 - 4ac}) (2ax + b + \sqrt{b^2 - 4ac})$$

$$= \frac{1}{4a} (2ax + b - \sqrt{b^2 - 4ac}) (2ax + b + \sqrt{b^2 - 4ac}).$$

31. Let $3x^2 - 6xy + 14x - 4y + 8 = 3x^2 - (6y - 14)x - (4y - 8) = 0.$

Then
$$x = \frac{6y - 14 \pm \sqrt{36y^2 - 168y + 196 + 48y - 96}}{6}$$

$$= \frac{6y - 14 \pm \sqrt{36y^2 - 120y + 100}}{6}$$

$$= \frac{6y - 14 \pm (6y - 10)}{6}$$

$$= 2y - 4, -\frac{2}{3}.$$

Therefore $3x^2 - 6xy + 14x - 4y + 8 = (x - 2y + 4)(3x + 2).$

32. Let $x^2 - xy - 2y^2 + 3x - 6y = x^2 - (y - 3)x - (2y^2 + 6y) = 0$.

Then
$$\begin{aligned} x &= \frac{y - 3 \pm \sqrt{y^2 - 6y + 9 + 8y^2 + 24y}}{2} \\ &= \frac{y - 3 \pm \sqrt{9y^2 + 18y + 9}}{2} \\ &= \frac{y - 3 \pm (3y + 3)}{2} \\ &= 2y, -y - 3. \end{aligned}$$

Therefore $x^2 - xy - 2y^2 + 3x - 6y = (x - 2y)(x + y + 3)$.

33. Let $x^2 - 4xy - y + 3y^2 - 2 - x = x^2 - (4y + 1)x + (3y^2 - y - 2) = 0$.

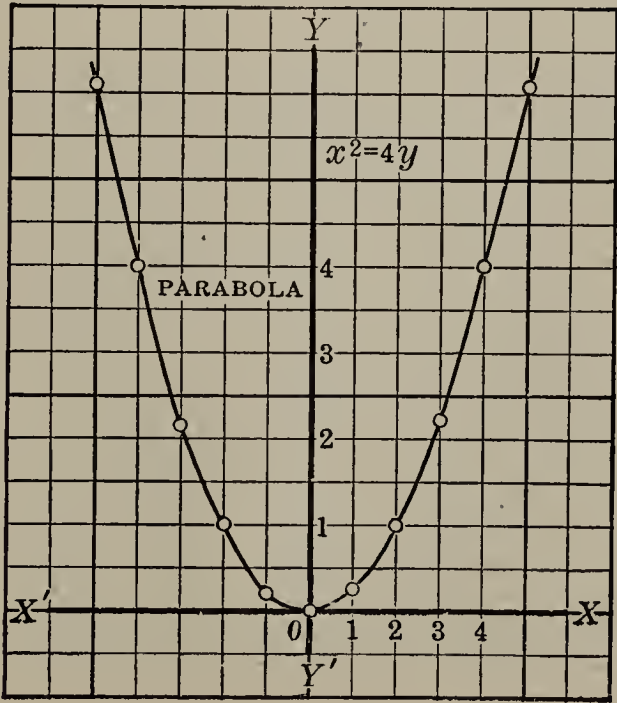
Then
$$\begin{aligned} x &= \frac{4y + 1 \pm \sqrt{16y^2 + 8y + 1 - 12y^2 + 4y + 8}}{2} \\ &= \frac{4y + 1 \pm \sqrt{4y^2 + 12y + 9}}{2} \\ &= \frac{4y + 1 \pm (2y + 3)}{2} \\ &= 3y + 2, y - 1. \end{aligned}$$

Therefore $x^2 - 4xy - y + 3y^2 - 2 - x = (x - 3y - 2)(x - y + 1)$.

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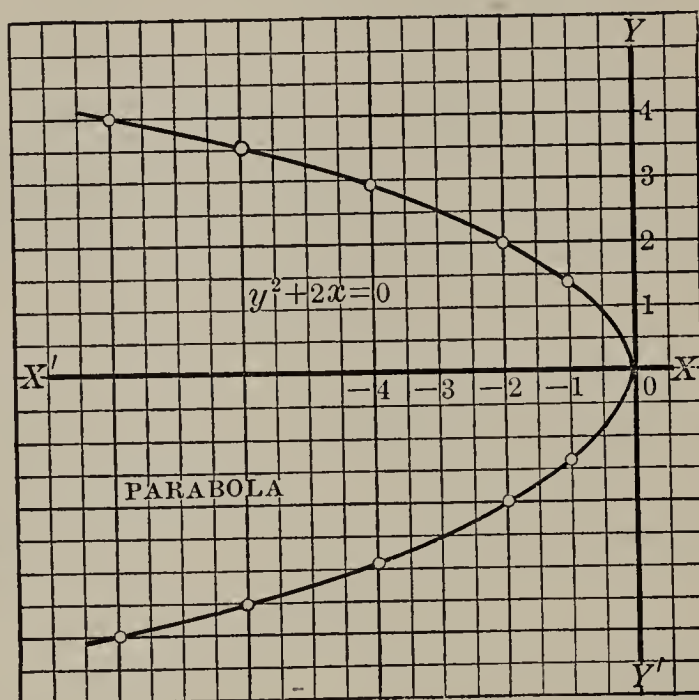
1. $x^2 = 4y$. Solving for y , $y = \frac{x^2}{4}$.

If $x =$	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
then $y =$	9	6.25	4	2.25	1	.25	0	.25	1	2.25	4	6.25	9



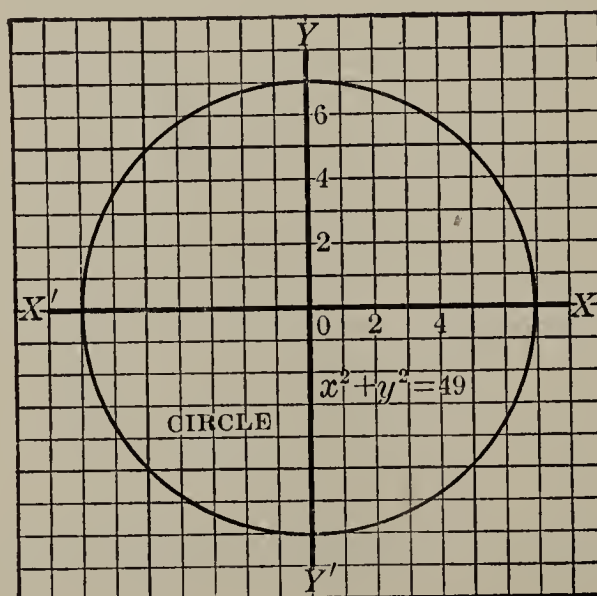
2. $y^2 + 2x = 0$. Solving for y , $y = \pm \sqrt{-2x}$.

If $x =$	- 8	- 6	- 4	- 2	- 1	0	Positive values
then $y =$	± 4	± 3.46	± 2.82	± 2	± 1.41	0	Imaginary



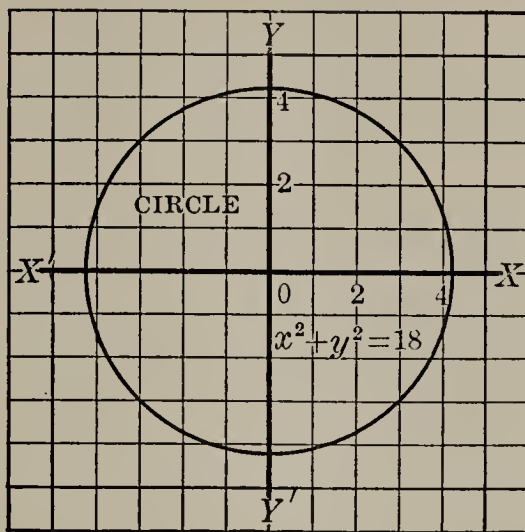
3. $x^2 + y^2 = 49$.

Circle, center at $(0, 0)$, and $r = 7$.



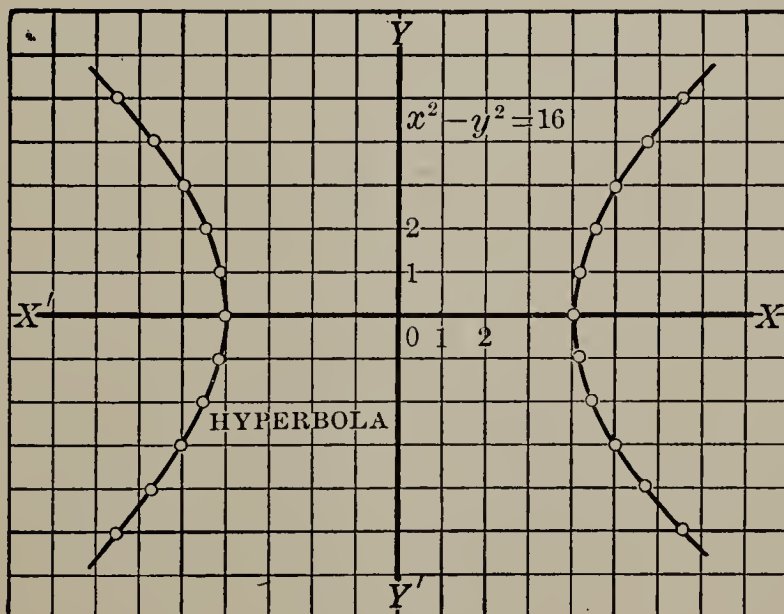
4. $x^2 + y^2 = 18$.

Circle, center at $(0, 0)$, and $r = 4.2$.



5. $x^2 - y^2 = 16$. Solving for x , $x = \pm \sqrt{16 + y^2}$.

If $y =$	± 5	± 4	± 3	± 2	± 1	0	± 1	± 2	± 3	± 4	± 5
then $x =$	-6.4	-5.6	-5	-4.4	-4.123	± 4	4.123	4.4	5	5.6	6.4

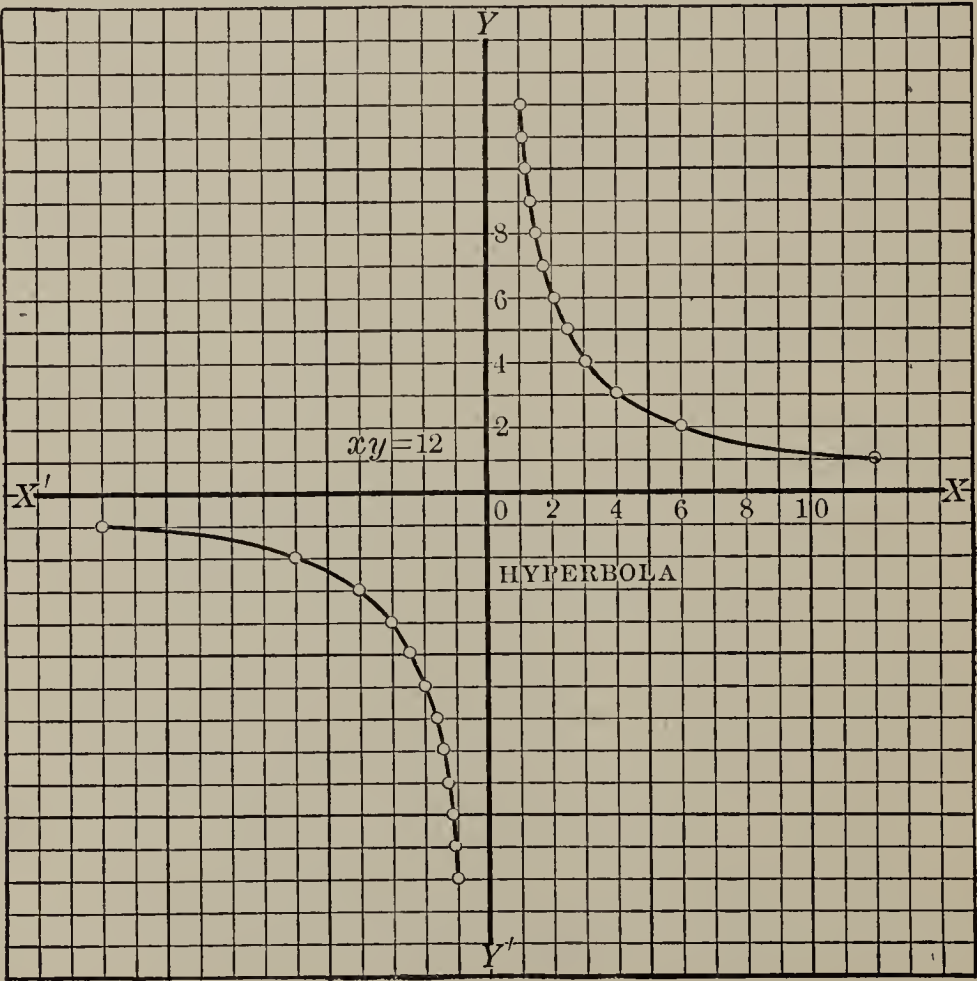


6. $xy = 12$. Solving for x , $x = \frac{12}{y}$.

If $y =$	-12	-11	-10	-9	-8	-7
then $x =$	-1	-1.09	-1.2	-1.3	-1.5	-1.7

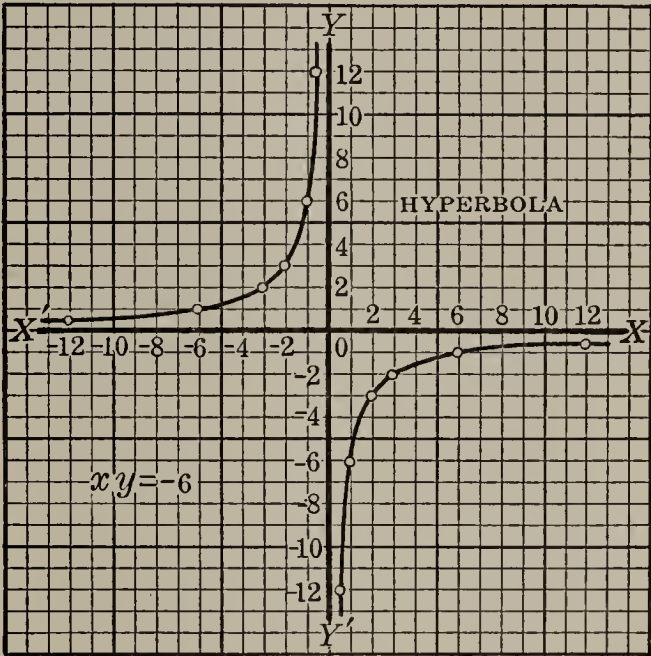
If $y =$	-6	-5	-4	-3	-2	-1
then $x =$	-2	-2.4	-3	-4	-6	-12

If $y =$	1	2	3	4	5	6	7	8	9	10	11	12
then $x =$	12	6	4	3	2.4	2	1.7	1.5	1.3	1.2	1.09	1



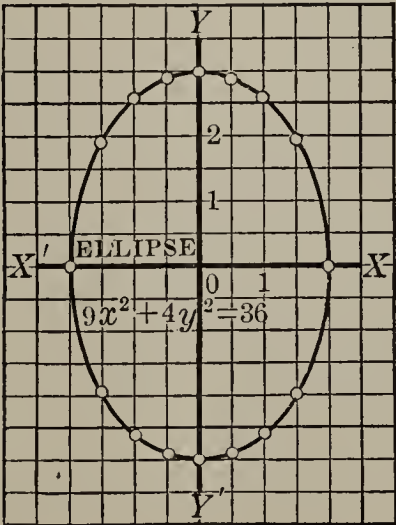
7. $xy = -6$. Solving for y , $y = -\frac{6}{x}$.

If $x =$	12	6	3	2	1	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-2	-3	-6	-12
then $y =$	$-\frac{1}{2}$	-1	-2	-3	-6	-12	12	6	3	2	1	$\frac{1}{2}$



8. $9x^2 + 4y^2 = 36$. Solving for y , $y = \pm \frac{3}{2}\sqrt{4 - x^2}$.

If $x =$	-2	-1.5	-1	0	1	1.5	2
then $y =$	0	± 1.98	± 2.59	± 3	± 2.59	± 1.98	0

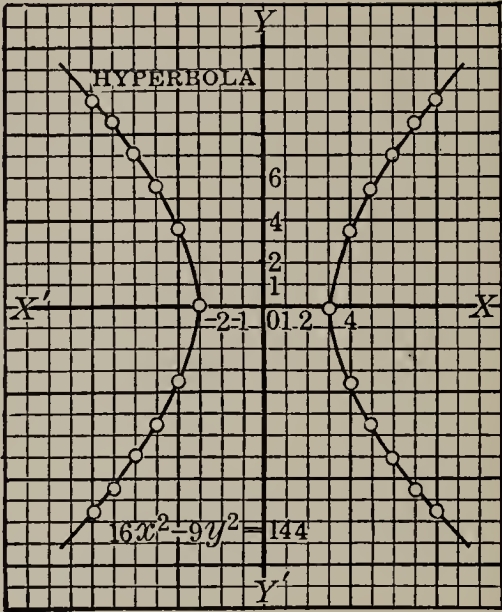


9. $16x^2 - 9y^2 = 144$. Solving for y , $y = \pm \frac{4}{3} \sqrt{x^2 - 9}$.

If $x =$	- 8	- 7	- 6
then $y =$	± 9.88	± 8.43	± 6.92

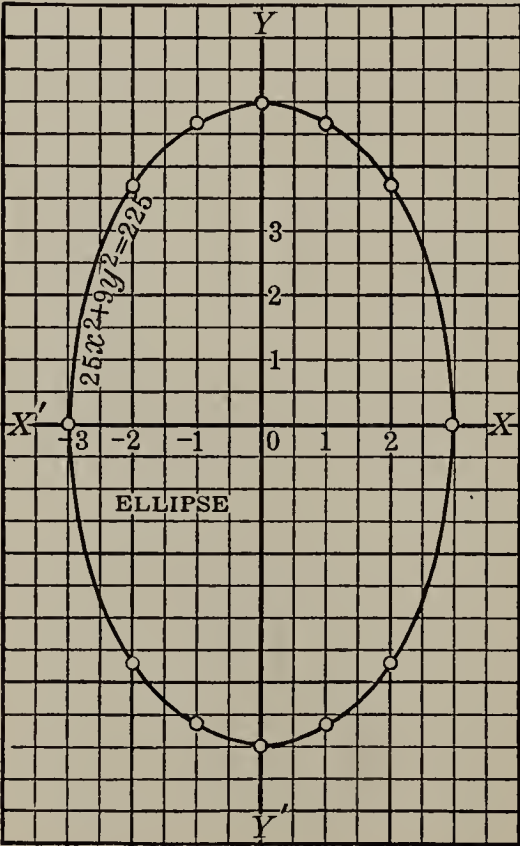
If $x =$	- 5	- 4	± 3	4
then $y =$	± 5.33	± 3.52	0	± 3.52

If $x =$	5	6	7	8
then $y =$	± 5.33	± 6.92	± 8.43	± 9.88



10. $25x^2 + 9y^2 = 225$. Solving for y , $y = \pm \frac{5}{3} \sqrt{9 - x^2}$.

If $x =$	- 3	- 2	- 1	0	1	2	3
then $y =$	0	± 3.72	± 4.71	± 5	± 4.71	± 3.72	0



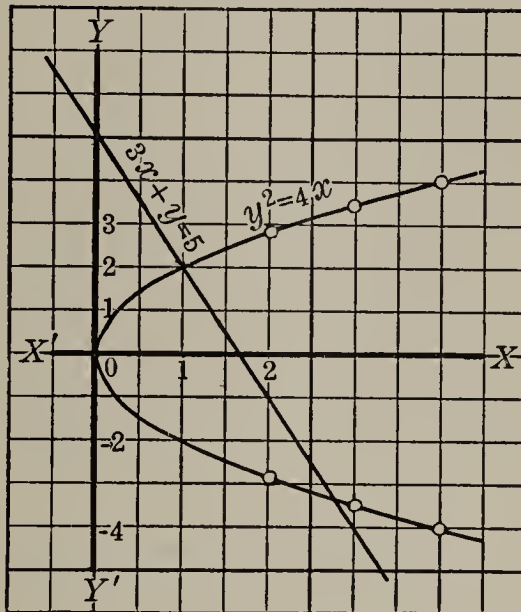
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1. $y^2 = 4x$. Solving for y , $y = \pm 2\sqrt{x}$.

$$3x + y = 5.$$

If $x =$	0	1	2	3	4
then $y =$	0	± 2	± 2.8	± 3.4	± 4

If $x =$	0	1
then $y =$	5	2



From the graphs

If $x =$	1	2.8
then $y =$	2	-3.4

2. $x^2 + y^2 = 25$.

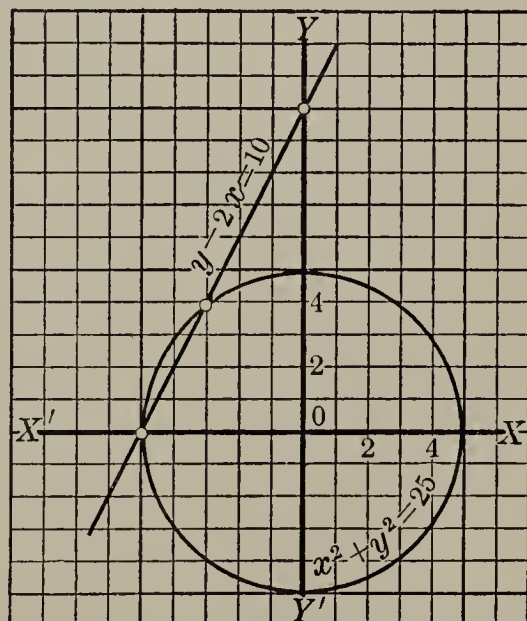
Circle, center $(0, 0)$, and $r = 5$.

$$y - 2x = 10.$$

If $x =$	0	-5
then $y =$	10	0

From the graphs

If $x =$	-3	-5
then $y =$	4	0



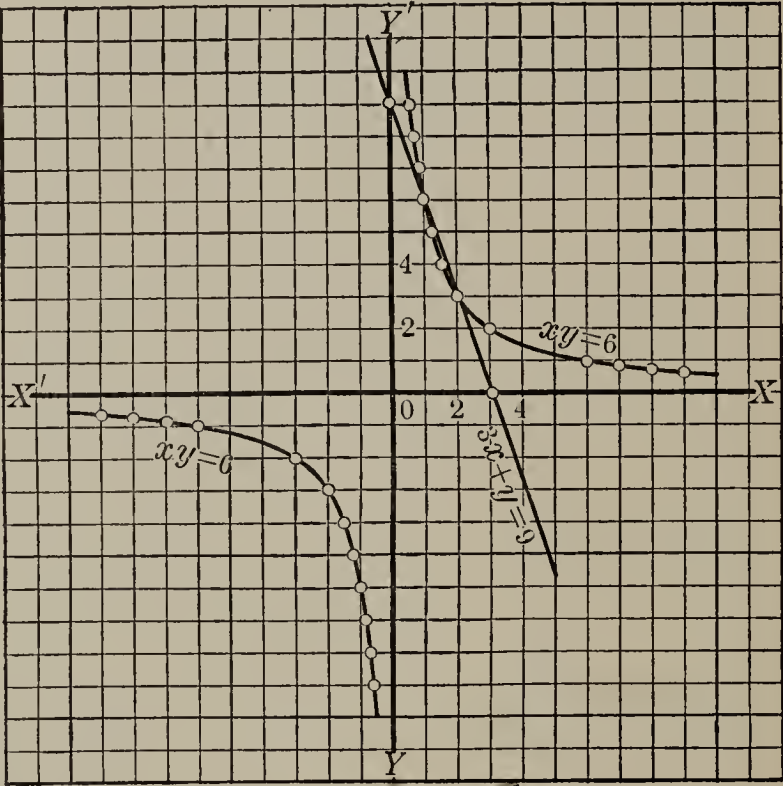
3. $xy = 6$. Solving for y , $y = \frac{6}{x}$.

If $x =$	-9	-8	-6	-3	-2	$-\frac{3}{2}$	-1.2	-1	$-\frac{2}{3}$
then $y =$	$-\frac{2}{3}$	$-\frac{3}{4}$	-1	-2	-3	-4	-5	-6	-9

If $x =$	$\frac{2}{3}$	1	1.2	1.5	2	3	6	8	9
then $y =$	9	6	5	4	3	2	1	$\frac{3}{4}$	$\frac{2}{3}$

$3x + y = 9.$

If $x =$	0	3
then $y =$	9	0



From the graphs

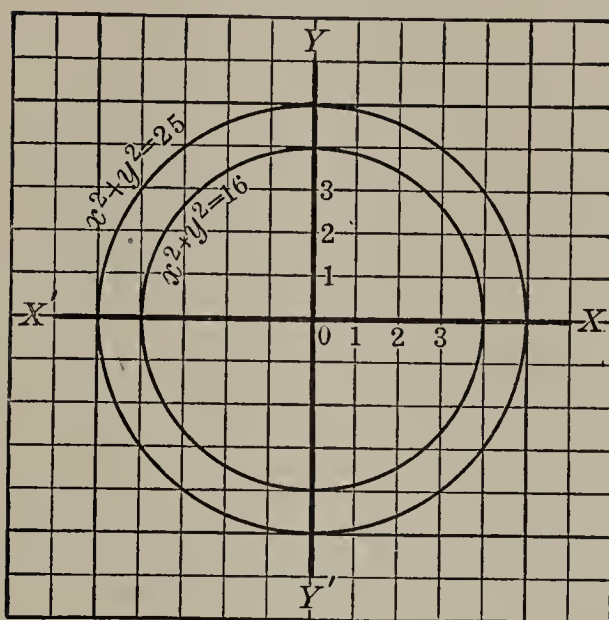
If $x =$	2	1
then $y =$	3	6

4. $x^2 + y^2 = 25$.

Circle, center at $(0, 0)$, and $r = 5$.

$x^2 + y^2 = 16$.

Circle, center at $(0, 0)$, and $r = 4$.



The graphs do not intersect.

Hence, no real roots.

5. $x^2 + y^2 = 9$.

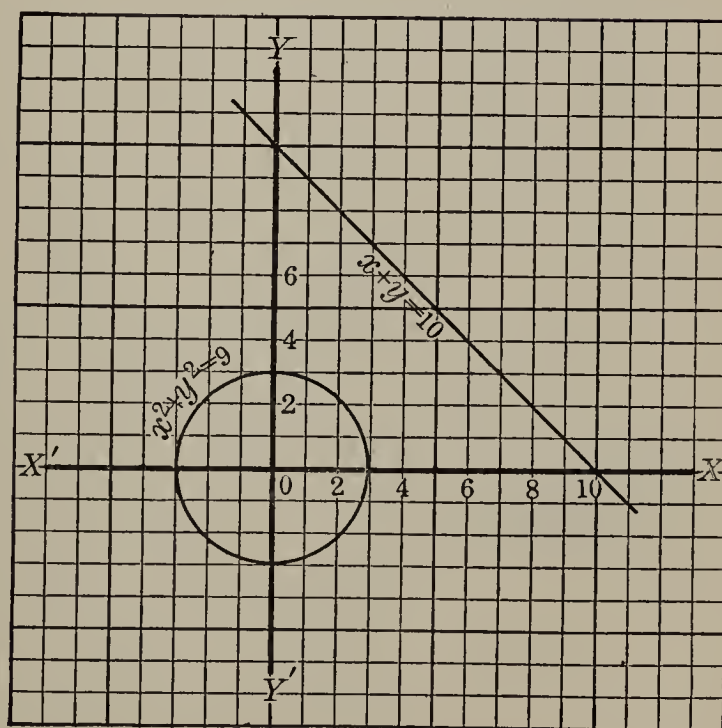
Circle, center at $(0, 0)$,
and $r = 3$.

$x + y = 10$.

If $x =$	0	10
then $y =$	10	0

The graphs do not intersect.

Hence, no real roots.

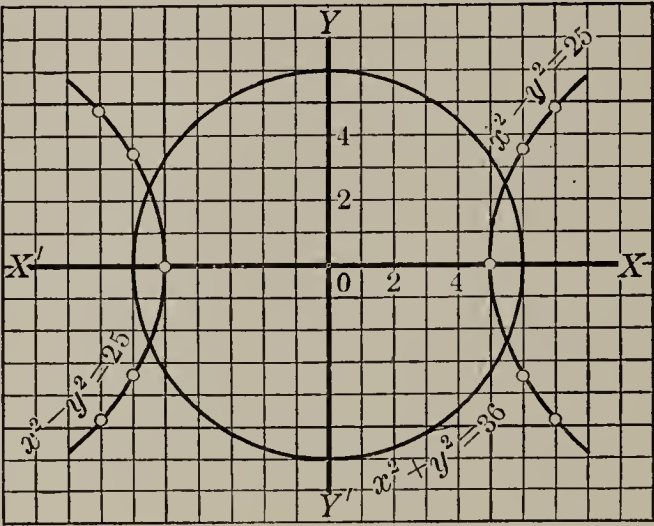


6. $x^2 + y^2 = 36$.

Circle, center at (0, 0), and $r = 6$.

$x^2 - y^2 = 25$. Solving for y , $y = \pm \sqrt{x^2 - 25}$.

If $x =$	- 7	- 6	- 5	5	6	7
then $y =$	± 4.89	± 3.31	0	0	± 3.31	± 4.89



From the
graphs

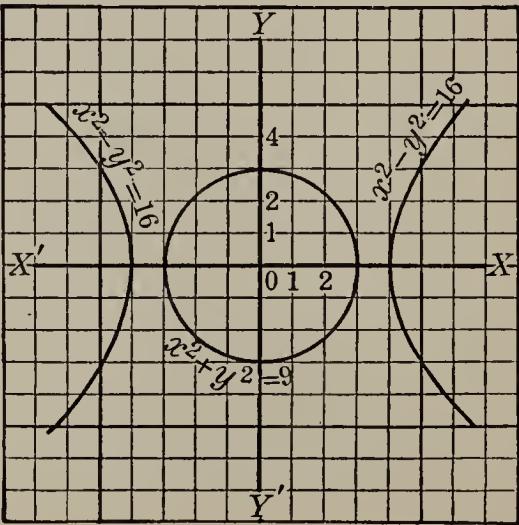
If $x =$	5.5	- 5.5	5.5	- 5.5
then $y =$	2.5	2.5	- 2.5	- 2.5

7. $x^2 + y^2 = 9$.

Circle, center at (0, 0), and $r = 3$.

$x^2 - y^2 = 16$. Solving for y , $y = \pm \sqrt{x^2 - 16}$.

If $x =$	- 6	- 5	- 4	4	5	6
then $y =$	± 4.47	± 3	0	0	± 3	± 4.47



The graphs do
not intersect.

Hence, no real
roots.

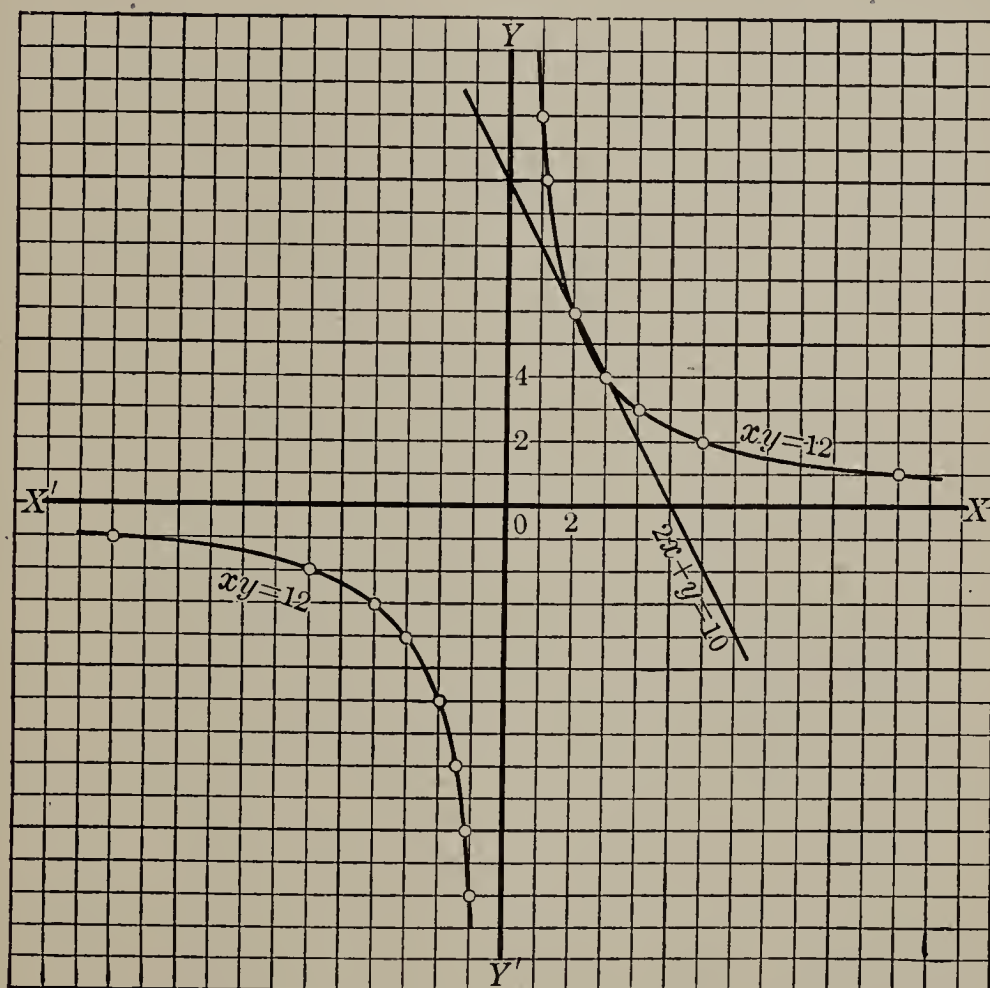
8. $xy = 12$. Solving for y , $y = \frac{12}{x}$.

If $x =$	12	6	4	3	2	1.5	1
then $y =$	1	2	3	4	6	8	12

If $x =$	-1	-1.5	-2	-3	-4	-6	-12
then $y =$	-12	-8	-6	-4	-3	-2	-1

$2x + y = 10$.

If $x =$	0	5
then $y =$	10	0



From the graphs

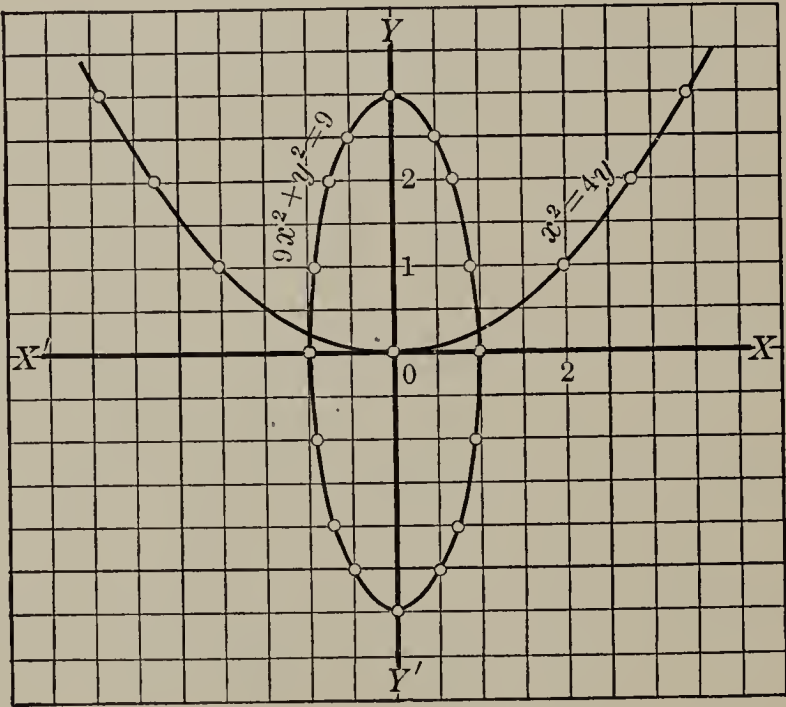
If $x =$	2	3
then $y =$	6	4

9. $x^2 = 4y$. Solving for x , $x = \pm 2\sqrt{y}$.

If $y =$	4	3	2	1	.0
then $x =$	± 4	± 3.46	± 2.82	± 2	0

$9x^2 + y^2 = 9$. Solving for x , $x = \pm \frac{1}{3}\sqrt{9 - y^2}$.

If $y =$	-3	-2.5	-2	-1	0	1	2	2.5	3
then $x =$	0	$\pm .55$	$\pm .74$	$\pm .94$	± 1	$\pm .94$	$\pm .74$	$\pm .55$	0



From the graphs

If $x =$	1	-1
then $y =$	$\frac{1}{4}$	$\frac{1}{4}$

10. $x^2 + 4y = 17$. Solving for y , $y = \frac{17 - x^2}{4}$.

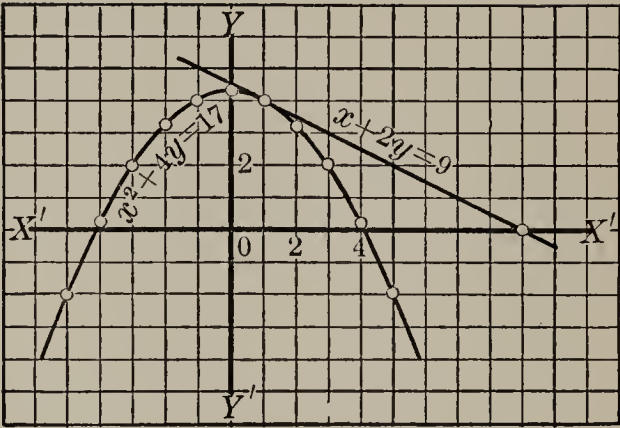
If $x =$	± 5	± 4	± 3	± 2	± 1	0
then $y =$	- 2	.25	2	3.25	4	4.25

$x + 2y = 9$.

If $x =$	9	0
then $y =$	0	$4\frac{1}{2}$

From the graphs

If $x =$	1
then $y =$	4

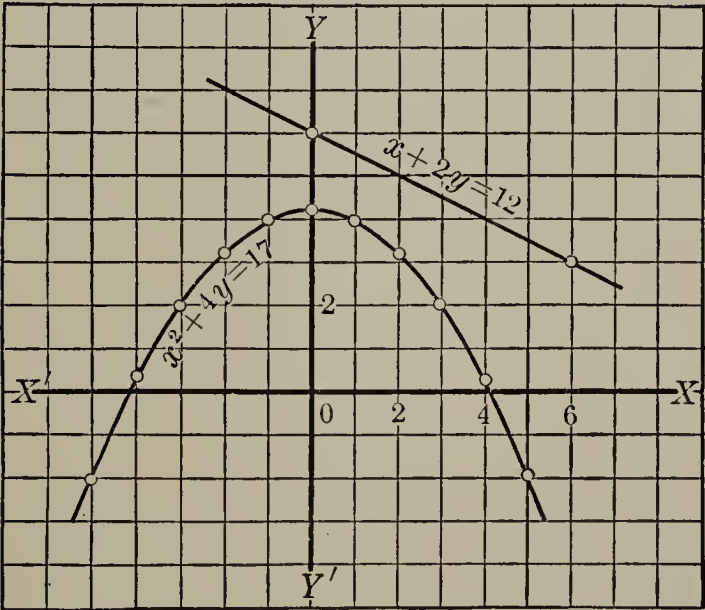


11. $x^2 + 4y = 17$. Solving for y , $y = \frac{17 - x^2}{4}$.

If $x =$	± 5	± 4	± 3	± 2	± 1	0
then $y =$	- 2	.25	2	3.25	4	4.25

$x + 2y = 12$.

If $x =$	0	6
then $y =$	6	3



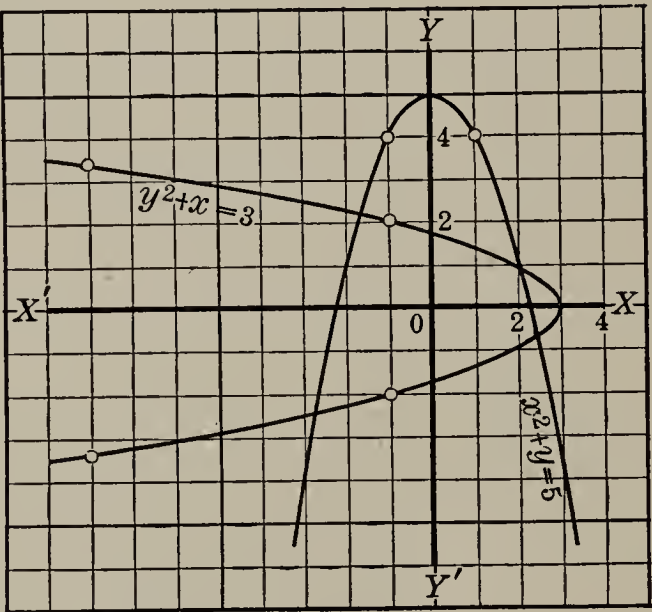
The graphs do not intersect. Hence, no real roots.

12. $x^2 + y = 5$. Solving for y , $y = 5 - x^2$.

If $x =$	0	± 1	± 2	± 3	± 4
then $y =$	5	4	1	- 4	- 11

$y^2 + x = 3$. Solving for y , $y = \pm \sqrt{3 - x}$.

If $x =$	- 13	- 8	- 6	- 1	2	3
then $y =$	± 4	± 3.3	± 3	± 2	± 1	0



From the graphs

If $x =$	2	2.4	- 1.7	- 2.7
then $y =$	1	- .7	2.2	- 2.4

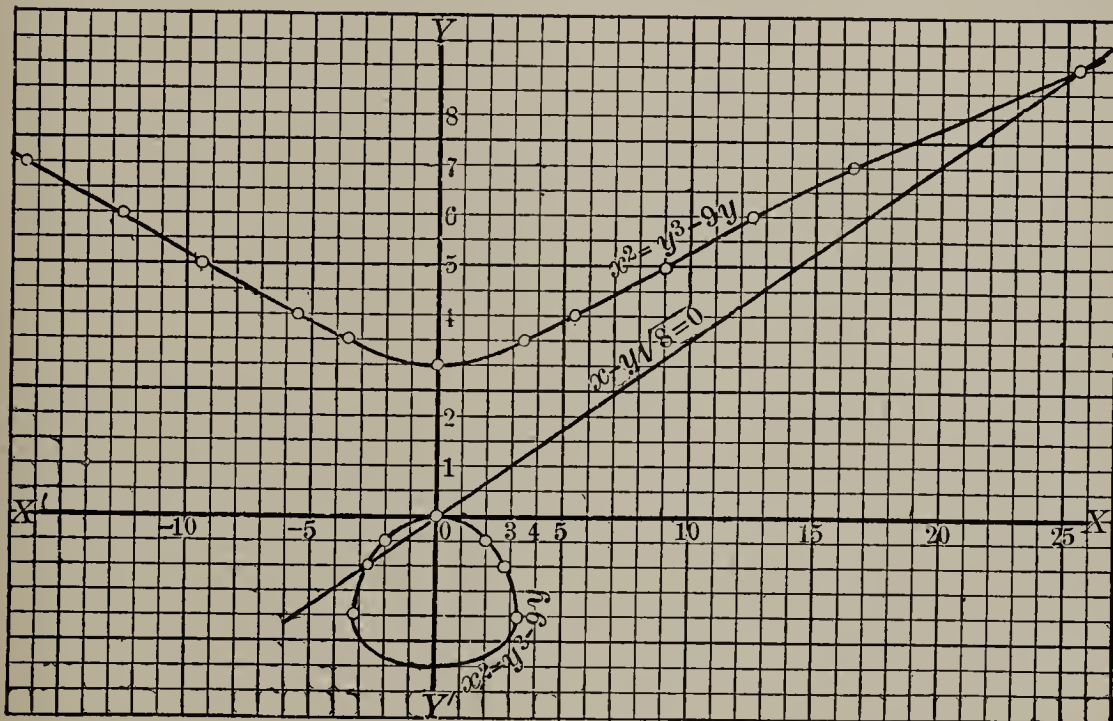
13. $x - y\sqrt{8} = 0$. Solving for x , $x = y\sqrt{8}$.

If $y =$	0	-1	8
then $x =$	0	-2.8	22.6

$x^2 = y^3 - 9y$. Solving for x , $x = \pm \sqrt{y^3 - 9y}$.

If $y =$	-3	$-\frac{5}{2}$	-2	-1	$-\frac{1}{2}$	0	3
then $x =$	0	± 2.6	± 3.16	± 2.82	± 2.09	0	0

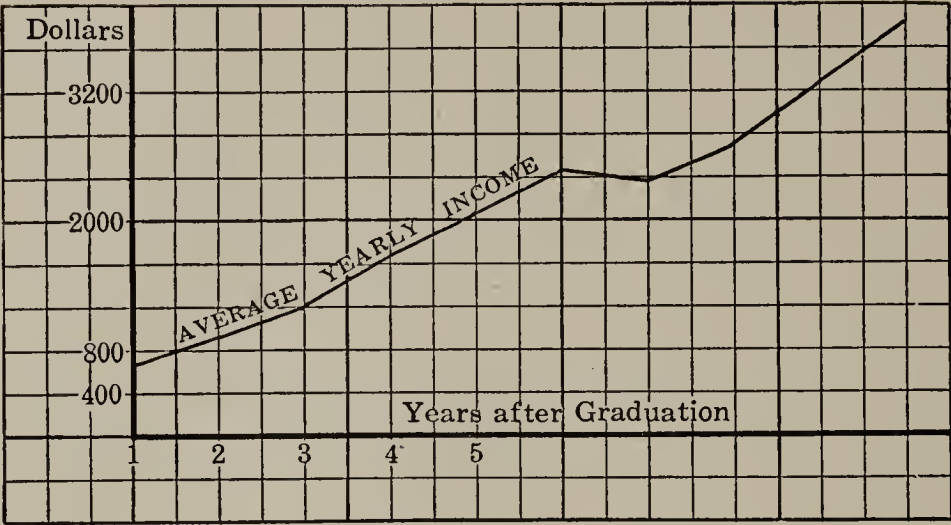
If $y =$	3.5	4	5	6	7	8
then $x =$	± 3.37	± 5.29	± 8.94	± 12.72	± 16.7	± 20.9



From the graphs

If $x =$	0	-2.8	25.4
then $y =$	0	-1	9

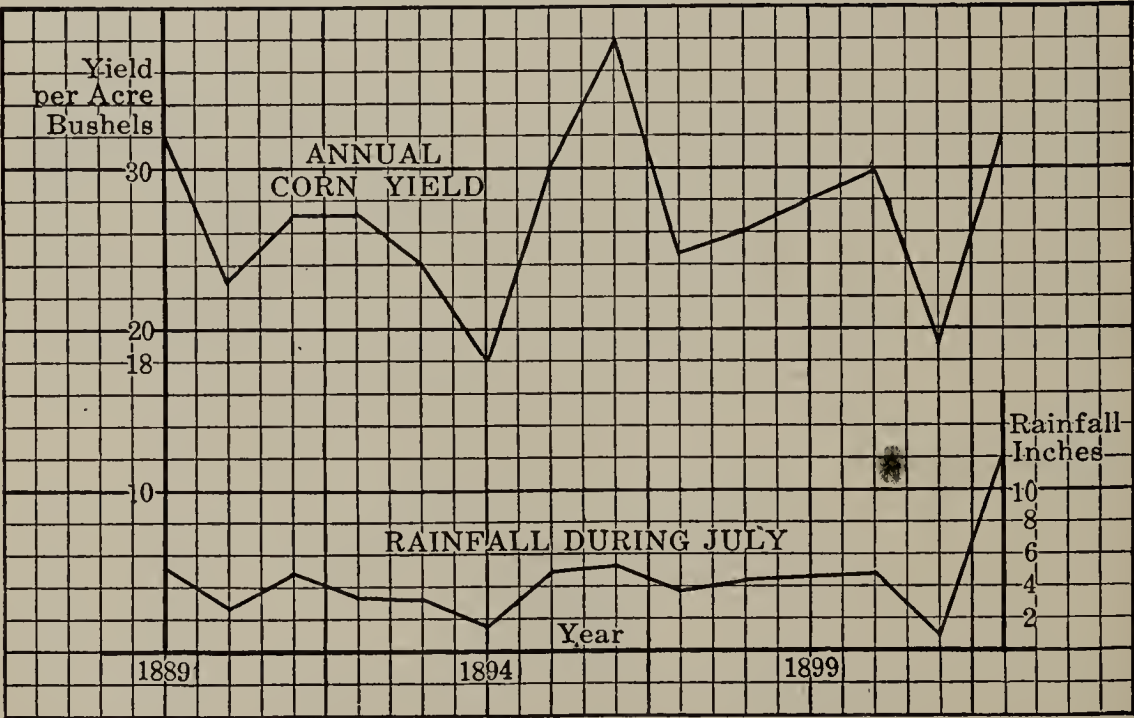
Years after graduation	1	2	3	4	5	6	7	8	9	10
Income in dollars . . .	706	902	1199	1651	2039	2408	2382	2709	3222	3804



The annual income increases almost directly with the years after graduation.

2.

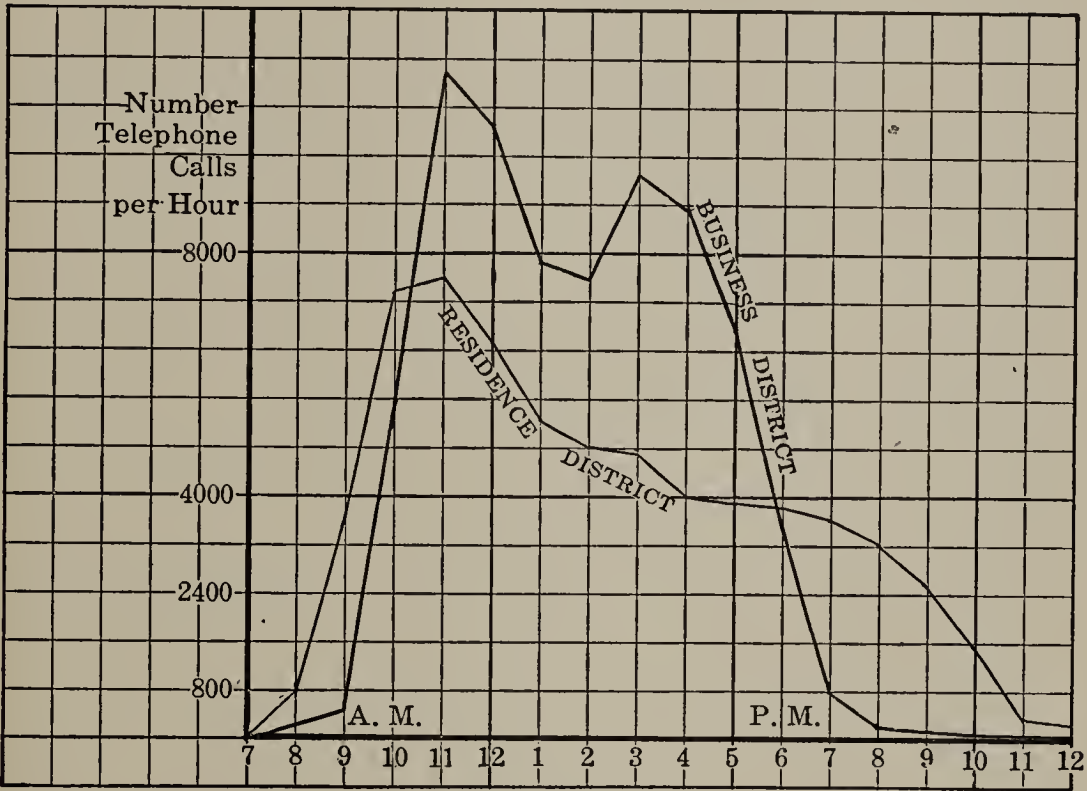
Year	'89	'90	'91	'92	'93	'94	'95	'96	'97	'98	'99	'00	'01	'02
Rainfall in inches	5.4	2.6	5.1	3.7	3.4	1.9	4.8	5.4	3.7	4.3	4.6	4.7	1.2	6.0
Corn yield in bushels	32	23	27	27	24	18	30	38	25	26	28	30	19	32



An increased corn yield accompanies an increased rainfall, and vice versa. The largest crops are harvested in years of greatest July rainfall.

3.

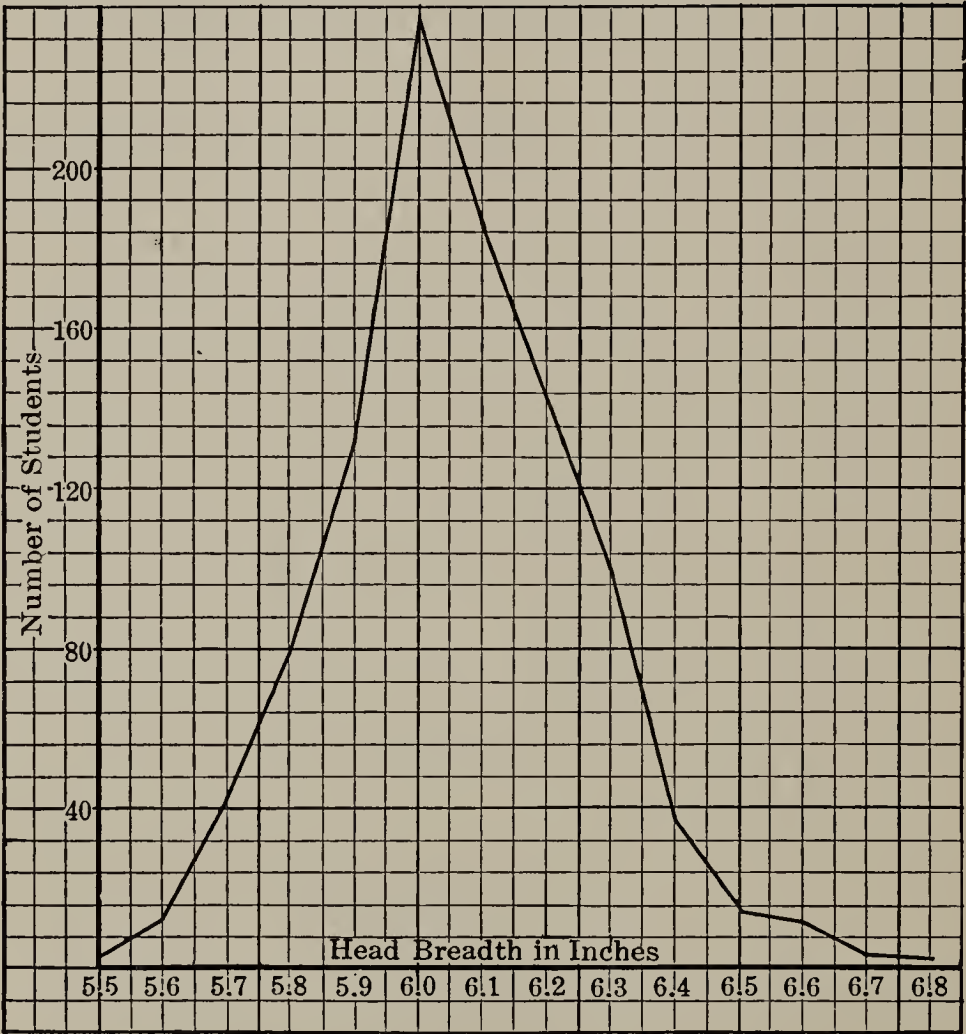
TELEPHONE CALLS PER HOUR IN HUNDREDS																		
Time of day	7	8	9	10	11	Noon	1	2	3	4	5	6	7	8	9	10	11	12
Business district .	1	2	4	55	108	101	78	75	93	87	67	35	8	2	1.5	1	1	1
Residence district	.25	8	35	73	76	65	52	48	47	40	38	38	36	32	25	15	4	3



The number of calls in the residence district is greater than the number in the business district in the early morning and in the evening hours. In the business district the curve shows a decreased number of calls during the lunch hours. In both curves the peak load is at 11 A.M.

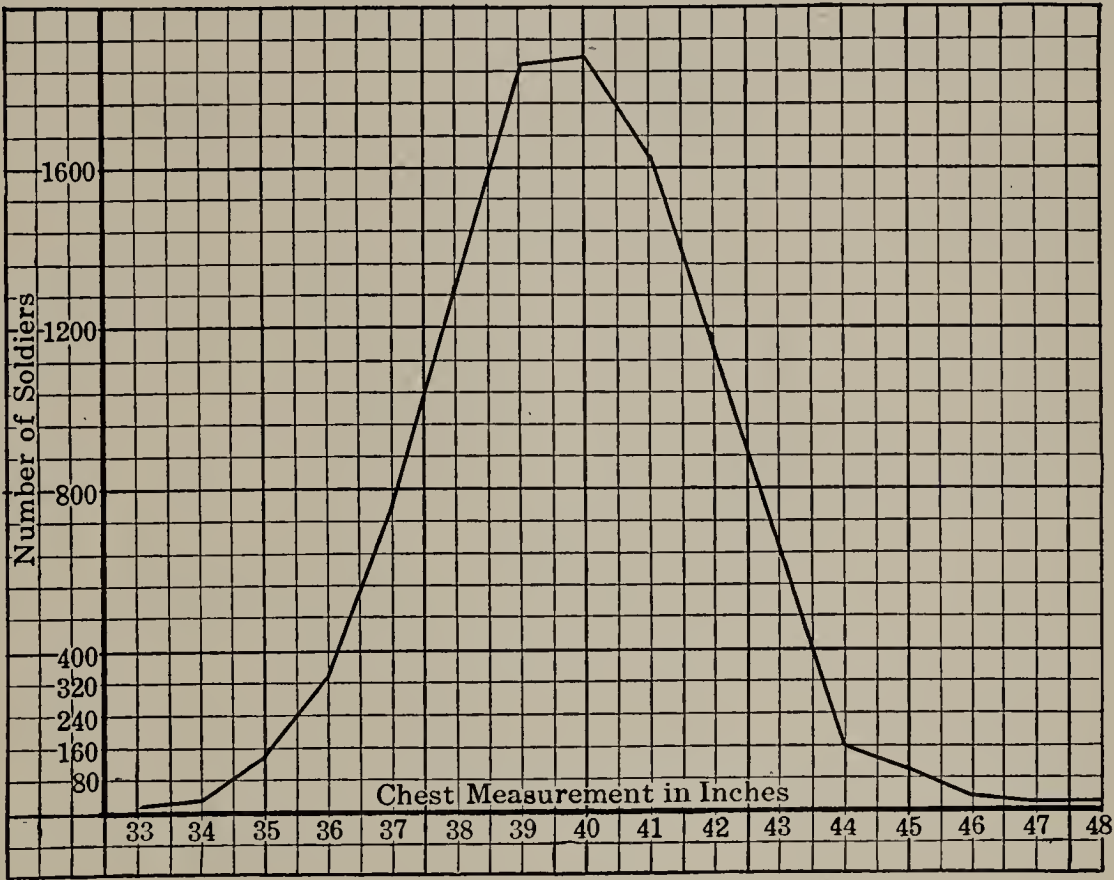
4.

Head breadth in inches	5.5	5.6	5.7	5.8	5.9	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8
Number of students	3	12	43	80	131	236	185	142	99	37	15	12	3	2



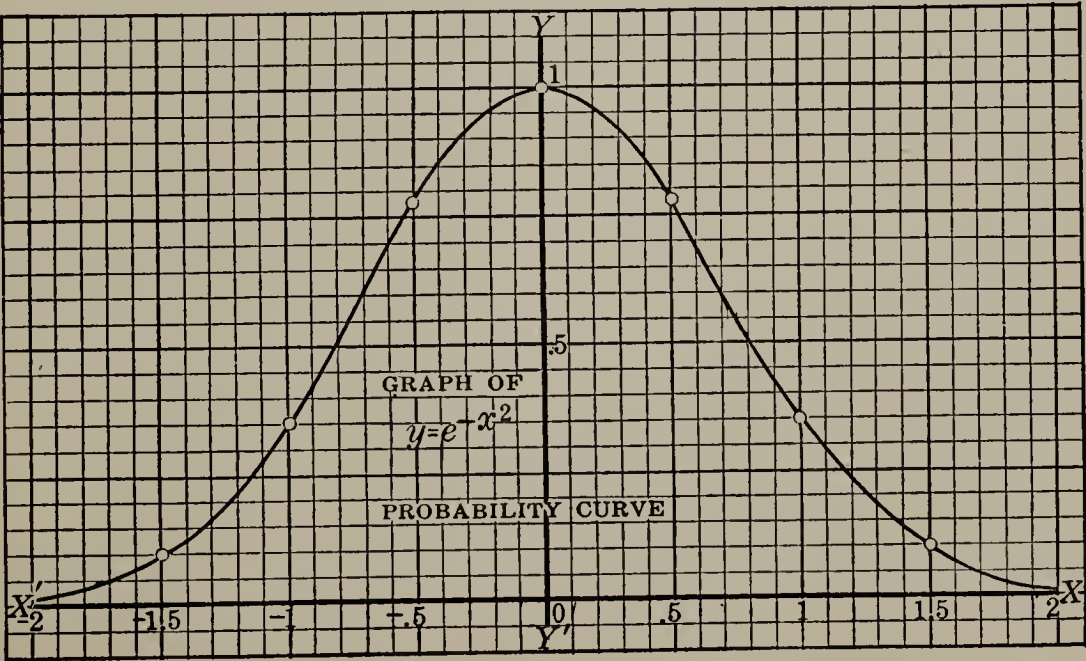
5.

Chest measurement in inches }	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
Number of soldiers	5	31	141	322	732	1305	1867	1882	1628	1148	645	160	87	38	7	2



6. For $y = e^{-x^2}$ in which $e = 2.7$.

If $x =$	0	$\pm .5$	± 1	± 1.5	± 2
then $y =$	1	.78	.35	.11	.02



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$$1. \quad x + y = 5, \quad (1)$$

$$x^2 + y^2 = 13. \quad (2)$$

$$\text{From (1),} \quad x = 5 - y. \quad (3)$$

$$\text{Substituting } (5 - y) \text{ for } x \text{ in (2),} \quad (5 - y)^2 + y^2 = 13. \quad (4)$$

$$\text{From (4),} \quad y^2 - 5y + 6 = 0. \quad (5)$$

$$\text{From (5),} \quad y = 3, 2. \quad (6)$$

$$\text{From (6) and (3),} \quad x = 2, 3.$$

$$2. \quad x^2 - y^2 = 8, \quad (1)$$

$$2x + y = 7. \quad (2)$$

$$\text{From (2),} \quad y = 7 - 2x. \quad (3)$$

$$\text{Substituting } (7 - 2x) \text{ for } y \text{ in (1),} \quad x^2 - (-2x + 7)^2 = 8. \quad (4)$$

$$\text{From (4),} \quad 3x^2 - 28x + 57 = 0. \quad (5)$$

$$\text{From (5),} \quad x = \frac{28 \pm \sqrt{784 - 4(3)(57)}}{6} \\ = +\frac{19}{3}, 3. \quad (6)$$

$$\text{From (6) and (3),} \quad y = -\frac{17}{3}, 1.$$

$$3. \quad x^2 + xy = 15, \quad (1)$$

$$x + y = 3. \quad (2)$$

$$\text{From (2),} \quad y = 3 - x. \quad (3)$$

$$\text{Substituting } (3 - x) \text{ for } y \text{ in (1),} \quad x^2 + x(3 - x) = 15. \quad (4)$$

$$\text{Whence} \quad x = 5. \quad (5)$$

$$\text{From (5) and (3),} \quad y = -2.$$

$$4. \quad x^2 + 2xy = 21, \quad (1)$$

$$x + 2y = 3. \quad (2)$$

$$\text{From (2),} \quad y = \frac{3 - x}{2}. \quad (3)$$

$$\text{Substituting } \frac{3 - x}{2} \text{ for } y \text{ in (1),} \quad x^2 + 2x\left(\frac{3 - x}{2}\right) = 21. \quad (4)$$

$$\text{Whence} \quad x = 7. \quad (5)$$

$$\text{From (5) and (3),} \quad y = -2.$$

$$5. \quad x^2 + 2y = 17, \quad (1)$$

$$2x - y = 2. \quad (2)$$

$$\text{From (2),} \quad y = 2x - 2. \quad (3)$$

$$\text{Substituting } (2x - 2) \text{ for } y \text{ in (1),} \quad x^2 + 2(2x - 2) = 17. \quad (4)$$

$$\text{Whence} \quad x = -7, 3. \quad (5)$$

$$\text{From (5) and (3),} \quad y = -16, 4.$$

$$6. \quad y^2 - 10x = 6, \quad (1)$$

$$x + 3y = 13. \quad (2)$$

$$\text{From (2),} \quad x = 13 - 3y. \quad (3)$$

$$\text{Substituting } (13 - 3y) \text{ for } x \text{ in (1),} \quad y^2 - 10(13 - 3y) = 6. \quad (4)$$

$$y^2 + 30y - 136 = 0. \quad (5)$$

$$\text{Whence} \quad y = \frac{-30 \pm \sqrt{900 + 544}}{2} \quad (6)$$

$$= 4, -34. \quad (7)$$

$$\text{From (7) and (3),} \quad x = 1, +115.$$

$$7. \quad y^2 - 2x^2 = 7, \quad (1)$$

$$2y + 3x = 1. \quad (2)$$

$$\text{From (2),} \quad y = \frac{1 - 3x}{2}. \quad (3)$$

$$\text{Substituting } \frac{1 - 3x}{2} \text{ for } y \text{ in (1),} \quad \left(\frac{1 - 3x}{2}\right)^2 - 2x^2 = 7. \quad (4)$$

$$x^2 - 6x - 27 = 0. \quad (5)$$

$$\text{Whence} \quad x = -3, +9. \quad (6)$$

$$\text{From (6) and (3),} \quad y = +5, -13.$$

$$8. \quad 2x^2 - xy = 70, \quad (1)$$

$$x + 4y = 23. \quad (2)$$

$$\text{From (2),} \quad y = \frac{23 - x}{4}. \quad (3)$$

$$\text{Substituting } \frac{23 - x}{4} \text{ for } y \text{ in (1),} \quad 2x^2 - x\left(\frac{23 - x}{4}\right) = 70. \quad (4)$$

$$9x^2 - 23x - 280 = 0. \quad (5)$$

$$\text{Whence} \quad x = \frac{23 \pm \sqrt{529 - 4(9)(-280)}}{18} \quad (6)$$

$$= 7, -\frac{49}{9}. \quad (7)$$

$$\text{From (7) and (3),} \quad y = 4, +\frac{247}{36}.$$

$$9. \quad s^2 + 2st = 108, \quad (1)$$

$$4s - 3t = 6. \quad (2)$$

$$\text{From (2),} \quad t = \frac{4s - 6}{3}. \quad (3)$$

$$\text{Substituting } \frac{4s - 6}{3} \text{ for } t \text{ in (1),}$$

$$s^2 + 2s\left(\frac{4s - 6}{3}\right) = 108. \quad (4)$$

$$11s^2 - 12s - 324 = 0. \quad (5)$$

$$\text{Whence} \quad s = \frac{12 \pm \sqrt{144 - 4(11)(-324)}}{22} \quad (6)$$

$$= 6, -\frac{54}{11}. \quad (7)$$

$$\text{From (7) and (3),} \quad t = 6, -\frac{24}{11}.$$

$$10. \quad s^2 - 3st = 22, \quad (1)$$

$$4s + 2t = 2. \quad (2)$$

$$\text{From (2),} \quad t = 1 - 2s. \quad (3)$$

$$\text{Substituting } (1 - 2s) \text{ for } t \text{ in (1),}$$

$$s^2 - 3s(1 - 2s) = 22. \quad (4)$$

$$7s^2 - 3s - 22 = 0. \quad (5)$$

$$\text{Whence} \quad s = \frac{3 \pm \sqrt{9 - 4(7)(-22)}}{14} \quad (6)$$

$$= +2, -\frac{11}{7}. \quad (7)$$

$$\text{From (7) and (3),} \quad t = -3, +\frac{22}{7}.$$

$$11. \quad s^2 + t^2 = 169, \quad (1)$$

$$2s + t = 22. \quad (2)$$

$$\text{From (2),} \quad t = 22 - 2s. \quad (3)$$

$$\text{Substituting } (22 - 2s) \text{ for } t \text{ in (1),}$$

$$s^2 + (22 - 2s)^2 = 169. \quad (4)$$

$$5s^2 - 88s + 315 = 0. \quad (5)$$

$$\text{Whence} \quad s = \frac{88 \pm \sqrt{7744 - 4(5)(315)}}{10} \quad (6)$$

$$= +\frac{63}{5}, 5. \quad (7)$$

$$\text{From (7) and (3),} \quad t = -\frac{16}{5}, 12.$$

$$12. \quad s^2 + 2t^2 = 27, \quad (1)$$

$$4t - 2s = 18. \quad (2)$$

$$\text{From (2),} \quad s = 2t - 9. \quad (3)$$

$$\text{Substituting } (2t - 9) \text{ for } s \text{ in (1),}$$

$$(2t - 9)^2 + 2t^2 = 27. \quad (4)$$

$$t^2 - 6t + 9 = 0. \quad (5)$$

$$\text{Whence} \quad t = 3. \quad (6)$$

$$\text{From (6) and (3),} \quad s = -3.$$

$$13. \quad s^2 + 3ts + t^2 = 44, \quad (1)$$

$$2s - t = 0. \quad (2)$$

$$\text{From (2),} \quad t = 2s. \quad (3)$$

Substituting $2s$ for t in (1),

$$s^2 + 3s(2s) + (2s)^2 = 44. \quad (4)$$

$$s^2 = 4. \quad (5)$$

$$\text{Whence} \quad s = 2, -2. \quad (6)$$

$$\text{From (6) and (3),} \quad t = 4, -4.$$

$$14. \quad s^2 + ts + t^2 = 12, \quad (1)$$

$$s + t = 2. \quad (2)$$

$$\text{From (2),} \quad t = 2 - s. \quad (3)$$

Substituting $(2 - s)$ for t in (1),

$$s^2 + (2 - s)s + (2 - s)^2 = 12. \quad (4)$$

$$s^2 - 2s - 8 = 0. \quad (5)$$

$$\text{Whence} \quad s = +4, -2. \quad (6)$$

$$\text{From (6) and (3),} \quad t = -2, +4.$$

$$15. \quad 2s^2 - ts + t^2 = 16, \quad (1)$$

$$2s - t = 5. \quad (2)$$

$$\text{From (2),} \quad t = 2s - 5. \quad (3)$$

Substituting $(2s - 5)$ for t in (1),

$$2s^2 - (2s - 5)s + (2s - 5)^2 = 16. \quad (4)$$

$$4s^2 - 15s + 9 = 0. \quad (5)$$

$$\text{Whence} \quad s = +\frac{3}{4}, 3. \quad (6)$$

$$\text{From (6) and (3),} \quad t = -\frac{7}{2}, 1.$$

$$16. \quad xy + 10 = 0, \quad (1)$$

$$4x + y = 6. \quad (2)$$

$$\text{From (2),} \quad y = 6 - 4x. \quad (3)$$

Substituting $(6 - 4x)$ for y in (1),

$$x(6 - 4x) + 10 = 0. \quad (4)$$

$$2x^2 - 3x - 5 = 0. \quad (5)$$

$$\text{Factoring,} \quad (x + 1)(2x - 5) = 0. \quad (6)$$

$$\text{Whence} \quad x = -1, +\frac{5}{2}. \quad (7)$$

$$\text{From (7) and (3),} \quad y = +10, -4.$$

$$17. \quad 4x + y = 28, \quad (1)$$

$$2x^2 + 3xy = 98. \quad (2)$$

$$\text{From (1),} \quad y = 28 - 4x. \quad (3)$$

Substituting $(28 - 4x)$ for y in (2),

$$2x^2 + 3x(28 - 4x) = 98. \quad (4)$$

$$5x^2 - 42x + 49 = 0. \quad (5)$$

$$\text{Whence} \quad x = \frac{42 \pm \sqrt{1764 - 4(5)(49)}}{10}, \quad (6)$$

$$= 7, \frac{7}{5}. \quad (7)$$

$$\text{From (7) and (3),} \quad y = 0, \frac{1}{5}^2.$$

$$18. \quad 3x_1 + 4x_2 = 5, \quad (1)$$

$$2x_1x_2 - 6x_1 = -3. \quad (2)$$

$$\text{From (1),} \quad x_2 = \frac{5 - 3x_1}{4}. \quad (3)$$

$$\text{Substituting } \frac{5 - 3x_1}{4} \text{ for } x_2 \text{ in (2),}$$

$$2x_1\left(\frac{5 - 3x_1}{4}\right) - 6x_1 = -3. \quad (4)$$

$$3x_1^2 + 7x_1 - 6 = 0. \quad (5)$$

$$\text{Factoring,} \quad (x_1 + 3)(3x_1 - 2) = 0. \quad (6)$$

$$\text{Whence} \quad x_1 = -3, \frac{2}{3}. \quad (7)$$

$$\text{From (7) and (3),} \quad x_2 = +\frac{7}{2}, \frac{3}{4}.$$

$$19. \quad \frac{x}{y} + \frac{y}{x} = 2, \quad (1)$$

$$3x + 2y = 5. \quad (2)$$

$$\text{From (1),} \quad x^2 + y^2 = 2xy. \quad (3)$$

$$\text{From (2),} \quad y = \frac{5 - 3x}{2}. \quad (4)$$

$$\text{Substituting } \frac{5 - 3x}{2} \text{ for } y \text{ in (3),}$$

$$x^2 + \left(\frac{5 - 3x}{2}\right)^2 = 2x\left(\frac{5 - 3x}{2}\right). \quad (5)$$

$$x^2 - 2x + 1 = 0. \quad (6)$$

$$\text{Whence} \quad x = 1. \quad (7)$$

$$\text{From (7) and (4),} \quad y = 1.$$

$$20. \quad \frac{1}{x} - \frac{1}{y} = \frac{1}{4}, \quad (1)$$

$$3x - y = 2. \quad (2)$$

$$\text{From (1),} \quad 4y - 4x = xy. \quad (3)$$

$$\text{From (2),} \quad y = 3x - 2. \quad (4)$$

$$\text{Substituting } (3x - 2) \text{ for } y \text{ in (3),}$$

$$4(3x - 2) - 4x = x(3x - 2). \quad (5)$$

$$3x^2 - 10x + 8 = 0. \quad (6)$$

$$\text{Factoring,} \quad (x - 2)(3x - 4) = 0. \quad (7)$$

$$\text{Whence} \quad x = 2, \frac{4}{3}. \quad (8)$$

$$\text{From (8) and (4),} \quad y = 4, 2.$$

$$21. \quad \frac{x}{4} + \frac{y}{5} = 6, \quad (1)$$

$$\frac{10}{y} - \frac{10}{x} = \frac{3}{2}. \quad (2)$$

$$\text{From (2),} \quad 20x - 20y = 3xy. \quad (3)$$

$$\text{From (1),} \quad y = \frac{120 - 5x}{4}. \quad (4)$$

$$\text{Substituting } \frac{120 - 5x}{4} \text{ for } y \text{ in (3),}$$

$$20x - 20\left(\frac{120 - 5x}{4}\right) = 3x\left(\frac{120 - 5x}{4}\right). \quad (5)$$

$$x^2 - 12x - 160 = 0. \quad (6)$$

$$\text{Whence} \quad x = 20, -8. \quad (7)$$

$$\text{From (7) and (4),} \quad y = 5, +40.$$

$$22. \quad \frac{1}{x} + \frac{y}{2} = 3, \quad (1)$$

$$\frac{1}{y} + \frac{x}{2} = \frac{5}{4}. \quad (2)$$

$$\text{From (1),} \quad 2 + xy = 6x. \quad (3)$$

$$\text{From (2),} \quad 4 + 2xy = 5y. \quad (4)$$

$$2 \cdot (3) - (4), \quad 12x - 5y = 0. \quad (5)$$

$$\text{From (5),} \quad y = \frac{12x}{5}. \quad (6)$$

$$\text{Substituting } \frac{12x}{5} \text{ for } y \text{ in (3),}$$

$$2 + x\left(\frac{12x}{5}\right) = 6x. \quad (7)$$

$$6x^2 - 15x + 5 = 0. \quad (8)$$

$$\text{Whence} \quad x = \frac{15 \pm \sqrt{225 - 4(6)(5)}}{12} \quad (9)$$

$$= \frac{15 + \sqrt{105}}{12}, \frac{15 - \sqrt{105}}{12}. \quad (10)$$

$$\text{From (10) and (6),} \quad y = \frac{15 + \sqrt{105}}{5}, \frac{15 - \sqrt{105}}{5}.$$

$$23. \quad \frac{s}{3} + \frac{t}{4} = 5, \quad (1)$$

$$\frac{6}{s} - \frac{4}{t} = \frac{2}{3}. \quad (2)$$

$$\text{From (1),} \quad 4s + 3t = 60. \quad (3)$$

$$\text{From (2),} \quad 9t - 6s = st. \quad (4)$$

From (3),
$$t = \frac{60 - 4s}{3}. \quad (5)$$

Substituting $\frac{60 - 4s}{3}$ for t in (4),

$$9\left(\frac{60 - 4s}{3}\right) - 6s = s\left(\frac{60 - 4s}{3}\right). \quad (6)$$

$$2s^2 - 57s + 270 = 0. \quad (7)$$

Whence

$$s = \frac{57 \pm \sqrt{3249 - 4(2)(270)}}{4}. \quad (8)$$

$$= + \frac{45}{2}, 6. \quad (9)$$

From (9) and (5),

$$t = -10, 12.$$

24.

$$\frac{t+1}{s+1} = \frac{3}{2}, \quad (1)$$

$$\frac{t+s^2}{s+t^2} = \frac{1}{2}. \quad (2)$$

From (1),

$$2t+2 = 3s+3. \quad (3)$$

From (2),

$$2t+2s^2 = s+t^2. \quad (4)$$

From (3),

$$t = \frac{3s+1}{2}. \quad (5)$$

Substituting $\frac{3s+1}{2}$ for t in (4),

$$2s^2 - \frac{9s^2 + 6s + 1}{4} + 3s + 1 - s = 0. \quad (6)$$

$$s^2 - 2s - 3 = 0. \quad (7)$$

Whence

$$s = 3, -1. \quad (8)$$

From (8) and (5),

$$t = 5, -1.$$

25.

$$\frac{x}{y} - \frac{y}{x} = \frac{5}{6}, \quad (1)$$

$$3y+2x = 2. \quad (2)$$

From (1),

$$6x^2 - 6y^2 = 5xy. \quad (3)$$

From (2),

$$x = \frac{2-3y}{2}. \quad (4)$$

Substituting $\frac{2-3y}{2}$ for x in (3),

$$6\left(\frac{2-3y}{2}\right)^2 - 6y^2 = 5\left(\frac{2-3y}{2}\right)y. \quad (5)$$

$$15y^2 - 23y + 6 = 0. \quad (6)$$

Factoring,

$$(5y-6)(3y-1) = 0. \quad (7)$$

Whence

$$y = +\frac{6}{5}, \frac{1}{3}. \quad (8)$$

From (8) and (4),

$$x = -\frac{4}{5}, \frac{1}{2}.$$

$$26. \quad s^2 + t^2 + 2t = 40, \quad (1)$$

$$s + t + 2 = 0. \quad (2)$$

$$\text{From (2),} \quad s = -t - 2. \quad (3)$$

$$\text{Substituting } (-t - 2) \text{ for } s \text{ in (1),}$$

$$(-t - 2)^2 + t^2 + 2t = 40. \quad (4)$$

$$t^2 + 3t - 18 = 0. \quad (5)$$

$$\text{Whence} \quad t = -6, +3. \quad (6)$$

$$\text{From (6) and (3),} \quad s = +4, -5.$$

$$27. \quad 2x^2 + xy = 8, \quad (1)$$

$$y - x = \sqrt{2}. \quad (2)$$

$$\text{From (2),} \quad y = x + \sqrt{2}. \quad (3)$$

$$\text{Substituting } (x + \sqrt{2}) \text{ for } y \text{ in (1),}$$

$$2x^2 + x(x + \sqrt{2}) = 8. \quad (4)$$

$$3x^2 + \sqrt{2}x - 8 = 0. \quad (5)$$

$$\text{Whence} \quad x = \frac{-\sqrt{2} \pm \sqrt{2 - 4(3)(-8)}}{6} \quad (6)$$

$$= \sqrt{2}, -\frac{4\sqrt{2}}{3}. \quad (7)$$

$$\text{From (7) and (3),} \quad y = 2\sqrt{2}, -\frac{\sqrt{2}}{3}.$$

$$28. \quad x^2 + y^2 + 4x + 6y = 40, \quad (1)$$

$$x - 10 = y. \quad (2)$$

$$\text{From (2),} \quad x = y + 10. \quad (3)$$

$$\text{Substituting } (y + 10) \text{ for } x \text{ in (1),}$$

$$(y + 10)^2 + y^2 + 4(y + 10) + 6y = 40. \quad (4)$$

$$y^2 + 15y + 50 = 0. \quad (5)$$

$$\text{Whence} \quad y = -5, -10. \quad (6)$$

$$\text{From (6) and (3),} \quad x = +5, \quad 0.$$

$$29. \quad y + x\sqrt{15} = 0, \quad (1)$$

$$y^2 + x^3 = 16x. \quad (2)$$

$$\text{From (1),} \quad y = -x\sqrt{15}. \quad (3)$$

$$\text{Substituting } -x\sqrt{15} \text{ for } y \text{ in (2),}$$

$$15x^2 + x^3 = 16x. \quad (4)$$

$$\text{Factoring,} \quad x(x + 16)(x - 1) = 0. \quad (5)$$

$$\text{Whence} \quad x = 0, \quad -16, \quad +1. \quad (6)$$

$$\text{From (6) and (3),} \quad y = 0, +16\sqrt{15}, -\sqrt{15}.$$

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1. $x^2 + y^2 = 20,$ (1)
 $3y^2 + x^2 = 28.$ (2)
 (2) - (1), $2y^2 = 8.$ (3)
 From (3), $y = \pm 2.$ (4)
 From (1) and (4), $x^2 + 4 = 20.$ (5)
 From (5), $x = \pm 4.$ (6)
 Therefore $x = 4, +4, -4, -4,$
 and $y = 2, -2, +2, -2.$
2. $3x^2 + 2xy = 7,$ (1)
 $2x^2 - 3xy = -4.$ (2)
 (1) · 3, $9x^2 + 6xy = 21.$ (3)
 (2) · 2, $4x^2 - 6xy = -8.$ (4)
 (3) + (4), $13x^2 = 13.$ (5)
 From (5), $x = 1, -1.$ (6)
 From (1) and (6), $y = 2, -2.$
3. $3y^2 - xy = 2,$ (1)
 $2y^2 + 3xy = 38.$ (2)
 (1) · 3, $9y^2 - 3xy = 6.$ (3)
 (2) + (3), $11y^2 = 44.$ (4)
 From (4), $y = 2, -2.$ (5)
 From (1) and (5), $x = 5, -5.$
4. $y^2 - xy = 2x^2,$ (1)
 $2x^2 + xy = 16.$ (2)
 From (1), $x = \frac{-y \pm \sqrt{y^2 + 8y^2}}{4}$ (3)
 $= \frac{y}{2}, -y.$ (4)
 Substituting $\frac{y}{2}$ for x in (2),
 $2\left(\frac{y}{2}\right)^2 + \left(\frac{y}{2}\right)y = 16.$ (5)
 Whence $y = 4, -4.$ (6)
 Then from $x = \frac{y}{2}$ in (4), $x = 2, -2.$
 Substituting $(-y)$ for x in (2),
 $2y^2 - y^2 = 16.$
 Whence $y = +4, -4.$
 Then from $x = -y$ in (4), $x = -4, +4.$
 Therefore $x = 2, -2, -4, +4,$
 and $y = 4, -4, +4, -4.$

$$5. \quad x^2 - 3y + 3 = 0, \quad (1)$$

$$y^2 + x^2 = 25. \quad (2)$$

$$\text{From (1),} \quad x^2 = 3y - 3. \quad (3)$$

$$\text{Substituting } (3y - 3) \text{ for } x^2 \text{ in (2),} \quad y^2 + 3y - 3 = 25. \quad (4)$$

$$\text{Whence} \quad y = -7, 4. \quad (5)$$

$$\text{From (5) and (2),} \quad 49 + x^2 = 25. \quad (6)$$

$$\text{Whence} \quad x = \pm 2\sqrt{-6}. \quad (7)$$

$$\text{Also from (5) and (2),} \quad 16 + x^2 = 25. \quad (8)$$

$$\text{Whence} \quad x = \pm 3. \quad (9)$$

$$\text{Therefore} \quad x = 2\sqrt{-6}, -2\sqrt{-6}, 3, -3,$$

$$\text{and} \quad y = -7, -7, 4, +4.$$

$$6. \quad 3x^2 + 2y^2 = 7xy, \quad (1)$$

$$x^2 + y^2 - 5x = 3. \quad (2)$$

$$\text{From (1),} \quad x = 2y, \frac{y}{3}. \quad (3)$$

$$\text{Substituting } 2y \text{ for } x \text{ in (2),} \quad 5y^2 - 10y - 3 = 0. \quad (4)$$

$$\text{Whence} \quad y = \frac{5 \pm 2\sqrt{10}}{5}. \quad (5)$$

$$\text{From (3) and (5),} \quad x = \frac{10 \pm 4\sqrt{10}}{5}. \quad (6)$$

$$\text{Substituting } \frac{y}{3} \text{ for } x \text{ in (2),} \quad 10y^2 - 15y - 27 = 0. \quad (7)$$

$$\text{Whence} \quad y = \frac{15 \pm 3\sqrt{145}}{20}. \quad (8)$$

$$\text{From (3) and (8),} \quad x = \frac{5 \pm \sqrt{145}}{20}.$$

Therefore

$$x = \frac{10 + 4\sqrt{10}}{5}, \frac{10 - 4\sqrt{10}}{5}, \frac{5 + \sqrt{145}}{20}, \frac{5 - \sqrt{145}}{20},$$

$$\text{and} \quad y = \frac{5 + 2\sqrt{10}}{5}, \frac{5 - 2\sqrt{10}}{5}, \frac{15 + 3\sqrt{145}}{20}, \frac{15 - 3\sqrt{145}}{20}.$$

$$7. \quad x^2 + 2xy - y^2 = 32, \quad (1)$$

$$2x^2 - 3xy + y^2 = 0. \quad (2)$$

$$\text{From (2),} \quad x = y, \quad (3)$$

$$\text{and} \quad x = \frac{y}{2}. \quad (4)$$

Substituting y for x in (1),

$$y^2 + 2y^2 - y^2 = 32. \quad (5)$$

Whence

$$y = \pm 4. \quad (6)$$

From (3) and (6),

$$x = \pm 4. \quad (7)$$

Substituting $\frac{y}{2}$ for x in (1),

$$\left(\frac{y}{2}\right)^2 + 2\left(\frac{y}{2}\right)y - y^2 = 32. \quad (8)$$

Whence

$$y = \pm 8\sqrt{2}. \quad (9)$$

From (4) and (9),

$$x = \pm 4\sqrt{2}.$$

Therefore

$$x = 4, -4, 4\sqrt{2}, -4\sqrt{2},$$

and

$$y = 4, -4, 8\sqrt{2}, -8\sqrt{2}.$$

$$8. \quad 2x^2 - xy + 2y^2 = 12, \quad (1)$$

$$2x^2 + xy + 2y^2 = 8. \quad (2)$$

$$3 \cdot (2) - 2 \cdot (1), \quad 2x^2 + 5xy + 2y^2 = 0. \quad (3)$$

From (3), $x = -\frac{y}{2}, -2y.$

Substituting $\left(-\frac{y}{2}\right)$ for x in (2) and solving,

$$y = +2, -2.$$

Whence $x = -1, +1.$

Substituting $(-2y)$ for x in (2) and solving,

$$y = +1, -1.$$

Whence $x = -2, +2.$

Therefore $x = -1, +1, -2, +2,$

and $y = +2, -2, +1, -1.$

$$9. \quad 2x^2 + y^2 = 3xy, \quad (1)$$

$$x^2 + 3xy = 16. \quad (2)$$

Solving (1) for y , $y = 2x, x.$

Substituting $2x$ for y in (2) and solving,

$$x = \pm \frac{4}{7}\sqrt{7}.$$

Whence $y = \pm \frac{8}{7}\sqrt{7}.$

Substituting x for y in (2) and solving,

$$x = \pm 2.$$

Whence $y = \pm 2.$

Therefore $x = \frac{4}{7}\sqrt{7}, -\frac{4}{7}\sqrt{7}, 2, -2,$

and $y = \frac{8}{7}\sqrt{7}, -\frac{8}{7}\sqrt{7}, 2, -2.$

10. $s^2 - 3st = 4,$ (1)
 $t^2 + s^2 = 4.$ (2)
 (2) - (1), $t^2 + 3st = 0.$ (3)
 Solving for $t,$ $t = 0, -3s.$
 Substituting 0 for t in (2) and solving,
 $s = \pm 2.$
 Substituting $(-3s)$ for t in (2) and solving,
 $s = \pm \frac{1}{5}\sqrt{10}.$
 Whence $t = \mp \frac{3}{5}\sqrt{10}.$
 Therefore $s = 2, -2, +\frac{1}{5}\sqrt{10}, -\frac{1}{5}\sqrt{10},$
 and $t = 0, 0, -\frac{3}{5}\sqrt{10}, +\frac{3}{5}\sqrt{10}.$
11. $s^2 + 2st + 4t^2 = 13,$ (1)
 $t^2 + 8 = -2st.$ (2)
 (1) $\cdot 8 +$ (2) $\cdot 13,$ $8s^2 + 42st + 45t^2 = 0.$ (3)
 Solving for $s,$ $s = -\frac{3t}{2}, -\frac{15t}{4}.$
 Substituting $\left(-\frac{3t}{2}\right)$ for s in (2) and solving,
 $t = +2, -2.$
 Whence $s = -3, +3.$
 Substituting $\left(-\frac{15t}{4}\right)$ for s in (2) and solving,
 $t = +\frac{4}{13}\sqrt{13}, -\frac{4}{13}\sqrt{13}.$
 Whence $s = -\frac{15}{13}\sqrt{13}, +\frac{15}{13}\sqrt{13}.$
 Therefore $s = -3, +3, -\frac{15}{13}\sqrt{13}, +\frac{15}{13}\sqrt{13},$
 and $t = +2, -2, +\frac{4}{13}\sqrt{13}, -\frac{4}{13}\sqrt{13}.$

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1. Solving the four systems given in the Hints,
 $x = 6, 1, -1, -6,$
 and $y = 1, 6, -6, -1.$
2. $x^2 + y^2 = 100,$ (1)
 $xy = 48.$ (2)
 (1) + (2) $\cdot 2,$ $x^2 + 2xy + y^2 = 196.$ (3)
 Whence $x + y = \pm 14.$ (4), (5)
 (1) - (2) $\cdot 2,$ $x^2 - 2xy + y^2 = 4.$ (6)
 Whence $x - y = \pm 2.$ (7), (8)
 Solving (4), (5), (7), and (8), $x = 8, 6, -6, -8,$
 and $y = 6, 8, -8, -6.$

3. $x^2 + xy + y^2 = 19,$ (1)
 $xy = 6.$ (2)
 (1) + (2), $x^2 + 2xy + y^2 = 25.$ (3)
 Whence $x + y = \pm 5.$ (4), (5)
 (1) - (2) $\cdot 3,$ $x^2 - 2xy + y^2 = 1.$ (6)
 Whence $x - y = \pm 1.$ (7), (8)
 Solving (4), (5), (7), and (8), $x = 3, 2, -2, -3,$
 and $y = 2, 3, -3, -2.$
4. $x^2 + 4y^2 = 20,$ (1)
 $xy = 4.$ (2)
 (1) + (2) $\cdot 4,$ $x^2 + 4xy + 4y^2 = 36.$ (3)
 Whence $x + 2y = \pm 6.$ (4), (5)
 (1) - (2) $\cdot 4,$ $x^2 - 4xy + 4y^2 = 4.$ (6)
 Whence $x - 2y = \pm 2.$ (7), (8)
 Solving (4), (5), (7), and (8), $x = 4, 2, -2, -4,$
 and $y = 1, 2, -2, -1.$
5. $9x^2 + y^2 = 61,$ (1)
 $xy = 10.$ (2)
 (1) + (2) $\cdot 6,$ $9x^2 + 6xy + y^2 = 121.$ (3)
 Whence $3x + y = \pm 11.$ (4), (5)
 (1) - (2) $\cdot 6,$ $9x^2 - 6xy + y^2 = 1.$ (6)
 Whence $3x - y = \pm 1.$ (7), (8)
 Solving (4), (5), (7), and (8), $x = 2, \frac{5}{3}, -\frac{5}{3}, -2,$
 and $y = 5, 6, -6, -5.$
6. $4x^2 + xy + y^2 = 85,$ (1)
 $xy = 12.$ (2)
 (1) + (2) $\cdot 3,$ $4x^2 + 4xy + y^2 = 121.$ (3)
 Whence $2x + y = \pm 11.$ (4), (5)
 (1) - (2) $\cdot 5,$ $4x^2 - 4xy + y^2 = 25.$ (6)
 Whence $2x - y = \pm 5.$ (7), (8)
 Solving (4), (5), (7), and (8), $x = 4, \frac{3}{2}, -\frac{3}{2}, -4,$
 and $y = 3, 8, -8, -3.$
7. $4x^2 + 25y^2 = 41,$ (1)
 $xy = 2.$ (2)
 (1) + (2) $\cdot 20,$ $4x^2 + 20xy + 25y^2 = 81.$ (3)
 Whence $2x + 5y = \pm 9.$ (4), (5)
 (1) - (2) $\cdot 20,$ $4x^2 - 20xy + 25y^2 = 1.$ (6)
 Whence $2x - 5y = \pm 1.$ (7), (8)
 Solving (4), (5), (7), and (8), $x = \frac{5}{2}, 2, -2, -\frac{5}{2},$
 and $y = \frac{4}{5}, 1, -1, -\frac{4}{5}.$

$$8. \quad x^2 - xy + y^2 = 8, \quad (1)$$

$$xy + 1 = 0. \quad (2)$$

$$\text{From (2),} \quad xy = -1. \quad (3)$$

$$(1) - (3), \quad x^2 - 2xy + y^2 = 9. \quad (4)$$

$$\text{Whence} \quad x - y = \pm 3. \quad (5), (6)$$

$$(1) + (3) \cdot 3, \quad x^2 + 2xy + y^2 = 5. \quad (7)$$

$$\text{Whence} \quad x + y = \pm \sqrt{5}. \quad (8), (9)$$

$$\text{Solving (5), (6), (8), and (9),} \quad x = \frac{3 + \sqrt{5}}{2}, \frac{\sqrt{5} - 3}{2}, \frac{3 - \sqrt{5}}{2}, \frac{-3 - \sqrt{5}}{2},$$

$$\text{and} \quad y = \frac{\sqrt{5} - 3}{2}, \frac{\sqrt{5} + 3}{2}, \frac{-\sqrt{5} - 3}{2}, \frac{3 - \sqrt{5}}{2}.$$

$$9. \quad \frac{x^2}{3} - xy + \frac{4y^2}{3} = \frac{7}{3}, \quad (1)$$

$$x = \frac{6}{y}. \quad (2)$$

$$\text{From (1),} \quad x^2 - 3xy + 4y^2 = 7. \quad (3)$$

$$\text{From (2),} \quad xy = 6. \quad (4)$$

$$(3) - (4), \quad x^2 - 4xy + 4y^2 = 1. \quad (5)$$

$$\text{Whence} \quad x - 2y = \pm 1. \quad (6), (7)$$

$$(3) + (4) \cdot 7, \quad x^2 + 4xy + 4y^2 = 49. \quad (8)$$

$$\text{Whence} \quad x + 2y = \pm 7. \quad (9), (10)$$

$$\text{Solving (6), (7), (9), and (10),} \quad x = 4, 3, -3, -4,$$

$$\text{and} \quad y = \frac{3}{2}, 2, -2, -\frac{3}{2}.$$

$$10. \quad \frac{1}{x^2} + \frac{1}{y^2} = 13. \quad (1)$$

$$\frac{1}{xy} = 6. \quad (2)$$

$$(1) + (2) \cdot 2, \quad \frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 25. \quad (3)$$

$$\text{Whence} \quad \frac{1}{x} + \frac{1}{y} = \pm 5. \quad (4), (5)$$

$$(1) - (2) \cdot 2, \quad \frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \quad (6)$$

$$\text{Whence} \quad \frac{1}{x} - \frac{1}{y} = \pm 1. \quad (7), (8)$$

$$\text{Solving (4), (5), (7), and (8),} \quad x = \frac{1}{3}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{3},$$

$$\text{and} \quad y = \frac{1}{2}, \frac{1}{3}, -\frac{1}{3}, -\frac{1}{2}.$$

$$11. \quad \frac{1}{x^2} + \frac{1}{y^2} = 25, \quad (1)$$

$$\frac{1}{xy} = 12. \quad (2)$$

$$(1) + (2) \cdot 2, \quad \frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 49. \quad (3)$$

$$\text{Whence} \quad \frac{1}{x} + \frac{1}{y} = \pm 7. \quad (4), (5)$$

$$(1) - (2) \cdot 2, \quad \frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \quad (6)$$

$$\text{Whence} \quad \frac{1}{x} - \frac{1}{y} = \pm 1. \quad (7), (8)$$

$$\begin{array}{l} \text{Solving (4), (5), (7), and (8),} \\ \text{and} \end{array} \quad \begin{array}{l} x = \frac{1}{4}, \frac{1}{3}, -\frac{1}{3}, -\frac{1}{4}, \\ y = \frac{1}{3}, \frac{1}{4}, -\frac{1}{4}, -\frac{1}{3}. \end{array}$$

$$12. \quad \frac{1}{x^2} + \frac{1}{y^2} = \frac{5}{4}, \quad (1)$$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{2}. \quad (2)$$

$$(2) \text{ squared,} \quad \frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = \frac{1}{4}. \quad (3)$$

$$(1) - (3), \quad \frac{2}{xy} = 1. \quad (4)$$

$$(1) + (4), \quad \frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = \frac{9}{4}. \quad (5)$$

$$\text{Whence} \quad \frac{1}{x} + \frac{1}{y} = \pm \frac{3}{2}. \quad (6), (7)$$

$$\begin{array}{l} \text{Solving (2), (6), and (7),} \\ \text{and} \end{array} \quad \begin{array}{l} x = 1, -2, \\ y = 2, -1. \end{array}$$

$$13. \quad \frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}, \quad (1)$$

$$\frac{1}{9x^2} + \frac{1}{4y^2} = \frac{97}{36}. \quad (2)$$

$$\text{Squaring (1),} \quad \frac{1}{9x^2} + \frac{2}{6xy} + \frac{1}{4y^2} = \frac{169}{36}. \quad (3)$$

$$(3) - (2), \quad \frac{2}{6xy} = \frac{72}{36}. \quad (4)$$

$$(2) - (4), \quad \frac{1}{9x^2} - \frac{2}{6xy} + \frac{1}{4y^2} = \frac{25}{36}. \quad (5)$$

$$\text{Whence} \quad \frac{1}{3x} - \frac{1}{2y} = \pm \frac{5}{6}. \quad (6), (7)$$

$$\begin{array}{l} \text{Solving (1), (6), and (7),} \\ \text{and} \end{array} \quad \begin{array}{l} x = \frac{1}{2}, \frac{2}{9}, \\ y = \frac{1}{3}, \frac{3}{4}. \end{array}$$

$$14. \quad \frac{1}{x^2} - \frac{1}{xy} + \frac{1}{y^2} = 7, \quad (1)$$

$$\frac{1}{xy} = 6. \quad (2)$$

$$(1) + (2) \cdot 3, \quad \frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 25. \quad (3)$$

$$\text{Whence} \quad \frac{1}{x} + \frac{1}{y} = \pm 5. \quad (4), (5)$$

$$(1) - (2), \quad \frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \quad (6)$$

$$\text{Whence} \quad \frac{1}{x} - \frac{1}{y} = \pm 1. \quad (7), (8)$$

$$\begin{array}{l} \text{Solving (4), (5), (7), and (8),} \\ \text{and} \end{array} \quad \begin{array}{l} x = \frac{1}{3}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{3}, \\ y = \frac{1}{2}, \frac{1}{3}, -\frac{1}{3}, -\frac{1}{2}. \end{array}$$

$$15. \quad \frac{1}{x^2} - \frac{1}{xy} + \frac{1}{y^2} = 13, \quad (1)$$

$$\frac{1}{x} + \frac{1}{y} = 7. \quad (2)$$

$$\text{Squaring (2),} \quad \frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 49. \quad (3)$$

$$(3) - (1), \quad \frac{3}{xy} = 36. \quad (4)$$

$$(4) \div 3, \quad \frac{1}{xy} = 12. \quad (5)$$

$$(1) - (5), \quad \frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \quad (6)$$

$$\text{Whence} \quad \frac{1}{x} - \frac{1}{y} = \pm 1. \quad (7), (8)$$

$$\begin{array}{l} \text{Solving (2), (7), and (8),} \\ \text{and} \end{array} \quad \begin{array}{l} x = \frac{1}{4}, \frac{1}{3}, \\ y = \frac{1}{3}, \frac{1}{4}. \end{array}$$

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1. Solving the system given in the Hint,

$$\begin{array}{l} x = 6, \\ y = -2. \end{array}$$

$$2. \quad x^2 - y^2 = 77, \quad (1)$$

$$x - y = 7. \quad (2)$$

$$(1) \div (2), \quad x + y = 11. \quad (3)$$

$$\begin{array}{l} \text{From (2) and (3),} \\ \text{and} \end{array} \quad \begin{array}{l} x = 9, \\ y = 2. \end{array}$$

3. $x^2 - 2xy = 187$, (1) Squaring (2),
 $2y - x = -17$. (2) $x^2 - 4xy + 4y^2 = 25$. (4)
 (1) \div (2), $x = 11$. (3) (3) - (4), $6xy = -18$. (5)
 From (2) and (3), $y = -3$. (5) \div 3, $2xy = -6$. (6)
 4. $4x^2 - y^2 = 16$, (1) (3) + (6),
 $2x + y = 8$. (2) $x^2 + 4xy + 4y^2 = 1$. (7)
 (1) \div (2), $2x - y = 2$. (3) Whence $x + 2y = \pm 1$. (8), (9)
 From (2) and (3), $x = \frac{5}{2}$,
 and $y = 3$. Solving (2), (8), and (9),
 $x = +3, +2$,
 $y = -1, -\frac{3}{2}$.
 5. $R^2h - 75 = 0$, (1) and
 $Rh = 15$. (2) 9. $\frac{1}{x} + \frac{1}{y} = 1$, (1)
 (1) \div (2), $R = 5$. $\frac{1}{y^2} - \frac{1}{x^2} = 5$. (2)
 Whence $h = 3$. (2) \div (1), $\frac{1}{y} - \frac{1}{x} = 5$. (3)
 6. $\frac{1}{x^2} - \frac{1}{y^2} = 15$, (1) Solving (1) and (3),
 $\frac{1}{x} + \frac{1}{y} = 3$. (2) $x = -\frac{1}{2}$,
 (1) \div (2), $\frac{1}{x} - \frac{1}{y} = 5$. (3) and $y = \frac{1}{3}$.
 Solving (2) and (3),
 $x = \frac{1}{4}$,
 and $y = -1$.
 7. $x^3 + y^3 = 28$, (1)
 $x + y = 4$. (2)
 (1) \div (2),
 $x^2 - xy + y^2 = 7$. (3)
 Squaring (2),
 $x^2 + 2xy + y^2 = 16$. (4)
 (4) - (3), $3xy = 9$. (5)
 (5) \div 3, $xy = 3$. (6)
 (3) - (6),
 $x^2 - 2xy + y^2 = 4$. (7)
 Whence $x - y = \pm 2$. (8), (9)
 Solving (2), (8), and (9),
 $x = 3, 1$,
 and $y = 1, 3$.
 8. $x^3 - 8y^3 = 35$, (1)
 $x - 2y = 5$. (2)
 (1) \div (2),
 $x^2 + 2xy + 4y^2 = 7$. (3) and
10. $\frac{1}{x^3} + \frac{1}{y^3} = 152$, (1)
 $\frac{1}{x} + \frac{1}{y} = 8$. (2)
 (1) \div (2), $\frac{1}{x^2} - \frac{1}{xy} + \frac{1}{y^2} = 19$. (3)
 Squaring (2),
 $\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 64$. (4)
 (4) - (3), $\frac{3}{xy} = 45$. (5)
 (5) \div 3, $\frac{1}{xy} = 15$. (6)
 (3) - (6), $\frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 4$. (7)
 Whence $\frac{1}{x} - \frac{1}{y} = \pm 2$. (8), (9)
 Solving (2), (8), and (9),
 $x = \frac{1}{5}, \frac{1}{3}$,
 $y = \frac{1}{3}, \frac{1}{5}$.

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$$1. \quad y^2 + xy = 8, \quad (1)$$

$$x + y = 4. \quad (2)$$

$$\text{From (2),} \quad x = 4 - y. \quad (3)$$

$$\text{Substituting in (1) and solving,} \quad y = 2.$$

$$\text{Whence} \quad x = 2.$$

$$2. \quad x^2 - 9y^2 = 7, \quad (1)$$

$$x - 3y = 1. \quad (2)$$

$$(1) \div (2), \quad x + 3y = 7. \quad (3)$$

$$\text{Solving (2) and (3),} \quad x = 4,$$

$$\text{and} \quad y = 1.$$

$$3. \quad x^2 - x - 2y = 0, \quad (1)$$

$$y^2 - 4y + x = 0. \quad (2)$$

$$(1) - (2), \quad x^2 - 2x + 2y - y^2 = 0. \quad (3)$$

$$\text{From (3),} \quad x = 2 - y, \quad (4)$$

$$\text{and} \quad x = y. \quad (5)$$

$$\text{From (4) and (1),}$$

$$4 - 4y + y^2 - 2 + y - 2y = 0. \quad (6)$$

$$\text{Whence} \quad y = \frac{5 \pm \sqrt{17}}{2}. \quad (7)$$

$$\text{From (4) and (7),} \quad x = \frac{-1 \mp \sqrt{17}}{2}. \quad (8)$$

$$\text{From (1) and (5),} \quad y^2 - y - 2y = 0. \quad (9)$$

$$\text{Whence} \quad y = 0, 3. \quad (10)$$

$$\text{From (5) and (10),} \quad x = 0, 3.$$

$$\text{Therefore} \quad x = \frac{-1 - \sqrt{17}}{2}, \frac{-1 + \sqrt{17}}{2}, 0, 3,$$

$$\text{and} \quad y = \frac{5 + \sqrt{17}}{2}, \frac{5 - \sqrt{17}}{2}, 0, 3.$$

$$4. \quad x^2 + y^2 = 25, \quad (1)$$

$$x + y = 7. \quad (2)$$

$$\text{From (2),} \quad x = 7 - y. \quad (3)$$

$$\text{Substituting in (1) and solving,} \quad y = 4, 3. \quad (4)$$

$$\text{From (3) and (4),} \quad x = 3, 4.$$

$$5. \quad \frac{x}{y} + 3 = 0, \quad (1)$$

$$xy + 12 = 0. \quad (2)$$

Solving (1) for x , $x = -3y$.

Substituting in (2) and solving, $y = +2, -2$,
and $x = -6, +6$.

$$6. \quad (x + y)^2 = 9, \quad (1)$$

$$(x - y)^2 = 49. \quad (2)$$

From (1), $x + y = \pm 3. \quad (3), (4)$

From (2), $x - y = \pm 7. \quad (5), (6)$

Solving (3), (4), (5), and (6), $x = +5, -2, +2, -5$,
and $y = -2, +5, -5, +2$.

$$7. \quad 2x^2 + y^2 = 33, \quad (1)$$

$$x^2 + 2y^2 = 54. \quad (2)$$

$$(1) \cdot 2 - (2), \quad 3x^2 = 12. \quad (3)$$

Whence $x = \pm 2$.

$$\text{Substituting in (2),} \quad 4 + 2y^2 = 54. \quad (4)$$

Whence $y = \pm 5$.

Therefore $x = 2, +2, -2, -2$,

and $y = 5, -5, +5, -5$.

$$8. \quad 3h^2 - 8k^2 = 40, \quad (1)$$

$$5h^2 + k^2 = 81. \quad (2)$$

$$(1) + (2) \cdot 8, \quad 43h^2 = 688.$$

Whence $h = \pm 4$.

$$\text{Substituting in (2),} \quad 80 + k^2 = 81.$$

Whence $k = \pm 1$.

Therefore $h = 4, +4, -4, -4$,

and $k = 1, -1, +1, -1$.

$$9. \quad 4R_1^2 + 3 = 9R_2^2, \quad (1)$$

$$12R_1^2 + R_2^2 = \frac{31}{9}. \quad (2)$$

$$(1) + (2) \cdot 9, \quad 112R_1^2 = 28.$$

Whence $R_1 = \pm \frac{1}{2}$.

$$\text{Substituting in (1),} \quad 1 + 3 = 9R_2^2.$$

Whence $R_2 = \pm \frac{2}{3}$.

Therefore $R_1 = \frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$,

and $R_2 = \frac{2}{3}, -\frac{2}{3}, +\frac{2}{3}, -\frac{2}{3}$.

$$10. \quad xy + x = 18, \quad (1)$$

$$xy + y = 20. \quad (2)$$

$$(2) - (1), \quad y - x = 2. \quad (3)$$

$$\text{Solving for } x \text{ in (3),} \quad x = y - 2. \quad (4)$$

$$\text{Substituting in (2),} \quad y^2 - 2y + y = 20.$$

$$\text{Whence} \quad y = 5, -4.$$

$$\text{From (4),} \quad x = 3, -6.$$

$$11. \quad x^2 + y^2 = 169, \quad (1)$$

$$xy = 60. \quad (2)$$

$$(1) + (2) \cdot 2, \quad x^2 + 2xy + y^2 = 289. \quad (3)$$

$$\text{Whence} \quad x + y = \pm 17. \quad (4), (5)$$

$$(1) - (2) \cdot 2, \quad x^2 - 2xy + y^2 = 49. \quad (6)$$

$$\text{Whence} \quad x - y = \pm 7. \quad (7), (8)$$

$$\text{Solving (4), (5), (7), and (8),} \quad x = 12, 5, -5, -12,$$

$$\text{and} \quad y = 5, 12, -12, -5.$$

$$12. \quad x^2 = y, \quad (1)$$

$$xy = 8. \quad (2)$$

$$\text{Substituting } x^2 \text{ for } y \text{ in (2),} \quad x^2 \cdot x = 8. \quad (3)$$

$$\text{Whence} \quad x = 2, -1 \pm \sqrt{-3}. \quad (4)$$

$$\text{From (2) and (4),} \quad y = 4, -2 \mp 2\sqrt{-3}.$$

$$\text{Therefore} \quad x = 2, -1 + \sqrt{-3}, -1 - \sqrt{-3},$$

$$\text{and} \quad y = 4, -2 - 2\sqrt{-3}, -2 + 2\sqrt{-3}.$$

$$13. \quad x - xy = 5, \quad (1)$$

$$2y + xy = 6. \quad (2)$$

$$(1) + (2), \quad x + 2y = 11. \quad (3)$$

$$\text{Solving for } y, \quad y = \frac{11 - x}{2}. \quad (4)$$

$$\text{Substituting in (1),} \quad x - \frac{11x - x^2}{2} = 5.$$

$$\text{Whence} \quad x = 10, -1.$$

$$\text{From (4),} \quad y = \frac{1}{2}, +6.$$

$$14. \quad x^3 - y^3 = 19, \quad (1)$$

$$x^2 + xy + y^2 = 19. \quad (2)$$

$$(1) \div (2), \quad x - y = 1. \quad (3)$$

$$\text{Squaring (3),} \quad x^2 - 2xy + y^2 = 1. \quad (4)$$

$$[(2) - (4)] \div 3, \quad xy = 6. \quad (5)$$

$$(2) + (5), \quad x^2 + 2xy + y^2 = 25. \quad (6)$$

$$\text{Whence} \quad x + y = \pm 5. \quad (7), (8)$$

$$\text{Solving (3), (7), and (8),} \quad x = 3, -2,$$

$$\text{and} \quad y = 2, -3.$$

$$15. \quad 4n^2 + 7m^2 = 9, \quad (1)$$

$$2n^2 - \frac{9}{2} = m^2. \quad (2)$$

$$(1) - (2) \cdot 2,$$

$$9m^2 = 0.$$

Whence

$$m = 0.$$

Substituting in (1),

$$4n^2 = 9.$$

Whence

$$n = \frac{3}{2}, -\frac{3}{2},$$

and

$$m = 0, 0.$$

$$16. \quad 5W_1^2 - 6.8W_2^2 = 99.55, \quad (1)$$

$$W_1^2 - W_2^2 = 20. \quad (2)$$

$$(2) \cdot 5 - (1),$$

$$1.8W_2^2 = .45.$$

Whence

$$W_2 = \pm .5.$$

Substituting in (2),

$$W_1^2 - .25 = 20.$$

Whence

$$W_1 = \pm 4.5.$$

Therefore

$$W_1 = 4.5, -4.5, +4.5, -4.5,$$

and

$$W_2 = .5, +.5, -.5, -.5.$$

$$17. \quad xy + 2y^2 = 2, \quad (1)$$

$$3xy + 5y^2 = 2. \quad (2)$$

$$(2) - (1),$$

$$3y^2 + 2xy = 0. \quad (3)$$

Solving (3),

$$y = -\frac{2x}{3}, 0. \quad (4)$$

Substituting $\left(-\frac{2x}{3}\right)$ for y in (1),

$$\frac{-2x^2}{3} + \frac{8x^2}{9} = 2.$$

Whence

$$x = +3, -3.$$

From (4),

$$y = -2, +2.$$

Substituting 0 for y in (1) or (2), x is infinite.

$$18. \quad x^2 + 2xy + 2y^2 = 10, \quad (1)$$

$$3x^2 - xy - y^2 = 51. \quad (2)$$

$$(1) + (2) \cdot 2,$$

$$7x^2 = 112.$$

Whence

$$x = \pm 4.$$

Substituting $(+4)$ for x in (1),

$$16 + 8y + 2y^2 = 10.$$

Whence

$$y = -3, -1.$$

Substituting (-4) for x in (1),

$$16 - 8y + 2y^2 = 10.$$

Whence

$$y = 3, 1.$$

Therefore

$$x = +4, +4, -4, -4,$$

and

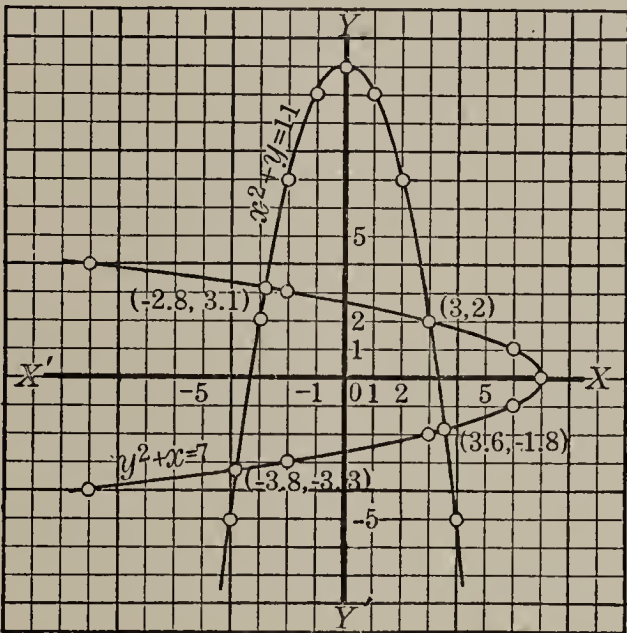
$$y = -3, -1, +3, +1.$$

19. $y^2 + x = 7,$
 $x^2 + y = 11.$
 $y^2 + x = 7.$ Then $y = \pm \sqrt{7 - x}.$

If $x =$	7	6	3	- 2	- 9	0
then $y =$	0	± 1	± 2	± 3	± 4	± 2.64

$x^2 + y = 11.$ Then $y = 11 - x^2.$

If $x =$	0	± 1	± 2	± 3	± 4	± 5
then $y =$	11	10	7	2	- 5	- 14



The sets of roots are

x	- 2.8	3	3.6	- 3.8
y	3.1	2	- 1.8	- 3.3

20. $x^2 + xy + y^2 = 7,$ (1)
 $x^2 + y^2 = 10.$ (2)
(1) - (2), $xy = - 3.$ (3)
(1) + (3), $x^2 + 2xy + y^2 = 4.$ (4)
Whence $x + y = \pm 2.$ (5), (6)
(1) - (3) $\cdot 3,$ $x^2 - 2xy + y^2 = 16.$ (7)
Whence $x - y = \pm 4.$ (8), (9)
Solving (5), (6), (8), and (9), $x = + 3, - 1, + 1, - 3,$
and $y = - 1, + 3, - 3, + 1.$

$$21. \quad x^2 + xy + y = 0, \quad (1)$$

$$x^2 + xy + x = 0. \quad (2)$$

$$(1) - (2), \quad y = x. \quad (3)$$

$$\text{Substituting in (1),} \quad x^2 + x^2 + x = 0.$$

$$\text{Whence} \quad x = 0, -\frac{1}{2}.$$

$$\text{From (3),} \quad y = 0, -\frac{1}{2}.$$

$$22. \quad \frac{1}{x^2} + \frac{1}{y^2} = 13, \quad (1)$$

$$\frac{1}{x} - \frac{1}{y} = 1. \quad (2)$$

$$\text{Squaring (2),} \quad \frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \quad (3)$$

$$(1) - (3), \quad \frac{2}{xy} = 12. \quad (4)$$

$$(1) + (4), \quad \frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 25. \quad (5)$$

$$\text{Whence} \quad \frac{1}{x} + \frac{1}{y} = \pm 5. \quad (6), (7)$$

$$\begin{array}{l} \text{Solving (2), (6), and (7),} \\ \text{and} \end{array} \quad \begin{array}{l} x = \frac{1}{3}, -\frac{1}{2}, \\ y = \frac{1}{2}, -\frac{1}{3}. \end{array}$$

$$23. \quad \frac{1}{x^3} - \frac{1}{y^3} = 7, \quad (1)$$

$$\frac{1}{x} - \frac{1}{y} = 1. \quad (2)$$

$$(1) \div (2), \quad \frac{1}{x^2} + \frac{1}{xy} + \frac{1}{y^2} = 7. \quad (3)$$

$$\text{Squaring (2),} \quad \frac{1}{x^2} - \frac{2}{xy} + \frac{1}{y^2} = 1. \quad (4)$$

$$[(3) - (4)] \div 3, \quad \frac{1}{xy} = 2. \quad (5)$$

$$(3) + (5), \quad \frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2} = 9. \quad (6)$$

$$\text{Whence} \quad \frac{1}{x} + \frac{1}{y} = \pm 3. \quad (7), (8)$$

$$\begin{array}{l} \text{Solving (2), (7), and (8),} \\ \text{and} \end{array} \quad \begin{array}{l} x = \frac{1}{2}, -1, \\ y = 1, -\frac{1}{2}. \end{array}$$

$$24. \quad 4x^2 + y^2 = 289, \quad (1)$$

$$xy = 60. \quad (2)$$

$$(1) + (2) \cdot 4, \quad 4x^2 + 4xy + y^2 = 529. \quad (3)$$

$$\text{Whence} \quad 2x + y = \pm 23. \quad (4), (5)$$

$$(1) - (2) \cdot 4, \quad 4x^2 - 4xy + y^2 = 49. \quad (6)$$

$$\text{Whence} \quad 2x - y = \pm 7. \quad (7), (8)$$

$$\text{Solving (4), (5), (7), and (8),} \quad x = \frac{15}{2}, 4, -4, -\frac{15}{2},$$

$$\text{and} \quad y = 8, 15, -15, -8.$$

$$25. \quad x^{\frac{1}{2}} + y^{\frac{1}{2}} = 9, \quad (1) \quad 27. \quad \frac{9x}{y} = 18 = xy. \quad (1), (2)$$

$$x^{\frac{1}{2}}y^{\frac{1}{2}} = 20. \quad (2)$$

$$\text{Squaring (1),} \quad \text{From (1),} \quad x = 2y. \quad (3)$$

$$x + 2\sqrt{xy} + y = 81. \quad (3) \quad xy = 18. \quad (2)$$

$$(3) - (2) \cdot 4, \quad \text{Substituting in (2),} \quad 2y^2 = 18. \quad (4)$$

$$x - 2\sqrt{xy} + y = 1. \quad (4) \quad \text{Whence} \quad y = 3, -3.$$

$$\text{From (4),} \quad \sqrt{x} - \sqrt{y} = \pm 1. \quad (5) \quad \text{From (3),} \quad x = 6, -6.$$

$$\text{From (5) and (1),} \quad x = 25, 16, \quad 28. \quad 4x + \frac{1}{y} = 46. \quad (1)$$

$$\text{and} \quad y = 16, 25.$$

$$26. \quad x^2 + y^2 + x + y = 58, \quad (1) \quad \frac{26x}{5} - \frac{1}{y} = 46. \quad (2)$$

$$x + y = 5. \quad (2)$$

$$(1) - (2), \quad x^2 + y^2 = 53. \quad (3) \quad (1) + (2), \quad \frac{46x}{5} = 92. \quad (3)$$

$$\text{Solving (2) for } x, \quad x = 5 - y. \quad (4) \quad x = 10. \quad (4)$$

$$\text{Substituting in (3),} \quad \text{From (1) and (4),}$$

$$(5 - y)^2 + y^2 = 53. \quad (5) \quad 40 + \frac{1}{y} = 46.$$

$$\text{Whence} \quad y = +7, -2.$$

$$\text{From (4),} \quad x = -2, +7. \quad \text{Whence} \quad y = \frac{1}{6}.$$

$$29. \quad \frac{1}{x-2} + \frac{1}{y-2} = \frac{3}{4}, \quad (1)$$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{12}. \quad (2)$$

$$\text{From (2),} \quad 12y - 12x = xy. \quad (3)$$

$$\text{From (1),} \quad 10x + 10y = 3xy + 28. \quad (4)$$

$$(3) \cdot 3, \quad -36x + 36y = 3xy. \quad (5)$$

$$(4) - (5), \quad 46x - 26y = 28. \quad (6)$$

$$\text{Whence} \quad x = \frac{13y + 14}{23}. \quad (7)$$

$$\text{From (7) and (3),} \quad 12y - \frac{156y + 168}{23} = \frac{13y^2 + 14y}{23}. \quad (8)$$

$$\text{From (8),} \quad y = 6, \frac{2}{3}. \quad (9)$$

$$\text{From (9) and (7),} \quad x = 4, \frac{4}{3}.$$

$$30. \quad 3xy = x^2y^2 - 88, \quad (1)$$

$$x - y = 6. \quad (2)$$

$$\text{From (1),} \quad (xy - 11)(xy + 8) = 0. \quad (3)$$

From (3) and (2),

$$(y^2 + 6y - 11)(y^2 + 6y + 8) = 0. \quad (4)$$

$$\text{Whence} \quad y = -3 + 2\sqrt{5}, -3 - 2\sqrt{5}, -2, -4.$$

$$\text{From (2) and (4),} \quad x = +3 + 2\sqrt{5}, +3 - 2\sqrt{5}, +4, +2.$$

$$31. \quad xy = c, \quad (1)$$

$$x + y = a. \quad (2)$$

$$\text{From (2),} \quad x = a - y. \quad (3)$$

$$\text{From (1) and (3),} \quad ay - y^2 = c. \quad (4)$$

$$\text{Whence} \quad y = \frac{a \pm \sqrt{a^2 - 4c}}{2}. \quad (5)$$

$$\text{From (3) and (5),} \quad x = \frac{a \mp \sqrt{a^2 - 4c}}{2}.$$

$$\text{Therefore} \quad x = \frac{a + \sqrt{a^2 - 4c}}{2}, \frac{a - \sqrt{a^2 - 4c}}{2},$$

$$\text{and} \quad y = \frac{a - \sqrt{a^2 - 4c}}{2}, \frac{a + \sqrt{a^2 - 4c}}{2}.$$

$$32. \quad x^{-2} - y^{-2} = 6, \quad (1) \quad 33. \quad x - y = 16, \quad (1)$$

$$x^{-1} + y^{-1} = 2. \quad (2) \quad x^{\frac{1}{2}} - y^{\frac{1}{2}} = 2. \quad (2)$$

$$(1) \div (2), \quad x^{-1} - y^{-1} = 3. \quad (3) \quad (1) \div (2), \quad x^{\frac{1}{2}} + y^{\frac{1}{2}} = 8. \quad (3)$$

$$(3) + (2), \quad 2x^{-1} = 5. \quad (2) + (3), \quad 2x^{\frac{1}{2}} = 10.$$

$$\text{Whence} \quad x = \frac{5}{2}. \quad \text{Whence} \quad x = 25.$$

$$(2) - (3), \quad 2y^{-1} = -1. \quad (3) - (2), \quad 2y^{\frac{1}{2}} = 6.$$

$$\text{Whence} \quad y = -2. \quad \text{Whence} \quad y = 9.$$

$$34. \quad \frac{x-1}{y-1} = 3, \quad (1)$$

$$\frac{y^2 + y + 1}{x^2 - x + 1} = \frac{13}{43}. \quad (2)$$

$$\text{From (1),} \quad x = 3y - 2. \quad (3)$$

$$\text{From (2),} \quad 43y^2 + 43y + 43 = 13x^2 - 13x + 13. \quad (4)$$

$$\text{From (3) and (4),} \quad 43y^2 + 43y + 30 = 117y^2 - 156y + 52 - 39y + 26. \quad (5)$$

$$\text{From (5),} \quad 37y^2 - 119y + 24 = 0. \quad (6)$$

$$\text{From (6),} \quad y = 3, +\frac{8}{37}. \quad (7)$$

$$\text{From (3) and (7),} \quad x = 7, -\frac{59}{37}.$$

$$35. \quad 4x^2 - 13xy + 9y^2 = 9, \quad (1)$$

$$xy - y^2 = 3. \quad (2)$$

$$(2) \cdot 3, \quad 3xy - 3y^2 = 9. \quad (3)$$

$$(1) - (3), \quad 4x^2 - 16xy + 12y^2 = 0. \quad (4)$$

$$\text{From (4),} \quad x = 3y. \quad (5)$$

$$x = y. \quad (6)$$

$$\text{From (2) and (5),} \quad 3y^2 - y^2 = 3. \quad (7)$$

$$\text{From (7),} \quad y = \pm \frac{1}{2}\sqrt{6}. \quad (8)$$

$$\text{From (5),} \quad x = \pm \frac{3}{2}\sqrt{6}. \quad (9)$$

No results can be obtained by using (6) and (1) or (6) and (2).

$$\text{Therefore} \quad x = \frac{3}{2}\sqrt{6}, -\frac{3}{2}\sqrt{6},$$

$$\text{and} \quad y = \frac{1}{2}\sqrt{6}, -\frac{1}{2}\sqrt{6}.$$

$$36. \quad x^2 + 2xy = 16, \quad (1)$$

$$3x^2 - 4xy + 2y^2 = 6. \quad (2)$$

$$(1) \cdot 3, \quad 3x^2 + 6xy = 48. \quad (3)$$

$$(2) \cdot 8, \quad 24x^2 - 32xy + 16y^2 = 48. \quad (4)$$

$$(4) - (3), \quad 16y^2 - 38xy + 21x^2 = 0. \quad (5)$$

$$\text{From (5),} \quad y = \frac{3}{2}x, \quad (6)$$

$$\text{and} \quad y = \frac{7}{8}x. \quad (7)$$

$$\text{From (1) and (6),} \quad x^2 + 3x^2 = 16. \quad (8)$$

$$\text{From (8),} \quad x = \pm 2. \quad (9)$$

$$\text{From (6) and (9),} \quad y = \pm 3. \quad (10)$$

$$\text{From (1) and (7),} \quad x^2 + \frac{7x^2}{4} = 16. \quad (11)$$

$$\text{From (11),} \quad x = \pm \frac{8}{11}\sqrt{11}. \quad (12)$$

$$\text{From (7) and (12),} \quad y = \pm \frac{7}{11}\sqrt{11}.$$

$$\text{Therefore} \quad x = 2, -2, \frac{8}{11}\sqrt{11}, -\frac{8}{11}\sqrt{11},$$

$$\text{and} \quad y = 3, -3, \frac{7}{11}\sqrt{11}, -\frac{7}{11}\sqrt{11}.$$

$$37. \quad x^3 = y^3 + 37, \quad (1)$$

$$x^2y = xy^2 + 12. \quad (2)$$

$$\text{From (1),} \quad x^3 - y^3 = 37. \quad (3)$$

$$\text{From (2),} \quad x^2y - xy^2 = 12. \quad (4)$$

$$(3) \div (4), \quad \frac{x^2 + xy + y^2}{xy} = \frac{37}{12}. \quad (5)$$

$$\text{Clearing and solving for } x, \quad x = \frac{4y}{3}, \quad (6)$$

$$\text{and} \quad x = \frac{3y}{4}. \quad (7)$$

From (2) and (7),
$$\frac{9y^3}{16} = \frac{3y^3}{4} + 12. \quad (8)$$

Whence
$$y = -4, 2 \pm 2\sqrt{-3}. \quad (9)$$

From (7) and (9),
$$x = -3, \frac{3 \pm 3\sqrt{-3}}{2}. \quad (10)$$

From (2) and (6),
$$\frac{16y^3}{9} = \frac{4y^3}{3} + 12. \quad (11)$$

Whence
$$y = 3, \frac{-3 \pm 3\sqrt{-3}}{2}. \quad (12)$$

From (6) and (12),
$$x = 4, -2 \pm 2\sqrt{-3}.$$

Therefore

$$x = 4, -3, -2 + 2\sqrt{-3}, -2 - 2\sqrt{-3}, \frac{3 + 3\sqrt{-3}}{2}, \frac{3 - 3\sqrt{-3}}{2},$$

and $y = 3, -4, \frac{-3 + 3\sqrt{-3}}{2}, \frac{-3 - 3\sqrt{-3}}{2}, 2 + 2\sqrt{-3}, 2 - 2\sqrt{-3}.$

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1. Let x equal one number and y the other.

Then
$$x - y = 6, \quad (1)$$

and
$$x^2 - y^2 = 120. \quad (2)$$

(2) \div (1),
$$x + y = 20. \quad (3)$$

From (1) and (3),
$$x = 13, y = 7.$$

2. Let x equal one number and y the other.

Then
$$x + y = 20, \quad (1)$$

and
$$x^2 + y^2 = 202. \quad (2)$$

From (1),
$$x = 20 - y. \quad (3)$$

Substituting $(20 - y)$ for x in (2),

$$(20 - y)^2 + y^2 = 202.$$

Whence
$$x = 11, y = 9.$$

3. Let x equal one number and y the other.

Then
$$x + y + xy = 132, \quad (1)$$

and
$$\frac{x}{y} = 3. \quad (2)$$

From (2),
$$x = 3y. \quad (3)$$

Substituting $3y$ for x in (1),

$$3y + y + 3y^2 = 132. \quad (4)$$

From (4) and (3),
$$x = 18, y = 6.$$

4. Let x and y equal the dimensions of the lot in rods.

Then $xy = 160,$ (1)

and $2x + 2y = 56.$ (2)

From (2), $x = 28 - y.$ (3)

Substituting $(28 - y)$ for x in (1),
 $y(28 - y) = 160.$ (4)

From (4), $y = 8.$ (5)

From (3) and (5), $x = 20.$

5. Let x equal one side and y the other.

Then $x^2 + y^2 = 1681,$ (1)

and $\frac{xy}{2} = 180.$ (2)

$(1) + (2) \cdot 4,$ $x^2 + 2xy + y^2 = 2401.$ (3)

Whence $x + y = \pm 49.$ (4), (5)

$(1) - (2) \cdot 4,$ $x^2 - 2xy + y^2 = 961.$ (6)

Whence $x - y = \pm 31.$ (7), (8)

From (4), (5), (7), and (8), $x = 40$ feet,

and $y = 9$ feet.

6. Let x and y equal the dimensions in rods.

Then $xy = 15 \cdot 160,$ (1)

and $(x + 20)(y + 20) = 30 \cdot 160.$ (2)

From (1), $xy = 2400.$ (3)

From (2), $xy + 20x + 20y = 4400.$ (4)

$(4) - (3),$ $20x + 20y = 2000.$ (5)

Whence $x = 100 - y.$ (6)

Substituting $(100 - y)$ for x in (3),
 $y(100 - y) = 2400.$ (7)

Whence $y = 40.$ (8)

From (6) and (8), $x = 60.$

Therefore the dimensions are 40 rods by 60 rods.

7. Let x and y equal the sides of the squares, respectively.

Then $x^2 - y^2 = 495,$ (1)

and $4x - 4y = 60.$ (2)

From (2), $x = y + 15.$ (3)

Substituting $(y + 15)$ for x in (1),
 $y^2 + 30y + 225 - y^2 = 495.$ (4)

Whence $y = 9.$ (5)

From (3) and (5), $x = 24.$

Therefore the sides are 24 feet and 9 feet.

8. Let x and y equal the dimensions in rods.

$$\begin{array}{ll}
 \text{Then} & xy = 6912, \quad (1) \\
 \text{and} & x^2 + y^2 = 14,400. \quad (2) \\
 (2) + (1) \cdot 2, & x^2 + 2xy + y^2 = 28,224. \quad (3) \\
 \text{Whence} & x + y = \pm 168. \quad (4), (5) \\
 (2) - (1) \cdot 2, & x^2 - 2xy + y^2 = 576. \quad (6) \\
 \text{Whence} & x - y = \pm 24. \quad (7), (8) \\
 \text{From (4), (5), (7), and (8),} & x = 96. \\
 & y = 72.
 \end{array}$$

Therefore the perimeter $2x + 2y$ is 336 rods.

9. Let x = the numerator,
and y = the denominator.

$$\text{Then} \quad \frac{x}{y} = \frac{2}{3}, \quad (1)$$

$$\text{and} \quad \frac{x^2 - 44}{y^2 - 44} = \frac{5}{14}. \quad (2)$$

$$\text{From (1),} \quad x = \frac{2y}{3}. \quad (3)$$

Substituting $\frac{2y}{3}$ for x in (2),

$$14 \cdot \frac{4y^2}{9} - 396 = 5y^2. \quad (4)$$

$$\text{From (4),} \quad y = \pm 18. \quad (5)$$

$$\text{From (3) and (5),} \quad x = \pm 12.$$

Therefore the fraction is $\frac{1}{1}\frac{2}{8}$.

10. Let x and y equal the number of days, respectively, required by each of the two men when working alone.

$$\text{Then} \quad \frac{1}{x} + \frac{1}{y} = \frac{5}{24}, \quad (1)$$

$$\text{and} \quad x - y = 4. \quad (2)$$

$$\text{From (2),} \quad x = y + 4. \quad (3)$$

Substituting $(y + 4)$ for x in (1),

$$\frac{1}{y + 4} + \frac{1}{y} = \frac{5}{24}. \quad (4)$$

$$\text{Whence} \quad y = 8. \quad (5)$$

$$\text{From (3) and (5),} \quad x = 12.$$

11. Let x and y equal the dimensions in feet.

$$2x + 2y = 250. \quad (1)$$

$$xy = 1926. \quad (2)$$

From (1), $x = 125 - y. \quad (3)$

Substituting $(125 - y)$ for x in (2),

$$y(125 - y) = 1926. \quad (4)$$

Whence $y = 18. \quad (5)$

From (3) and (5), $x = 107.$

Therefore the dimensions are 18 by 107 feet.

12. Let x and y equal the base and altitude in inches.

Then $x = y + 8, \quad (1)$

and $\frac{xy}{2} = 240. \quad (2)$

Substituting $(y + 8)$ for x in (2),

$$\frac{y(y + 8)}{2} = 240. \quad (3)$$

Whence $y = -4 + 4\sqrt{31}. \quad (4)$

From (1) and (4), $x = 4 + 4\sqrt{31}.$

Therefore the base is 26.27 inches and the altitude is 18.27 inches.

13. Let x and y equal the edges of the respective cubes.

Then $x^3 - y^3 = 316, \quad (1)$

and $x - y = 4. \quad (2)$

(1) \div (2), $x^2 + xy + y^2 = 79. \quad (3)$

Squaring (2), $x^2 - 2xy + y^2 = 16. \quad (4)$

From (3) and (4), $xy = 21. \quad (5)$

(3) + (5), $x^2 + 2xy + y^2 = 100. \quad (6)$

Whence $x + y = \pm 10. \quad (7), (8)$

From (2), (7), and (8), $x = 7,$

and $y = 3.$

Therefore the edges are 7 inches and 3 inches, respectively.

14. Let x and y equal the sides, respectively.

Then $x + y + \sqrt{x^2 + y^2} = 80, \quad (1)$

and $\frac{xy}{2} = 240. \quad (2)$

From (1), $x + y - 80 = -\sqrt{x^2 + y^2}. \quad (3)$

Squaring (3), $80x + 80y - xy = 3200. \quad (4)$

From (2), $xy = 480.$ (5)

(4) + (5), $x = 46 - y.$ (6)

Substituting $(46 - y)$ for x in (2),

$$46y - y^2 = 480. \quad (7)$$

Whence $y = 16.$ (8)

From (6) and (8), $x = 30.$

$$\sqrt{x^2 + y^2} = 34.$$

Therefore the sides are 16 feet and 30 feet ; the hypotenuse, 34 feet.

15. Let x and y equal the dimensions.

Then $xy = a^2,$ (1)

and $2x + 2y = 7a.$ (2)

From (1), $x = \frac{a^2}{y}.$ (3)

Substituting $\frac{a^2}{y}$ for x in (2),

$$\frac{2a^2}{y} + 2y = 7a. \quad (4)$$

Whence $y = \frac{7a \pm a\sqrt{33}}{4}.$ (5)

From (3) and (5), $x = \frac{7a \mp a\sqrt{33}}{4}.$

16. Let x equal the rate of the boat and y equal the rate of the first train.

Then $y + 10$ equals the rate of the second train.

Then $\frac{30}{x} + \frac{120}{y} = 6,$ (1)

and $\frac{120}{y + 10} + \frac{30}{x} = 5.$ (2)

(1) - (2), $\frac{120}{y} - \frac{120}{y + 10} = 1.$ (3)

Whence $y = 30.$ (4)

Then $y + 10 = 40.$

From (1) and (4), $x = 15.$

Therefore the rate of the boat is 15 miles per hour ; the rate of the first train, 30 miles per hour ; and the rate of the second train, 40 miles per hour.

17. Let x equal the number of reserved seats and y equal the price of each reserved seat.

$$\text{Then} \quad xy + (x + 1400)(y - 15) = 49,000, \quad (1)$$

$$\text{and} \quad xy = 25,000. \quad (2)$$

$$\text{From (1),} \quad 2xy - 15x + 1400y = 70,000. \quad (3)$$

$$(3) - (2) \cdot 2, \quad 280y - 3x = 4000. \quad (4)$$

$$\text{From (2),} \quad x = \frac{25000}{y}. \quad (5)$$

$$\text{Substituting } \frac{25000}{y} \text{ for } x \text{ in (4),} \quad 280y - 3 \cdot \frac{25000}{y} = 4000. \quad (6)$$

$$\text{Whence} \quad y = 25. \quad (7)$$

$$y - 15 = 10.$$

$$\text{From (2) and (7),} \quad x = 1000.$$

$$x + 1400 = 2400.$$

Therefore there were 1000 reserved seats at 25 cents each and 2400 unreserved seats at 15 cents each.

18. Let x equal the units' digit and y equal the tens' digit.

$$\text{Then} \quad (x + y)(10y + x) = 324, \quad (1)$$

$$\text{and} \quad 10y + x + 3y + 3x = 10x + y. \quad (2)$$

$$\text{From (1),} \quad x^2 + 11xy + 10y^2 = 324. \quad (3)$$

$$\text{From (2),} \quad 12y = 6x. \quad (4)$$

$$\text{From (3) and (4),} \quad x^2 + \frac{11x^2}{2} + \frac{5x^2}{2} = 324. \quad (5)$$

$$\text{Whence} \quad x = \pm 6. \quad (6)$$

$$\text{From (4) and (6),} \quad y = \pm 3.$$

Therefore the number is 36.

19. Let x and y equal the respective radii.

$$\text{Then} \quad x + y = 31, \quad (1)$$

$$\text{and} \quad \pi x^2 - \pi y^2 = 155\pi. \quad (2)$$

$$\text{From (2),} \quad x^2 - y^2 = 155. \quad (3)$$

$$\text{From (1),} \quad x = 31 - y. \quad (4)$$

$$\text{From (4) and (3),} \quad (31 - y)^2 - y^2 = 155. \quad (5)$$

$$\text{Whence} \quad y = 13. \quad (6)$$

$$\text{From (4) and (6),} \quad x = 18.$$

Therefore the radii of the two circles are 13 inches and 18 inches.

20. Let x equal the sum in dollars and y equal the rate per cent.

$$\text{Then} \quad \frac{y}{100}x = 42, \quad (1)$$

$$\text{and} \quad \frac{(x + 200)(y - 1)}{100} = 48. \quad (2)$$

$$\text{From (1),} \quad xy = 4200. \quad (3)$$

$$\text{From (2),} \quad xy + 200y - x - 200 = 4800. \quad (4)$$

$$\text{From (3) and (4),} \quad x = 200y - 800. \quad (5)$$

$$\text{From (5) and (3),} \quad 200y^2 - 800y = 4200. \quad (6)$$

$$\text{Whence} \quad y = 7. \quad (7)$$

$$\text{From (3) and (7),} \quad x = 600.$$

Therefore the principal is \$600, and the rate is 7%.

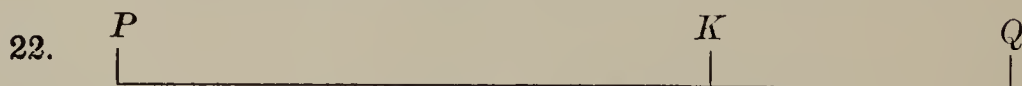
21. Let x equal the rate in miles of the wheelman going north and y equal the rate in miles of the wheelman going east.

$$\text{Then} \quad \left(\frac{4x}{3}\right)^2 + \left(3 + \frac{4y}{3}\right)^2 = 289, \quad (1)$$

$$\text{and} \quad \left(\frac{14x}{3}\right)^2 + \left(3 + \frac{14y}{3}\right)^2 = 2809. \quad (2)$$

$$\text{Solving (1) and (2),} \quad x = 6, y = 9.$$

Therefore the rate of the wheelman going north is 6 miles per hour, and the rate of the wheelman going east, 9 miles per hour.



First. A goes from P to K , and B goes from Q to K .

$$(A) \quad r = x \text{ (in miles per hour).}$$

$$t = y \text{ (in hours).}$$

$$d = PK = xy.$$

$$(B) \quad r = \frac{xy - 40}{y}.$$

$$t = y.$$

$$d = QK = xy - 40.$$

Second. A goes from K to Q , and B goes from K to P .

$$(A) \quad r = x.$$

$$t = \frac{xy - 40}{x} = \frac{d}{r}.$$

$$d = xy - 40.$$

$$(B) \quad r = \frac{xy - 40}{y}.$$

$$t = \frac{xy^2}{xy - 40} = \frac{d}{r}.$$

$$d = xy.$$

$$\text{Therefore} \quad \frac{xy - 40}{x} = 2, \quad (1)$$

$$\text{and} \quad \frac{xy^2}{xy - 40} = 8. \quad (2)$$

$$(1) \cdot (2), \quad y^2 = 16. \quad (3)$$

$$y = 4. \quad (\text{Root } -4 \text{ rejected.}) \quad (4)$$

$$\text{From (4) and (1),} \quad x = 20, \text{ A's rate.}$$

$$\text{Then} \quad \frac{xy - 40}{y} = 10, \text{ B's rate,}$$

$$\text{and} \quad PQ = 2xy - 40 = 120 \text{ miles.}$$



First. A goes from P to K , and B goes from Q to K .

$$(A) \ r = x \text{ (in miles per hour).}$$

$$t = y \text{ (in hours).}$$

$$d = xy.$$

$$(B) \ r = \frac{300 - xy}{y}.$$

$$t = y.$$

$$d = 300 - xy.$$

Second. A goes from K to Q , and B goes from K to P .

$$(A) \ r = x.$$

$$t = \frac{300 - xy}{x}.$$

$$d = 300 - xy.$$

$$(B) \ r = \frac{300 - xy}{y}.$$

$$t = \frac{xy^2}{300 - xy}.$$

$$d = xy.$$

Therefore

$$\frac{300 - xy}{x} = 6\frac{2}{3}, \quad (1)$$

and

$$\frac{xy^2}{300 - xy} = 15. \quad (2)$$

$$(1) \cdot (2),$$

$$y^2 = 100. \quad (3)$$

$$y = 10. \quad (\text{Root } -10 \text{ rejected.}) \quad (4)$$

From (4) and (1),

$$x = 18, \text{ A's rate.}$$

Therefore

$$\frac{300 - xy}{y} = 12, \text{ B's rate.}$$

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$$1. \quad x^2 + y^2 = 36, \quad (1)$$

$$(10 - x)^2 + y^2 = 64. \quad (2)$$

$$(2) - (1), \quad -20x + 100 = 28. \quad (3)$$

$$\text{Whence} \quad x = 3\frac{3}{5}. \quad (4)$$

$$\text{From (1) and (4),} \quad y = 4\frac{4}{5}.$$

$$2. \quad x^2 + y^2 = 64, \quad (1)$$

$$(17 - x)^2 + y^2 = 225. \quad (2)$$

$$(2) - (1), \quad 34x - 289 = -161. \quad (3)$$

$$\text{Whence} \quad x = 3\frac{1}{7}. \quad (4)$$

$$\text{From (1) and (4),} \quad y = 7\frac{1}{7}.$$

Therefore the altitude is $7\frac{1}{7}$, and the area is 60.

$$3. \quad x^2 + y^2 = 121, \quad (1)$$

$$(20 - x)^2 + y^2 = 169. \quad (2)$$

$$(2) - (1), \quad 400 - 40x = 48. \quad (3)$$

$$\text{Whence} \quad x = 8\frac{4}{5}. \quad (4)$$

$$\text{From (1) and (4),} \quad y = 6\frac{3}{5}.$$

Therefore the altitude is $6\frac{3}{5}$, and the area is 66.

$$4. \quad x^2 + y^2 = 169, \quad (1)$$

$$(14 - x)^2 + y^2 = 225. \quad (2)$$

$$(2) - (1), \quad 196 - 28x = 56. \quad (3)$$

$$\text{Whence} \quad x = 5. \quad (4)$$

$$\text{From (1) and (4),} \quad y = 12.$$

Therefore the altitude is 12, and the area is 84.

$$5. \quad x^2 + y^2 = 289, \quad (1)$$

$$(12 + x)^2 + y^2 = 625. \quad (2)$$

$$(2) - (1), \quad 144 + 24x = 336. \quad (3)$$

$$\text{Whence} \quad x = 8. \quad (4)$$

$$\text{From (1) and (4),} \quad y = 15.$$

Therefore the altitude is 15, and the area is 90.

$$6. \quad x^2 + y^2 = 400, \quad (1)$$

$$(16 - x)^2 + y^2 = 576. \quad (2)$$

$$(2) - (1), \quad 256 - 32x = 176. \quad (3)$$

$$\text{Whence} \quad x = \frac{5}{2}. \quad (4)$$

$$\text{From (1) and (4),} \quad y = 19.84.$$

Therefore the altitude is 19.84.

$$7. \quad x^2 + y^2 = 100, \quad (1)$$

$$(12 - x)^2 + y^2 = 100. \quad (2)$$

$$\text{Solving,} \quad x = 6, \text{ and } y = 8.$$

Therefore the altitude is 8.

$$8. \quad x^2 + y^2 = 100, \quad (1)$$

$$(21 - x)^2 + y^2 = 289. \quad (2)$$

$$\text{Solving,} \quad x = 6, \text{ and } y = 8.$$

Therefore the altitude is 8.

$$9. \quad x^2 + y^2 = 900, \quad (1)$$

$$(5 - x)^2 + y^2 = 841. \quad (2)$$

$$\text{Solving,} \quad x = 8.4, \text{ and } y = 28.8.$$

Therefore the altitude is 28.8, and the area is 504.

$$10. \quad x^2 + y^2 = 49, \quad (1)$$

$$(15 - x)^2 + y^2 = 400. \quad (2)$$

$$\text{Solving,} \quad x = -4.2.$$

(This means that x , or FE in the figure of Exercise 7 of the text, is laid off on FB to the right of F instead of to the left.)

$$\text{Then} \quad y = 5.6.$$

Therefore the altitude is 5.6, and the area is 92.4.

$$\begin{aligned} 11. \quad & x^2 + y^2 = 729, & (1) \\ & (19 - x)^2 + y^2 = 900. & (2) \end{aligned}$$

Solving, $y = 26.53 +$.

Therefore the altitude is 26.53, and the area is 809.2.

$$12. \quad x^2 + (100 - 23)^2 = 85^2.$$

Solving, $x = 36$.

Therefore the area is $\frac{1}{2} \cdot 36 \cdot 123$, or 2214.

13. Let x equal the third side, $x + 14$ the second, and y the projection of the third side on the first.

$$\text{Now} \quad \frac{16 \times \text{first side}}{2} = 144.$$

Whence the first side is 18.

$$\text{Then} \quad x^2 - y^2 = 256, \quad (1)$$

$$(14 + x)^2 = 256 + (18 - y)^2. \quad (2)$$

$$\text{From (1) and (2),} \quad 7x + 9y = 32. \quad (3)$$

$$\text{From (3) and (1), } x^2 + 14x - 680 = 0.$$

$$\text{Whence} \quad x = -34, 20. \quad (\text{Root } -34 \text{ rejected.})$$

$$x + 14 = 34.$$

Therefore the three sides are 18, 34, and 20.

$$\begin{aligned} 14. \text{ From the Hints,} \quad & \frac{x}{80 - y} = \frac{y}{102 - x}, \\ & x^2 + y^2 = 100. \end{aligned}$$

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$$2. \quad l = 1 + 7 \cdot 3 = 22.$$

$$8. \quad l = \sqrt{3} + 5 \cdot 3\sqrt{3} = 16\sqrt{3}.$$

$$3. \quad l = 15 + 10 \cdot (-6) = -45. \quad 9. \quad l = 3\sqrt{5} + 7(-2\sqrt{5}) = -11\sqrt{5}.$$

$$4. \quad l = a + 20 \cdot 3a = 61a. \quad 10. \quad l = 5 + a + 7(2 + 2a) = 19 + 15a.$$

$$5. \quad l = 9x + 15(-8)x = -111x. \quad 11. \quad l_1 = 0 + 11(-5x + 2) = -55x + 22.$$

$$6. \quad l = \frac{10}{3} + 5(-\frac{4}{3}) = -\frac{10}{3}. \quad l_2 = 0 + 19(-5x + 2) = -95x + 38.$$

$$7. \quad l = -\frac{1}{7} + 10 \cdot \frac{4}{7} = \frac{39}{7}. \quad 12. \quad l = \sqrt{a} + 3 + 9(4\sqrt{a} + 4) = 37\sqrt{a} + 39.$$

$$13. \quad l_1 = 9\sqrt{a} - 3 + 9(-4\sqrt{a} + 3) = -27\sqrt{a} + 24.$$

$$l_2 = 9\sqrt{a} - 3 + 19(-4\sqrt{a} + 3) = -67\sqrt{a} + 54.$$

$$14. \quad l = \frac{\sqrt{x}}{2} + 13 \cdot 2\sqrt{x} = 26\frac{1}{2}\sqrt{x}.$$

$$15. \quad l_1 = \sqrt{3} + 8\sqrt{3} = 9\sqrt{3}.$$

$$l_2 = \sqrt{3} + 11\sqrt{3} = 12\sqrt{3}.$$

$$16. \quad l_1 = \frac{\sqrt{x}}{2} + 1 + 7(\sqrt{x} + 1) = 7\frac{1}{2}\sqrt{x} + 8.$$

$$l_2 = \frac{\sqrt{x}}{2} + 1 + 15(\sqrt{x} + 1) = 15\frac{1}{2}\sqrt{x} + 16.$$

$$17. \quad l = \frac{2\sqrt{a}}{3} - 12 + 14\left(7 - \frac{2\sqrt{a}}{3}\right) = 86 - \frac{26\sqrt{a}}{3}.$$

18. $l = a + 28d.$

20. $l = a + (m - 2)d.$

19. $l = a + (m - 1)d.$

21. $l = a + (n - 3)5 = a + 5n - 15.$

22. $l = \frac{3}{\sqrt{3}} + (n - 6) \cdot 2\sqrt{3} = \sqrt{3} + 2\sqrt{3}n - 12\sqrt{3} = 2\sqrt{3}n - 11\sqrt{3}.$

23. $l = \sqrt{5} - 1 + (n - 4)(\sqrt{5} - 1)$
 $= \sqrt{5} - 1 + (n - 4)\sqrt{5} - n + 4$
 $= (n - 3)\sqrt{5} - n + 3.$

24. $l = \frac{1}{a} + (n - 1)\frac{a - 2}{a} = \frac{1}{a} + \frac{na - 2n - a + 2}{a} = \frac{(n - 1)a + 3 - 2n}{a}$

25. $d = \frac{22 - 2}{2} = 10.$

Seventh term $= 2 + 6 \cdot 10 = 62.$

n th term $= 2 + (n - 1)10 = 10n - 8.$

26. $d = s - r.$

Third term $= r + 2(s - r) = 2s - r.$

n th term $= r + (n - 1)(s - r) = 2r + n(s - r) - s.$

27. The edges of the box are n , $n + 2$, and $n + 4$, respectively.

Sum of the edges $= 4n + 4(n + 2) + 4(n + 4) = 12n + 24.$

Area of the faces $= 2n(n + 2) + 2n(n + 4) + 2(n + 2)(n + 4)$
 $= 2n^2 + 4n + 2n^2 + 8n + 2n^2 + 12n + 16$
 $= 6n^2 + 24n + 16.$

Volume $= n(n + 2)(n + 4) = n^3 + 6n^2 + 8n.$

28. $a = 16.$

$d = 32.$

$l_1 = 16 + 11 \cdot 32 = 368.$

$l_2 = 16 + (n - 1)32 = 32n - 16.$

29. Let the units' digit $= x - d$,
and the tens' digit $= x$,
and the hundreds' digit $= x + d$.
 $x - d + x + x + d = 24.$

$x = 8. \quad (1)$

$(x - d)^2 + x^2 + (x + d)^2 = 194.$

$x^2 - 2dx + d^2 + x^2 + x^2 + 2dx + d^2 = 194.$

$3x^2 + 2d^2 = 194. \quad (2)$

From (1) and (2),

$192 + 2d^2 = 194.$

$d^2 = 1.$

$d = \pm 1.$

$x - d = 7 \text{ or } 9.$

$x + d = 9 \text{ or } 7.$

Therefore the number is 789 or 987.

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1. Mean = $\frac{3 + 15}{2} = 9$.
Progression = 3, 9, 15.
2. Mean = $\frac{3 + 27}{2} = 15$.
Progression = 3, 15, 27.
3. Mean = $\frac{h + 5h}{2} = 3h$.
Progression = $h, 3h, 5h$.
4. Mean = $\frac{m + n}{2}$.
Progression = $m, \frac{m + n}{2}, n$.
5. $8 = 2 + (4 - 1)d$.
 $3d = 6$.
 $d = 2$.
Progression = 2, 4, 6, 8.
6. $29 = 5 + (4 - 1)d$.
 $3d = 24$.
 $d = 8$.
Progression = 5, 13, 21, 29.
7. $217 = 49 + (4 - 1)d$.
 $3d = 168$.
 $d = 56$.
Progression = 49, 105, 161, 217.
8. $x + 6y = x + (4 - 1)d$.
 $3d = 6y$.
 $d = 2y$.
Progression = $x, x + 2y, x + 4y, x + 6y$.
9. $3y = x + (4 - 1)d$.
 $3d = 3y - x$.
 $d = \frac{3y - x}{3}$.
Progression = $x, \frac{3y + 2x}{3}, \frac{6y + x}{3}, 3y$.
10. $k = h + (4 - 1)d$.
 $3d = k - h$.
 $d = \frac{k - h}{3}$.
Progression = $h, \frac{2h + k}{3}, \frac{h + 2k}{3}, k$.
11. $34 = 10 + (5 - 1)d$.
 $d = 6$.
Progression = 10, 16, 22, 28, 34.
12. $31 = -5 + 4d$.
 $d = 9$.
Progression = $-5, 4, 13, 22, 31$.
13. $44x = -12x + 4d$.
 $d = 14x$.
Progression = $-12x, 2x, 16x, 30x, 44x$.
14. $k = h + 4d$.
 $d = \frac{k - h}{4}$.
Progression = $h, \frac{3h + k}{4}, \frac{h + k}{2}, \frac{h + 3k}{4}, k$.
15. $131 = -13 + 8d$.
 $d = 18$.
Progression = $-13, 5, 23, 41, 59, 77, 95, 113, 131$.
16. $\frac{46}{3} = -\frac{8}{3} + 6d$.
 $d = 3$.
Progression = $-\frac{8}{3}, \frac{1}{3}, \frac{10}{3}, \frac{19}{3}, \frac{28}{3}, \frac{37}{3}, \frac{46}{3}$.
17. $18\sqrt{5} = -2\sqrt{5} + 5d$.
 $d = 4\sqrt{5}$.
Progression = $-2\sqrt{5}, 2\sqrt{5}, 6\sqrt{5}, 10\sqrt{5}, 14\sqrt{5}, 18\sqrt{5}$.

$$18. \quad 13a + 9b = 7a - 3b + 6d.$$

$$d = a + 2b.$$

Progression = $7a - 3b, 8a - b, 9a + b, 10a + 3b, 11a + 5b, 12a + 7b, 13a + 9b$.

$$19. \quad \frac{27}{2\sqrt{3}} = \sqrt{3} + 7d.$$

$$d = \frac{\sqrt{3}}{2}.$$

$$\text{Progression} = \sqrt{3}, \frac{3\sqrt{3}}{2}, 2\sqrt{3}, \frac{5\sqrt{3}}{2}, 3\sqrt{3}, \frac{7\sqrt{3}}{2}, 4\sqrt{3}, \frac{27}{2\sqrt{3}}.$$

$$20. \quad 4\sqrt{3} - 6 = \frac{3}{1 - \sqrt{3}} + 3d.$$

$$d = \frac{11\sqrt{3} - 9}{6}.$$

$$\text{Progression} = \frac{3}{1 - \sqrt{3}}, \frac{\sqrt{3} - 9}{3}, \frac{13\sqrt{3} - 27}{6}, 4\sqrt{3} - 6.$$

21. The average velocity equals the arithmetical mean.

$$m = \frac{25 + 15}{2} = 20 \text{ miles per hour.}$$

Since $\frac{\text{Distance}}{\text{Rate}} = \text{Time}, \frac{1}{20} = \frac{1}{20} \text{ hour, or } 3 \text{ minutes.}$

$$22. \quad \text{Mean velocity} = \frac{64 + 96}{2} = 80 \text{ feet per second.}$$

It will fall 80 feet during the third second.

$$23. \quad \text{Mean velocity} = \frac{0 + 32}{2} = 16 \text{ feet per second.}$$

The body falls 16 feet during the first second, and during the second second it falls $\frac{32 + 64}{2}$, or 48 feet.

$$24. \quad l = 1 + 19 \cdot 2 = 39.$$

$$\frac{1 + 39}{2} = 20 \text{ inches average.}$$

$$25. \quad l = 58 + 14 \cdot 2 = 86.$$

$$\text{Mean} = \frac{58 + 86}{2} = 72 \text{ inches.}$$

$$26. \quad l = 32 + 11 \cdot 32 = 384 \text{ feet.}$$

$$\text{Mean} = \frac{0 + 384}{2} = 192 \text{ feet per second.}$$

$$27. \quad 37 = 1 + 9d.$$

$$d = 4.$$

Progression = 1, 5, 9, 13, 17, 21, 25, 29, 33, 37 inches.

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1. $S = \frac{8}{2}(2 \cdot 2 + 7 \cdot 4) = 128.$
2. $S = \frac{1 \cdot 0}{2}(2 \cdot 8 + 9 \cdot 5) = 305.$
3. $S = \frac{9}{2}(2 + 8 \cdot 11) = 405.$
4. $S = \frac{1 \cdot 5}{2}(34 + 14 \cdot 3) = 570.$
5. $S = \frac{2 \cdot 0}{2}[200 + 19(-5)] = 1050.$
6. $S = \frac{8}{2}(4 + 7 \cdot 2) = 72.$
7. $S = \frac{1 \cdot 0}{2}(2 + 9 \cdot 8) = 370.$
8. $S = \frac{1 \cdot 0}{2}(-16 + 9 \cdot 4) = 100.$
9. $S = \frac{1 \cdot 8}{2}(2 + 17 \cdot 4) = 630.$
10. $S = \frac{2 \cdot 0}{2}[20 + 19(-2)] = -180.$
11. $S = \frac{1 \cdot 2}{2}(2 + 11 \cdot \frac{1}{2}) = 45.$
12. $S = \frac{1 \cdot 2}{2}[30 + 11 \cdot (-\frac{5}{2})] = 15.$
13. $S = \frac{1 \cdot 0 \cdot 0}{2}(2 + 99 \cdot 1) = 5050.$
14. $S = \frac{1 \cdot 0 \cdot 0}{2}(4 + 99 \cdot 2) = 10,100.$
15. $S = \frac{1 \cdot 0 \cdot 0}{2}(2 + 99 \cdot 2) = 10,000.$
16. $S = \frac{n}{2}[4 + (n-1)2] = n(n+1).$
17. $S = \frac{n}{2}[2 + (n-1)2] = n^2.$

$$18. \quad 538 = 248 + (n-1)2.$$

Whence

$$n = 146.$$

$$S = \frac{1 \cdot 4 \cdot 6}{2}(248 + 538) = 57,378.$$

$$19. \quad 861 = \frac{n}{2}[2 + (n-1)1].$$

$$n^2 + n = 1722.$$

$$n + \frac{1}{2} = \pm \frac{8 \cdot 3}{2}.$$

$$n = 41 \text{ or } -42. \quad (\text{Root } -42 \text{ rejected.})$$

$$20. \quad 242 = \frac{n}{2}[14 + (n-1)3].$$

$$3n^2 + 11n - 484 = 0.$$

$$(3n + 44)(n - 11) = 0.$$

$$n = 11 \text{ or } -\frac{44}{3}. \quad (\text{Root } -\frac{44}{3} \text{ rejected.})$$

$$21. \quad 372 = \frac{n}{2}[180 + (n-1)2].$$

$$2n^2 + 178n = 744.$$

$$n^2 + 89n - 372 = 0.$$

$$(n + 93)(n - 4) = 0.$$

$$n = 4 \text{ or } -93. \quad (\text{Root } -93 \text{ rejected.})$$

The terms are 90, 92, 94, 96.

$$22. \quad 22 = -2 + 6d.$$

$$d = 4.$$

$$l = -6 + 10 \cdot 4 = 34.$$

$$\begin{aligned}
 23. \quad S &= \frac{t}{2} \left(\frac{2}{a} + (t-1) \frac{a-2}{a} \right) \\
 &= \frac{t}{2} \left(\frac{2 + at - a - 2t + 2}{a} \right) \\
 &= \frac{t(at - a - 2t + 4)}{2a}.
 \end{aligned}$$

24. Given $l = 25$, $a = 1$, and $d = 4$; to find n and s .

$$\begin{aligned} 25 &= 1 + (n - 1)4. \\ n &= 7. \end{aligned}$$

$$\begin{aligned} s &= \frac{n}{2}(1 + 25). \\ s &= 91. \end{aligned}$$

25. Given $a = -20$, $d = 11$, and $s = 216$; to find n and l .

$$216 = \frac{n}{2}(-20 + l), \text{ or } l = \frac{432 + 20n}{n}. \quad (1)$$

$$l = -20 + (n - 1)11, \text{ or } l = 11n - 31. \quad (2)$$

From (1) and (2),

$$\frac{432 + 20n}{n} = 11n - 31.$$

$$11n^2 - 51n - 432 = 0.$$

$$(11n + 48)(n - 9) = 0.$$

$$n = 9 \text{ or } -\frac{48}{11}. \quad (\text{Root } -\frac{48}{11} \text{ rejected.})$$

Then from (2),

$$l = 68.$$

26. Given $d = -9$, $n = 15$, $s = 0$; to find a and l .

$$0 = \frac{1}{2}n(a + l), \text{ or } a = -l. \quad (1)$$

$$l = a + 14 \cdot (-9) \text{ or } a = l + 126. \quad (2)$$

From (1) and (2),

$$-l = l + 126.$$

$$l = -63.$$

From (1),

$$a = 63.$$

27. Given $a = h$, $n = n$, and $d = k - h$; to find S_n .

$$S_n = \frac{n}{2}[2h + (n - 1)(k - h)]$$

$$= \frac{3nh}{2} + \frac{kn^2}{2} - \frac{kn}{2} - \frac{hn^2}{2}.$$

28. Given $s = 9h$, $a = 12 - 10h$, and $n = 9$; to find l and d .

$$9h = \frac{9}{2}(12 - 10h + l). \quad (1)$$

From (1),

$$l = 12h - 12. \quad (2)$$

Again,

$$l = 12 - 10h + 8d. \quad (3)$$

Then from (2) and (3),

$$12 - 10h + 8d = 12h - 12. \quad (4)$$

$$8d = 22h - 24.$$

$$d = \frac{11h - 12}{4}.$$

From (2),

$$l = 12h - 12.$$

29. Given $s = 66\sqrt{3}$, $a = -4\sqrt{3}$, and $d = 2\sqrt{3}$; to find n and l .

$$66\sqrt{3} = \frac{n}{2}(-4\sqrt{3} + l) \text{ or } l = \frac{132\sqrt{3} + 4\sqrt{3}n}{n}.$$

$$l = -4\sqrt{3} + (n-1)2\sqrt{3}.$$

Then
$$\frac{132\sqrt{3} + 4\sqrt{3}n}{n} = -4\sqrt{3} + (n-1)2\sqrt{3}.$$

$$n^2 - 5n - 66 = 0.$$

$$(n-11)(n+6) = 0.$$

$$n = 11 \text{ and } -6. \quad (\text{Root } -6 \text{ rejected.})$$

$$l = -4\sqrt{3} + 10 \cdot 2\sqrt{3} = 16\sqrt{3}.$$

30.
$$S = \frac{1}{2}(1+12)$$

$$= 78 \text{ in a 12-hour day.}$$

$$2 \times 78 = 156 \text{ in a 24-hour day.}$$

31. (a) 15 miles per hour equals 22 feet per second.

$$l = a + (n-1)d.$$

$$\frac{1}{2} = 22 + (n-1)(-\frac{1}{2}).$$

$$\frac{1}{2}n = 22, n = 44.$$

(b)
$$S = \frac{1}{2}(22 + \frac{1}{2})$$

$$= 495 \text{ feet.}$$

32. (a) Given $a = 3$, $d = 8$, and $n = 30$.

$$l = 3 + 29 \cdot 8 = 235 \text{ inches passed over during the thirtieth second.}$$

The rate is, however, $235 + 4$ inches per second, or $19\frac{1}{2}$ feet per second at this time, for to gain 8 inches during the last second the car must be going 4 inches faster at the close of the second than its average rate during that second.

(b)
$$S = \frac{3}{2}(3 + 235)$$

$$= 3570 \text{ inches, or } 297\frac{1}{2} \text{ feet.}$$

33. Given $a = 24$, $d = -\frac{1}{2}$, and $l = \frac{1}{2}$.

$$\frac{1}{2} = 24 + (n-1)(-\frac{1}{2}).$$

$$1 = 48 + 1 - n.$$

$$n = 48.$$

$$S = \frac{1}{2}(24 + \frac{1}{2})$$

$$= 24 \cdot 24\frac{1}{2} = 588 \text{ inches fallen.}$$

In rebounding the ball covers 564 inches, making a total distance passed through of 1152 inches, or 96 feet.

34. Let the digits be 3, $3 + d$, and $3 + 2d$.

$$100 \cdot 3 + 10(3 + d) + 3 + 2d = 20\frac{1}{2}(9 + 3d).$$

$$666 + 24d = 369 + 123d.$$

$$99d = 297.$$

$$d = 3.$$

$$3 + d = 6.$$

$$3 + 2d = 9.$$

The number is 369.

35. Given $a = 60 \times 12$, $d = 75$, and $n = 10$.

$$l = a + (n - 1)d.$$

$$l = 720 + 9 \cdot 75 = 1395, \text{ or } \$1395.$$

$$S = \frac{1}{2}n(720 + 1395)$$

$$= 10,575, \text{ or } \$10,575.$$

36. Given $a = 200$, $d = .06$ of 200, and $n = 6$.

$$S = \frac{6}{2}[400 + 5(\frac{6}{100} \times 200)]$$

$$= 1380, \text{ or } \$1380.$$

37. Given $S = 555$ feet, $a = 16$ feet, and $d = 32$ feet.

$$S = \frac{n}{2}[2a + (n - 1)d].$$

$$555 = \frac{n}{2}[32 + (n - 1)32].$$

$$n^2 = \frac{555}{16}.$$

$$n = \pm 5.8896. \quad (\text{Root} - 5.8896 \text{ rejected.})$$

$$l = 16 + 4.8896 \cdot 32 = 172.45 +.$$

Velocity at end of time = distance passed during last second + 16 feet
= 188.45 feet per second.

38. Given $S = 256$, $a = 16$, and $d = 32$.

$$256 = \frac{n}{2}[32 + (n - 1)32].$$

$$32n^2 = 512.$$

$$n = \pm 4. \quad (\text{Root} - 4 \text{ rejected.})$$

$$l = 16 + 3(32) = 112.$$

From Exercise 37 the final velocity will be 128 feet per second.

39. In $S = \frac{n}{2}[2a + (n - 1)d]$, here $n = t$, $d = 32 = g$, and $a = 16 = \frac{g}{2}$.

Substituting these values in the above, we have

$$S = \frac{t}{2}[g + (t - 1)g] = \frac{t}{2}(g + tg - g) = \frac{t}{2}(tg) = \frac{1}{2}gt^2.$$

40. Given $n = \frac{6}{2} = 3$, $a = 16$, and $d = 32$.

$$l = 16 + 2 \cdot 32 = 80.$$

$$S = \frac{3}{2}(16 + 80) = 144.$$

Velocity $= 80 + 16 = 96$ feet per second.

41. A's rate $= 20$ miles per hour.

B's rate $= 30, 26, 22, 18$, etc., miles per hour.

Let $x =$ number of hours which must elapse.

Then $20x =$ number of miles traveled by A,

and $\frac{x}{2}[60 + (x - 1)(-4)] =$ number of miles traveled by B.

Therefore $20x = \frac{x}{2}[60 + (x - 1)(-4)].$

$$20x = 30x - 2x^2 + 2x.$$

$$2x^2 - 12x = 0.$$

$$x = 0 \text{ and } 6.$$

They are together at the start and again 6 hours later.

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2. Given $a = 1$, $r = 2$, and $n - 1 = 7$.

Then $t_8 = 1 \cdot 2^7 = 128.$

3. Given $a = 3$, $r = 2$, and $n - 1 = 9$.

Then $t_{10} = 3 \cdot 2^9 = 1536.$

4. Given $a = 3$, $r = \frac{2}{3}$, and $n - 1 = 7$.

Then $t_8 = 3 \cdot \left(\frac{2}{3}\right)^7 = \frac{1}{7} \frac{2^8}{3}.$

5. Given $a = 7$, $r = -2$, and $n - 1 = 11$.

Then $t_{12} = 7 \cdot (-2)^{11} = -14,336.$

6. Here $t_9 = 15 \cdot \left(-\frac{1}{3}\right)^8 = \frac{5}{21} \frac{1}{87}.$

7. Here $t_{12} = 12a^2 \cdot \left(\frac{3}{4}\right)^{11} = \frac{3^{12}}{4^{10}}a^2.$

8. Here $t_6 = -\frac{2c^3}{5} \cdot \left(\frac{5}{2c^3}\right)^5 = -\frac{625}{16c^{12}}.$

9. Here $t_8 = 4\sqrt{2} \cdot \left(\frac{\sqrt{2}}{2}\right)^7 = \frac{64}{128} = \frac{1}{2}.$

10. Here $t_7 = \frac{1}{2\sqrt{2}} \cdot (\sqrt{2})^6 = 2\sqrt{2}.$

11. Here $t_{20} = \$4.12(1.01)(1.01)^{19}$
 $= \$4.12(1.01)^{20}.$

12. Given $(n - 1)$ th term $= ar^{n-2}$.

$$(n - 2)\text{th term} = ar^{n-3}.$$

$$(n - 3)\text{th term} = ar^{n-4}.$$

$$(n + 1)\text{th term} = ar^n.$$

$$(n + 2)\text{th term} = ar^{n+1}.$$

13. Given $a = m$ and $r = \frac{n}{m}$.

$$\text{Progression} = m, n, \frac{n^2}{m}, \frac{n^3}{m^2}, \dots$$

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1. Given $a = 21$, $n = 4$, and $t_4 = 168$.

$$\text{Then } 168 = 21 \cdot r^3.$$

$$r^3 = 8.$$

$$r = 2, -1 \pm \sqrt{-3}.$$

The imaginary roots are rejected.

$$\text{Progression} = 21, 42, 84, 168.$$

2. Given $a = 15$, $n = 4$, and $t_4 = 405$.

$$\text{Then } 405 = 15 \cdot r^3.$$

$$r^3 = 27.$$

$$r = 3, \frac{-3 \pm \sqrt{-3}}{2}.$$

The imaginary roots are rejected.

$$\text{Progression} = 15, 45, 135, 405.$$

3. Given $a = 3$, $n = 5$, and $t_5 = 243$.

$$\text{Then } 243 = 3 \cdot r^4.$$

$$r^4 = 81.$$

$$r = \pm 3, \pm \sqrt{-9}.$$

The imaginary roots are rejected.

$$\text{Progression} = 3, \pm 9, 27, \pm 81, 243.$$

4. Given $a = 9$, $n = 3$, and $t_3 = 81$.

$$\text{Then } 81 = 9 \cdot r^2.$$

$$r^2 = 9.$$

$$r = \pm 3.$$

$$\text{Progression} = 9, \pm 27, 81.$$

5. Given $a = a^{12}$, $n = 3$, and $t_3 = a^{20}$.

$$\text{Then } a^{20} = a^{12}r^2.$$

$$r^2 = a^8.$$

$$r = \pm a^4.$$

$$\text{Progression} = a^{12}, \pm a^{16}, a^{20}.$$

6. Given $a = -9$, $n = 5$, and $t_5 = -144$.

$$\text{Then } -144 = (-9)r^4.$$

$$r^4 = \frac{144}{9}.$$

$$r = \pm 2, \pm 2\sqrt{-1}.$$

The imaginary roots are rejected.

$$\text{Progression} = -9, \mp 18, -36, \mp 72, -144.$$

7. Given $t_4 = 16$, $n = 5$, and $t_8 = 256$.

$$\text{Then } 256 = 16r^4.$$

$$r^4 = 16.$$

$$r = \pm 2, \pm 2\sqrt{-1}.$$

The imaginary roots are rejected.

$$\begin{aligned} \text{Tenth term} &= 256 \cdot (\pm 2)^2 \\ &= 1024. \end{aligned}$$

$$8. \quad t_2 = ar = 4\sqrt{3}. \quad (1)$$

$$t_5 = ar^4 = \frac{4}{9}. \quad (2)$$

$$(2) \div (1), \quad r^3 = \frac{\sqrt{3}}{27}.$$

$$r = \frac{\sqrt[6]{3}}{3} \text{ and two imaginary roots, which are rejected.}$$

First term $= 4\sqrt{3} \div \frac{\sqrt[6]{3}}{3} = 12\sqrt[3]{3}.$

$$\text{First term} = 4\sqrt{3} \div \frac{\sqrt[6]{3}}{3} = 12\sqrt[3]{3}.$$

9. Here $a = a$, $n = 3$, and $t_3 = c$.

$$l = c = ar^2.$$

$$r = \pm \sqrt{\frac{c}{a}} = \pm \frac{1}{a} \sqrt{ac}.$$

Therefore the means are

$$a \cdot \frac{\pm 1}{a} \sqrt{ac} = \pm \sqrt{ac}.$$

10. Given $a = a$, $n = 4$, and $t_4 = c$.

$$\text{Then } t_1 = a = a, \quad (1)$$

$$\text{and } t_4 = ar^3 = c. \quad (2)$$

$$(2) \div (1),$$

$$r^3 = \frac{c}{a}.$$

$$r = \sqrt[3]{\frac{c}{a}} = \frac{1}{a} \sqrt[3]{a^2 c}$$

and two imaginary roots, which are rejected.

$$\text{Progression} = a, \sqrt[3]{a^2 c}, \sqrt[3]{ac^2}, c.$$

11. Given $a = h$, $n = 5$, and $t_5 = k$.

$$k = hr^4.$$

$$r^4 = \frac{k}{h}.$$

$$r = \pm \sqrt[4]{\frac{k}{h}} = \pm \frac{1}{h} \sqrt[4]{h^3 k}$$

and two imaginary roots, which are rejected.

$$\text{Progression} = h, \pm \sqrt[4]{h^3 k}, \sqrt{hk}, \pm \sqrt[4]{h k^3}, k.$$

12. Let a = the first term,
and ar^3 = the fourth term.

$$a + ar^3 = 56. \quad (1)$$

$$ar = 6. \quad (2)$$

Substituting from (2) in (1),

$$\frac{6}{r} + 6r^2 = 56.$$

$$3r^3 - 28r + 3 = 0.$$

By factor theorem,

$$(r - 3)(3r^2 + 9r - 1) = 0.$$

$$r = 3, \frac{-9 \pm \sqrt{93}}{6}.$$

$$\text{Then } a = 2, 27 \pm 3\sqrt{93}.$$

The first progression = 2, 6, 18, 54,

The second and third progressions = $27 \pm 3\sqrt{93}$, 6, $-9 \pm \sqrt{93}$, $29 \mp 3\sqrt{93}$,

$$13. (a) AD = \sqrt{4 \cdot 9} = 6.$$

$$(b) DC = BC - BD = 18.$$

$$AD = \sqrt{3 \cdot 18} = 3\sqrt{6}.$$

$$(c) \text{ Let } BD = x,$$

$$\text{and } DC = 25 - x.$$

$$10 = \sqrt{x(25 - x)}.$$

$$100 = 25x - x^2.$$

$$x^2 - 25x + 100 = 0.$$

$$x = 20 \text{ or } 5.$$

$$\text{Then } 25 - x = 5 \text{ or } 20.$$

$$14. (a) AB = \sqrt{36 \cdot 4} = 12.$$

$$(b) AC = AD - DC.$$

$$\text{Let } x = AD;$$

$$\text{then } x - 90 = AC.$$

$$24 = \sqrt{x(x - 90)}.$$

$$x^2 - 90x - 576 = 0.$$

$$(x - 96)(x + 6) = 0.$$

$$x = 96 \text{ or } -6.$$

(Root -6 rejected.)

$$x - 90 = 6.$$

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2. Given $a = 1$, $r = 5$, and $n = 9$.

$$S_9 = \frac{1 - 1 \cdot 5^9}{1 - 5} = 488,281.$$

3. Given $a = 2$, $r = -2$, and $n = 8$.

$$S_8 = \frac{2 - 2(-2)^8}{1 - (-2)} = \frac{-510}{3} = -170.$$

4. Given $a = 40$, $r = \frac{1}{2}$, and $n = 7$.

$$S_7 = \frac{40 - 40 \cdot (\frac{1}{2})^7}{1 - \frac{1}{2}} = \frac{39\frac{1}{8}}{\frac{1}{2}} = 79\frac{3}{8}.$$

5. Given $a = -180$, $r = -\frac{1}{2}$, and $n = 8$.

$$S_8 = \frac{-180 - (-180)(-\frac{1}{2})^8}{1 - (-\frac{1}{2})} = \frac{-180 + \frac{45}{64}}{\frac{3}{2}} = \frac{-179\frac{9}{64}}{\frac{3}{2}} = -119\frac{1}{2}.$$

6. Given $a = \frac{2}{3}$, $r = \frac{3}{2}$, and $n = 6$.

$$S_6 = \frac{\frac{2}{3} - \frac{2}{3}(\frac{3}{2})^6}{1 - \frac{3}{2}} = \frac{-\frac{665}{96}}{-\frac{1}{2}} = \frac{665}{48}.$$

7. Given $a = a^3$, $r = a^2$, and $n = 10$.

$$S_{10} = \frac{a^3 - a^3 \cdot (a^2)^{10}}{1 - a^2} = \frac{a^3 - a^{23}}{1 - a^2}.$$

8. Given $a = 2\sqrt{3}$, $r = \sqrt{3}$, and $n = 7$.

$$S_7 = \frac{2\sqrt{3} - 2\sqrt{3}(\sqrt{3})^7}{1 - \sqrt{3}} = \frac{2\sqrt{3} - 162}{1 - \sqrt{3}} = 78 + 80\sqrt{3}.$$

9. Given $a = 4$, $r = 3$, and $n = n$.

$$S_n = \frac{4 - 4 \cdot 3^n}{1 - 3} = \frac{4(1 - 3^n)}{-2} = -2(1 - 3^n).$$

10. Given $a = 125$, $r = -\frac{\sqrt{5}}{5}$, and $n = n$.

$$S_n = \frac{125 \left[1 - \left(-\frac{\sqrt{5}}{5} \right)^n \right]}{1 + \frac{\sqrt{5}}{5}}.$$

11. Given $a = 3$, $r = 4$, and $n = n - 1$.

$$S_{n-1} = \frac{3 - 3 \cdot 4^{n-1}}{1 - 4} = \frac{3(1 - 4^{n-1})}{-3} = 4^{n-1} - 1.$$

12. Given $a = 3$, $r = -5$, and $n = n$.

$$S_n = \frac{3[1 - (-5)^n]}{1 + 5} = \frac{1 - (-5)^n}{2}.$$

13. Given $a = x$, $r = 4x^3$, and $n = n - 2$.

$$S_{n-2} = \frac{x[1 - (4x^3)^{n-2}]}{1 - 4x^3}.$$

14. Given $l = ar^{n-1}$ and $rl = ar^n$.

$$S_n = \frac{a - ar^n}{1 - r} = \frac{a - rl}{1 - r}.$$

15. Given $a = 60$, $r = \frac{1}{2}$, and $n = 6$.

$$t_6 = 60 \cdot \left(\frac{1}{2}\right)^5 \\ = 1\frac{5}{8} = 1\frac{7}{8}.$$

$$S_6 = \frac{60 - \frac{1}{2}\left(1\frac{5}{8}\right)}{1 - \frac{1}{2}} = \frac{59\frac{1}{8}}{\frac{1}{2}} = 118\frac{1}{8} \text{ inches.}$$

It rebounds five times in all, or $58\frac{1}{8}$ inches. Total distance covered is $176\frac{1}{4}$ inches.

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2. $S_\infty = \frac{a}{1-r}.$

$$S_\infty = \frac{1}{1 - \frac{1}{2}} = 2.$$

3. $S_\infty = \frac{3}{1 - (-\frac{1}{3})} = \frac{3}{\frac{4}{3}} = 2\frac{1}{4}.$

4. $S_\infty = \frac{5}{1 - (-\frac{2}{5})} = \frac{5}{\frac{7}{5}} = 3\frac{4}{7}.$

5. $S_\infty = \frac{2}{1 - \frac{\sqrt{2}}{2}} = \frac{2 \cdot 2}{2 - \sqrt{2}} = \frac{8 + 4\sqrt{2}}{2} = 4 + 2\sqrt{2}.$

6. $S_\infty = \frac{5a}{1 - \frac{1}{2}} = 10a.$

7. $S_\infty = \frac{1}{1-x}.$

8. $S_\infty = \frac{5}{1 - \frac{\sqrt{5}}{5}} = \frac{25}{5 - \sqrt{5}} = \frac{25(5 + \sqrt{5})}{20} = \frac{5(5 + \sqrt{5})}{4}.$

9. $S_\infty = \frac{1}{1 - \frac{1}{x}} = \frac{x}{x-1}.$

10. $.121212 \dots = \frac{12}{100} + \frac{12}{10000} + \frac{12}{1000000} \dots$

$$S_\infty = \frac{\frac{12}{100}}{1 - \frac{1}{100}} = \frac{4}{33}.$$

11. $.666 \dots = \frac{6}{10} + \frac{6}{100} + \frac{6}{1000} \dots$

$$S_\infty = \frac{\frac{6}{10}}{1 - \frac{1}{10}} = \frac{2}{3}.$$

12. $.151515 \dots = \frac{15}{100} + \frac{15}{10000} + \frac{15}{1000000} \dots$

$$S_\infty = \frac{\frac{15}{100}}{1 - \frac{1}{100}} = \frac{5}{33}.$$

$$13. \quad .3232 \dots = \frac{32}{100} + \frac{32}{10000} + \frac{32}{1000000} \dots$$

$$S_{\infty} = \frac{\frac{32}{100}}{1 - \frac{1}{100}} = \frac{32}{99}.$$

$$14. \quad 25.2727 \dots = 25 + \frac{27}{100} + \frac{27}{10000} + \frac{27}{1000000} \dots$$

$$S_{\infty} = \frac{\frac{27}{100}}{1 - \frac{1}{100}} = \frac{3}{11}.$$

$$25 + \frac{3}{11} = 25\frac{3}{11}.$$

$$15. \quad .71515 \dots = \frac{7}{10} + \frac{15}{1000} + \frac{15}{100000} + \frac{15}{10000000} \dots$$

$$S_{\infty} = \frac{\frac{15}{1000}}{1 - \frac{1}{1000}} = \frac{1}{66}.$$

$$\frac{7}{10} + \frac{1}{66} = \frac{118}{665}.$$

$$16. \quad .3108108 \dots = \frac{3}{10} + \frac{108}{10000} + \frac{108}{10000000} \dots$$

$$S_{\infty} = \frac{\frac{108}{10000}}{1 - \frac{1}{10000}} = \frac{2}{185}.$$

$$\frac{3}{10} + \frac{2}{185} = \frac{115}{370} = \frac{23}{74}.$$

$$17. \text{ Here } a = 5.80 \text{ and } r = \frac{99}{100}.$$

$$S_{\infty} = \frac{400}{1 - \frac{99}{100}} = 40,000 \text{ feet.}$$

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1. Let a , ar , and ar^2 be, respectively, the hundreds', the tens', and the units' digit.

$$a + ar + ar^2 = 13. \quad (1)$$

$$\frac{100a + 10ar + ar^2}{13} = 10 + \frac{9}{13}. \quad (2)$$

$$\text{From (1), } a(1 + r + r^2) = 13. \quad (3)$$

$$\text{From (2), } a(100 + 10r + r^2) = 139. \quad (4)$$

Dividing (3) by (4) and simplifying,

$$126r^2 + 9r - 1161 = 0. \quad (5)$$

$$14r^2 + r - 129 = 0. \quad (6)$$

$$(14r + 43)(r - 3) = 0.$$

$$r = 3, -\frac{43}{14}. \quad (\text{Root } -\frac{43}{14} \text{ rejected.})$$

Substituting in (1), $a + 3a + 9a = 13.$

$$a = 1.$$

The number is 139.

$$\begin{aligned}
 2. \quad t_{11} &= ar^{n-1}. \\
 t_{11} &= 1 \cdot 2^{10} = 1024. \\
 l_{11} &= a + (n-1)d. \\
 l_{11} &= 1 + 10 \cdot 100 = 1001. \\
 \frac{1}{1} \frac{0}{0} \frac{2}{2} \frac{3}{4} &= .977+, \text{ or } 97.7+ \%.
 \end{aligned}$$

$$\begin{aligned}
 3. \quad S_n &= \frac{a - ar^n}{1 - r}. \\
 S_{10} &= \frac{1 - 1 \cdot 2^{10}}{1 - 2} = \frac{-1023}{-1} = 1023. \\
 S &= \frac{n}{2} [2a + (n-1)d]. \\
 S &= 5(2 + 9 \cdot 100) = 4510.
 \end{aligned}$$

The ratio is 1023 : 4510.

$$\frac{1}{4} \frac{0}{5} \frac{2}{1} \frac{3}{0} = \frac{9}{410}, \text{ or } 22.6\%.$$

4. (a) The amount of \$500 put at interest for three years at 6%, interest compounded annually.

(b) The amount of \$500 at interest for five years at 5%, interest compounded annually.

(c) The amount of \$500 at interest for six years at 3%, interest compounded annually.

$$\begin{aligned}
 5. \quad 100 \cdot (1.04)^3 &= \$112.49. & 6. \quad 100(1.03)^4 &= \$112.55. \\
 100 \cdot (1.04)^5 &= \$121.67. & 100(1.03)^6 &= \$119.41.
 \end{aligned}$$

7. Here $a = 200$, $r = 1.04$, and $n = 5$.

$$\begin{aligned}
 S_n &= \frac{a - ar^n}{1 - r} = \frac{200 - 200(1.04)^5}{1 - 1.04} = \frac{-43.3305}{-.04} = 1083.26. \\
 S_{10} &= \frac{200 - 200(1.04)^{10}}{1 - 1.04}.
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \frac{Sr(1+r)^n}{(1+r)^n - 1} &= p. \\
 \frac{1000 \times .06(1+.06)^3}{(1+.06)^3 - 1} &= \$374.10.
 \end{aligned}$$

$$\begin{aligned}
 10. \quad p &= \frac{Sr(1+r)^n}{(1+r)^n - 1} \\
 &= \frac{2000 \times .05(1+.05)^5}{(1+.05)^5 - 1} \\
 &= \frac{100 \cdot 1.27628}{1.27628 - 1} \\
 &= \$461.95.
 \end{aligned}$$

11. Here $a = 10,000$, $r = 90\%$, and $n = 7$.

$$t_7 = 10,000 \cdot (.90)^6 = \$5314.41.$$

12. Here $a = 1$, $r = \frac{2}{3}$, and $n = 5$.

$$t_5 = 1 \cdot \left(\frac{2}{3}\right)^4 = \frac{16}{81}.$$

13. From geometry, $FD = \frac{BC}{2}$, $DE = \frac{AC}{2}$, $FE = \frac{AB}{2}$, $GH = \frac{FE}{2}$, etc.

It follows from the above that the perimeter of each successive triangle will be one half that of the triangle just outside it.

Hence the successive perimeters form the terms of a geometrical progression in which r is $\frac{1}{2}$.

If $AB = 5$,
then perimeter $ABC = 15$.

The progression is $15, \frac{15}{2}, \frac{15}{4}$, etc.

Then
$$S = \frac{15}{1 - \frac{1}{2}} = 30.$$

14. $HG = \frac{1}{2}$ of diagonal AC . (See Exercise 13, above.)

But $AC = DC\sqrt{2}$. (The diagonal of a square equals its side $\times \sqrt{2}$.)

Then $HG = \frac{DC\sqrt{2}}{2}$. Also $MN = \frac{HG\sqrt{2}}{2}$, etc.

Then perimeter $HGFE = \text{perimeter } \frac{ABCD \times \sqrt{2}}{2}$,

and perimeter $MNDL = \text{perimeter } \frac{HGFE\sqrt{2}}{2}$ etc.

The perimeter of each square after the first equals the perimeter of the preceding square multiplied by $\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$. Therefore the successive perimeters form the terms of a geometrical progression in which $r = \frac{1}{\sqrt{2}}$.

If $AB = 4$, then

$$\begin{aligned} S &= \frac{16}{1 - \frac{1}{\sqrt{2}}} = \frac{16}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{16\sqrt{2}}{\sqrt{2}-1} = \frac{16\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{32 + 16\sqrt{2}}{1} \\ &= 32 + 16(1.4142+) = 54.627. \end{aligned}$$

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1. $(a+b)^7 = a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7.$

2. $(a-1)^6 = a^6 - 6a^5 + 15a^4 - 20a^3 + 15a^2 - 6a + 1.$

3. $(a+1)^7 = a^7 + 7a^6 + 21a^5 + 35a^4 + 35a^3 + 21a^2 + 7a + 1.$

4. $(a+2)^5 = a^5 + 10a^4 + 40a^3 + 80a^2 + 80a + 32.$

5. $(a+3)^6 = a^6 + 18a^5 + 135a^4 + 540a^3 + 1215a^2 + 1458a + 729.$

6. $(2-a)^7 = 128 - 448a + 672a^2 - 560a^3 + 280a^4 - 84a^5 + 14a^6 - a^7.$

7. $(a + b)^{21} = a^{21} + 21 a^{20}b + 210 a^{19}b^2 + 1330 a^{18}b^3.$
8. $(a + b)^{30} = a^{30} + 30 a^{29}b + 435 a^{28}b^2 + 4060 a^{27}b^3.$
9. $(a + 1)^{32} = a^{32} + 32 a^{31} + 496 a^{30} + 4960 a^{29}.$
10. $(a - 2)^{25} = a^{25} - 50 a^{24} + 1200 a^{23} - 18,400 a^{22}.$
11. $(a^2 + 2b)^5 = (a^2)^5 + 5(a^2)^4(2b)^1 + 10(a^2)^3(2b)^2 + 10(a^2)^2(2b)^3$
 $+ 5(a^2)^1(2b)^4 + (2b)^5$
 $= a^{10} + 10 a^8b + 40 a^6b^2 + 80 a^4b^3 + 80 a^2b^4 + 32 b^5.$
12. $(a^2 + 2)^6 = (a^2)^6 + 6(a^2)^5 \cdot 2^1 + 15(a^2)^4 \cdot 2^2 + 20(a^2)^3 \cdot 2^3$
 $+ 15(a^2)^2 \cdot 2^4 + 6(a^2)^1 \cdot 2^5 + 2^6$
 $= a^{12} + 12 a^{10} + 60 a^8 + 160 a^6 + 240 a^4 + 192 a^2 + 64.$
13. $(a^2 - 2b)^7 = (a^2)^7 - 7(a^2)^6(2b) + 21(a^2)^5(2b)^2 - 35(a^2)^4(2b)^3$
 $+ 35(a^2)^3(2b)^4 - 21(a^2)^2(2b)^5 + 7(a^2)^1(2b)^6 - (2b)^7$
 $= a^{14} - 14 a^{12}b + 84 a^{10}b^2 - 280 a^8b^3 + 560 a^6b^4 - 672 a^4b^5$
 $+ 448 a^2b^6 - 128 b^7.$
14. $\left(\frac{a^2 + 1}{b}\right)^5 = \frac{(a^2 + 1)^5}{b^5} = \frac{a^{10} + 5 a^8 + 10 a^6 + 10 a^4 + 5 a^2 + 1}{b^5}.$
15. $\left(\frac{a^2 - 1}{a^3}\right)^6 = \frac{(a^2 - 1)^6}{a^{18}}$
 $= \frac{a^{12} - 6 a^{10} + 15 a^8 - 20 a^6 + 15 a^4 - 6 a^2 + 1}{a^{18}}$
 $= \frac{1}{a^6} - \frac{6}{a^8} + \frac{15}{a^{10}} - \frac{20}{a^{12}} + \frac{15}{a^{14}} - \frac{6}{a^{16}} + \frac{1}{a^{18}}.$
16. $(a^2 + 3b)^{20} = (a^2)^{20} + 20(a^2)^{19}(3b)^1 + 190(a^2)^{18}(3b)^2 + 1140(a^2)^{17}(3b)^3$
 $= a^{40} + 60 a^{38}b + 1710 a^{36}b^2 + 30,780 a^{34}b^3.$
17. $(a^2 - 3b^2)^{10} = (a^2)^{10} - 10(a^2)^9(3b^2)^1 + 45(a^2)^8(3b^2)^2 - 120(a^2)^7(3b^2)^3$
 $= a^{20} - 30 a^{18}b^2 + 405 a^{16}b^4 - 3240 a^{14}b^6.$
18. $\left(\frac{a^2 - 1}{a}\right)^{30} = \left(a - \frac{1}{a}\right)^{30}$
 $= a^{30} - 30 a^{29}\left(\frac{1}{a}\right)^1 + 435 a^{28}\left(\frac{1}{a}\right)^2 - 4060 a^{27}\left(\frac{1}{a}\right)^3$
 $= a^{30} - 30 a^{28} + 435 a^{26} - 4060 a^{24} \dots$
19. $\left(\frac{a}{b} + \frac{b}{a}\right)^{20} = \left(\frac{a}{b}\right)^{20} + 20\left(\frac{a}{b}\right)^{19}\left(\frac{b}{a}\right)^1 + 190\left(\frac{a}{b}\right)^{18}\left(\frac{b}{a}\right)^2 + 1140\left(\frac{a}{b}\right)^{17}\left(\frac{b}{a}\right)^3$
 $= \frac{a^{20}}{b^{20}} + \frac{20 a^{18}}{b^{18}} + \frac{190 a^{16}}{b^{16}} + \frac{1140 a^{14}}{b^{14}}.$
20. $\left(\frac{2x}{y^3} - \frac{y^4}{x^5}\right)^8 = \left(\frac{2x}{y^3}\right)^8 - 8\left(\frac{2x}{y^3}\right)^7\left(\frac{y^4}{x^5}\right) + 28\left(\frac{2x}{y^3}\right)^6\left(\frac{y^4}{x^5}\right)^2 - 56\left(\frac{2x}{y^3}\right)^5\left(\frac{y^4}{x^5}\right)^3$
 $= \frac{256 x^8}{y^{24}} - \frac{1024 x^2}{y^{17}} + \frac{1792}{x^4 y^{10}} - \frac{1792}{x^{10} y^3}.$

$$\begin{aligned}
 21. \quad & \left(\frac{a^2}{b^3} + \frac{2b^2}{a^4} \right)^{12} \\
 &= \left(\frac{a^2}{b^3} \right)^{12} + 12 \left(\frac{a^2}{b^3} \right)^{11} \left(\frac{2b^2}{a^4} \right) + 66 \left(\frac{a^2}{b^3} \right)^{10} \left(\frac{2b^2}{a^4} \right)^2 + 220 \left(\frac{a^2}{b^3} \right)^9 \left(\frac{2b^2}{a^4} \right)^3 \\
 &= \frac{a^{24}}{b^{36}} + 24 \frac{a^{18}}{b^{31}} + 264 \frac{a^{12}}{b^{26}} + 1760 \frac{a^6}{b^{21}}.
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & \left(\frac{3x^5}{y^3} - \frac{2y^{15}}{9x^{12}} \right)^5 \\
 &= \left(\frac{3x^5}{y^3} \right)^5 - 5 \left(\frac{3x^5}{y^3} \right)^4 \left(\frac{2y^{15}}{9x^{12}} \right) + 10 \left(\frac{3x^5}{y^3} \right)^3 \left(\frac{2y^{15}}{9x^{12}} \right)^2 - 10 \left(\frac{3x^5}{y^3} \right)^2 \left(\frac{2y^{15}}{9x^{12}} \right)^3 \\
 &= \frac{243x^{25}}{y^{15}} - 90x^8y^3 + \frac{40y^{21}}{3x^9} - \frac{80y^{39}}{81x^{26}}.
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \left(\frac{\sqrt{x}}{y} + \frac{\sqrt{y}}{x} \right)^{12} \\
 &= \left(\frac{\sqrt{x}}{y} \right)^{12} + 12 \left(\frac{\sqrt{x}}{y} \right)^{11} \left(\frac{\sqrt{y}}{x} \right) + 66 \left(\frac{\sqrt{x}}{y} \right)^{10} \left(\frac{\sqrt{y}}{x} \right)^2 + 220 \left(\frac{\sqrt{x}}{y} \right)^9 \left(\frac{\sqrt{y}}{x} \right)^3 \\
 &= \frac{x^6}{y^{12}} + \frac{12x^{\frac{9}{2}}}{y^{\frac{21}{2}}} + \frac{66x^3}{y^9} + \frac{220x^{\frac{3}{2}}}{y^{\frac{15}{2}}}.
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \left(1 + \frac{1}{n} \right)^n \\
 &= 1^n + n \cdot 1^{n-1} \left(\frac{1}{n} \right) + \frac{n(n-1)}{2} \cdot 1^{n-2} \left(\frac{1}{n} \right)^2 \\
 &\quad + \frac{n(n-1)(n-2)}{2 \cdot 3} \cdot 1^{n-3} \left(\frac{1}{n} \right)^3 \\
 &= 1 + 1 + \frac{n-1}{2n} + \frac{(n-1)(n-2)}{6n^2}.
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & (a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2} a^{n-2}b^2 \\
 &\quad + \frac{n(n-1)(n-2)}{2 \cdot 3} a^{n-3}b^3 \\
 &\quad + \frac{n(n-1)(n-2)(n-3)}{2 \cdot 3 \cdot 4} a^{n-4}b^4 \\
 &\quad + \frac{n(n-1)(n-2)(n-3)(n-4)}{2 \cdot 3 \cdot 4 \cdot 5} a^{n-5}b^5.
 \end{aligned}$$

$$\text{If } n = 1, (a+b)^n = a^1 + 1a^0b + \frac{1 \cdot 0}{2} a^{-1}b^2 = a + b.$$

$$\begin{aligned}
 \text{If } n = 2, (a+b)^n &= a^2 + 2a^{2-1}b + \frac{2 \cdot 1}{2} a^0b^2 + \frac{2 \cdot 1 \cdot 0}{2 \cdot 3} a^{-1}b^3 \\
 &= a^2 + 2ab + b^2.
 \end{aligned}$$

$$\begin{aligned}\text{If } n = 3, (a + b)^n &= a^3 + 3a^2b + \frac{3 \cdot 2}{2}a^1b^2 + \frac{3 \cdot 2 \cdot 1}{2 \cdot 3}a^0b^3 + \frac{3 \cdot 2 \cdot 1 \cdot 0}{2 \cdot 3 \cdot 4}a^{-1}b^4 \\ &= a^3 + 3a^2b + 3ab^2 + b^3.\end{aligned}$$

$$\begin{aligned}\text{If } n = 4, (a + b)^n &= a^4 + 4a^3b + \frac{4 \cdot 3}{2}a^2b^2 + \frac{4 \cdot 3 \cdot 2}{2 \cdot 3}ab^3 + \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 3 \cdot 4}a^0b^4 \\ &\quad + \frac{4 \cdot 3 \cdot 2 \cdot 1 \cdot 0}{2 \cdot 3 \cdot 4 \cdot 5}a^{-1}b^5 \\ &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4.\end{aligned}$$

The number of terms in each case is one more than n .

After the $(n + 1)$ th coefficient a zero factor occurs in the numerator, which makes the coefficient zero.

Where $n = 4$, the sixth term and all terms which follow it in the expansion contain the zero factor $(n - 4)$.

$$\begin{aligned}26. (1.1)^{10} &= (1 + .1)^{10} \\ &= 1^{10} + 10 \cdot 1^9(.1)^1 + 45 \cdot 1^8 \cdot (.1)^2 + 120 \cdot 1^7 \cdot (.1)^3 \\ &\quad + 210 \cdot 1^6(.1)^4 \\ &= 1 + 1 + .45 + .12 + .021 \\ &= 2.59 + .\end{aligned}$$

$$\begin{aligned}27. (5.2)^8 &= (5 + .2)^8 \\ &= 5^8 + 8 \cdot 5^7 \cdot (.2) + 28 \cdot 5^6(.2)^2 + 56 \cdot 5^5 \cdot (.2)^3 + 70 \cdot 5^4 \cdot (.2)^4 \\ &\quad + 56 \cdot 5^3 \cdot (.2)^5 + 28 \cdot 5^2 \cdot (.2)^6 \\ &= 390,625 + 125,000 + 17,500 + 1400 + 70 + 2.24 + .0448 \\ &= 534,597.28 + .\end{aligned}$$

$$\begin{aligned}28. (1.06)^6 &= (1 + .06)^6 \\ &= 1^6 + 6 \cdot 1^5 \cdot .1(.06) + 15 \cdot 1^4 \cdot (.06)^2 + \\ &= 1 + .360 + .054 + \\ &= 1.41 + .\end{aligned}$$

$$\begin{aligned}29. (.98)^{11} &= (1 - .02)^{11} \\ &= 1^{11} - 11 \cdot 1^{10} \cdot (.02) + 55 \cdot 1^9(.02)^2 - 165 \cdot 1^8 \cdot (.02)^3 \\ &= 1 - .220 + .022 - .00132 + \\ &= .80 + .\end{aligned}$$

$$\begin{aligned}30. (4.9)^9 &= (5 - .1)^9 \\ &= 5^9 - 9 \cdot 5^8 \cdot (.1)^1 + 36 \cdot 5^7(.1)^2 - 84 \cdot 5^6 \cdot (.1)^3 + 126 \cdot 5^5 \cdot (.1)^4 \\ &\quad - 126 \cdot 5^4(.1)^5 + 84 \cdot 5^3(.1)^6 \\ &= 1,953,125 - 351,562.5 + 28,125 - 1312.5 + 39.375 - .7875 \\ &\quad + .0105 \\ &= 1,628,413.59 + .\end{aligned}$$

$$\begin{aligned}31. (2.9)^8 &= (3 - .1)^8 \\ &= 3^8 - 8 \cdot 3^7 \cdot (.1) + 28 \cdot 3^6 \cdot (.1)^2 - 56 \cdot 3^5 \cdot (.1)^3 \\ &\quad + 70 \cdot 3^4 \cdot (.1)^4 - 56 \cdot 3^3 \cdot (.1)^5 \\ &= 6561 - 1749.6 + 204.12 - 13.608 + .567 - .01512 \\ &= 5002.46.\end{aligned}$$

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1. $(1 + x)^{\frac{1}{2}} = 1^{\frac{1}{2}} + \frac{1}{2} \cdot 1^{-\frac{1}{2}} \cdot x - \frac{1}{8} \cdot 1^{-\frac{3}{2}} x^2 + \frac{1}{16} \cdot 1^{-\frac{5}{2}} \cdot x^3$
 $= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}.$
2. $(2 + x)^{\frac{1}{2}} = 2^{\frac{1}{2}} + \frac{1}{2} \cdot 2^{-\frac{1}{2}} \cdot x - \frac{1}{8} \cdot 2^{-\frac{3}{2}} x^2 + \frac{1}{16} \cdot 2^{-\frac{5}{2}} \cdot x^3$
 $= \sqrt{2} + \frac{\sqrt{2}x}{4} - \frac{\sqrt{2}x^2}{32} + \frac{\sqrt{2}x^3}{128}.$
3. $(3 - x)^{\frac{1}{2}} = 3^{\frac{1}{2}} - \frac{1}{2} \cdot 3^{-\frac{1}{2}} x - \frac{1}{8} \cdot 3^{-\frac{3}{2}} x^2 - \frac{1}{16} \cdot 3^{-\frac{5}{2}} x^3$
 $= \sqrt{3} - \frac{\sqrt{3}x}{6} - \frac{\sqrt{3}x^2}{72} - \frac{\sqrt{3}x^3}{432}.$
4. $(1 + x)^{\frac{1}{3}} = 1^{\frac{1}{3}} + \frac{1}{3} \cdot 1^{-\frac{2}{3}} x - \frac{1}{9} \cdot 1^{-\frac{5}{3}} x^2 + \frac{5}{81} \cdot 1^{-\frac{8}{3}} x^3$
 $= 1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5x^3}{81}.$
5. $(2 + x)^{\frac{1}{3}} = 2^{\frac{1}{3}} + \frac{1}{3} \cdot 2^{-\frac{2}{3}} x - \frac{1}{9} \cdot 2^{-\frac{5}{3}} x^2 + \frac{5}{81} \cdot 2^{-\frac{8}{3}} \cdot x^3$
 $= \sqrt[3]{2} + \frac{\sqrt[3]{2}x}{6} - \frac{\sqrt[3]{2}x^2}{36} + \frac{5\sqrt[3]{2}x^3}{648}.$
6. $(3 - x)^{\frac{1}{3}} = 3^{\frac{1}{3}} - \frac{1}{3} \cdot 3^{-\frac{2}{3}} \cdot x - \frac{1}{9} \cdot 3^{-\frac{5}{3}} \cdot x^2 - \frac{5}{81} \cdot 3^{-\frac{8}{3}} \cdot x^3$
 $= \sqrt[3]{3} - \frac{\sqrt[3]{3}x}{9} - \frac{\sqrt[3]{3}x^2}{81} - \frac{5\sqrt[3]{3}x^3}{2187}.$
8. $(17)^{\frac{1}{2}} = (16 + 1)^{\frac{1}{2}}$
 $= 16^{\frac{1}{2}} + \frac{1}{2} \cdot 16^{-\frac{1}{2}} \cdot 1^1 - \frac{1}{8} \cdot 16^{-\frac{3}{2}} \cdot 1^2 + \frac{1}{16} \cdot 16^{-\frac{5}{2}} \cdot 1^3$
 $= 4 + .125 - .00195$
 $= 4.123.$
9. $(28)^{\frac{1}{2}} = (25 + 3)^{\frac{1}{2}}$
 $= 25^{\frac{1}{2}} + \frac{1}{2} \cdot 25^{-\frac{1}{2}} \cdot 3 - \frac{1}{8} \cdot 25^{-\frac{3}{2}} \cdot 3^2 + \frac{1}{16} \cdot 25^{-\frac{5}{2}} \cdot 3^3$
 $= 5 + .3 - .009 + .00054$
 $= 5.291.$
10. $(38)^{\frac{1}{2}} = (36 + 2)^{\frac{1}{2}}$
 $= 36^{\frac{1}{2}} + \frac{1}{2} \cdot 36^{-\frac{1}{2}} \cdot 2 - \frac{1}{8} \cdot 36^{-\frac{3}{2}} \cdot 2^2$
 $= 6 + .166 - .002$
 $= 6.164+.$
11. $(78)^{\frac{1}{2}} = (81 - 3)^{\frac{1}{2}}$
 $= 81^{\frac{1}{2}} - \frac{1}{2} \cdot 81^{-\frac{1}{2}} \cdot 3 - \frac{1}{8} \cdot 81^{-\frac{3}{2}} \cdot 3^2$
 $= 9 - .1666 - .0015$
 $= 8.831.$

12. $(125)^{\frac{1}{2}} = (121 + 4)^{\frac{1}{2}}$
 $= 121^{\frac{1}{2}} + \frac{1}{2} \cdot 121^{-\frac{1}{2}} \cdot 4^1 - \frac{1}{8} \cdot 121^{-\frac{3}{2}} \cdot 4^2 + \dots$
 $= 11 + .1818 - .0015 + \dots$
 $= 11.180.$
14. $(79)^{\frac{1}{2}} = (81 - 2)^{\frac{1}{2}}$
 $= 81^{\frac{1}{2}} - \frac{1}{2} \cdot 81^{-\frac{1}{2}} \cdot 2 + \dots$
 $= 9 - .1111 + \dots$
 $= 8.888.$
15. $(28)^{\frac{1}{3}} = (27 + 1)^{\frac{1}{3}}$
 $= 27^{\frac{1}{3}} + \frac{1}{3} \cdot 27^{-\frac{2}{3}} \cdot 1^1 - \frac{1}{9} \cdot 27^{-\frac{5}{3}} \cdot 1^2$
 $= 3 + .0370 - .0004$
 $= 3.037.$
16. $(66)^{\frac{1}{3}} = (64 + 2)^{\frac{1}{3}}$
 $= 64^{\frac{1}{3}} + \frac{1}{3} \cdot 64^{-\frac{2}{3}} \cdot 2 - \frac{1}{9} \cdot 64^{-\frac{5}{3}} \cdot 2^2 +$
 $= 4 + .0416 - .0004$
 $= 4.041.$
17. $(30)^{\frac{1}{3}} = (27 + 3)^{\frac{1}{3}}$
 $= 27^{\frac{1}{3}} + \frac{1}{3} \cdot 27^{-\frac{2}{3}} \cdot 3 - \frac{1}{9} \cdot 27^{-\frac{5}{3}} \cdot 3^2$
 $= 3 + .1111 - .0041$
 $= 3.1070.$
18. $(700)^{\frac{1}{3}} = (1000 - 300)^{\frac{1}{3}}$
 $= 1000^{\frac{1}{3}} - \frac{1}{3} \cdot 1000^{-\frac{2}{3}} \cdot 300 - \frac{1}{9} \cdot 1000^{-\frac{5}{3}} \cdot 300^2$
 $\quad - \frac{5}{81} \cdot 1000^{-\frac{8}{3}} \cdot 300^3 - \frac{10}{243} \cdot 1000^{-\frac{11}{3}} \cdot 300^4$
 $\quad - \frac{22}{729} \cdot 1000^{-\frac{14}{3}} \cdot 300^5$
 $= 10 - 1 - .1 - .0166 - .0033 - .0007$
 $= 8.879.$

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1. $6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720.$
2. $4! \cdot 3! = 4 \cdot 3 \cdot 2 \cdot 3 \cdot 2 = 144.$
3. $5! \cdot 2! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 2 = 240.$
4. $6! \div 2! = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{2} = 360.$
5. $4! - 3! \cdot 2! = 4 \cdot 3 \cdot 2 - 3 \cdot 2 \cdot 2 = 24 - 12 = 12.$
6. $n! \div (n-1)! = \frac{n(n-1)(n-2)(n-3) \text{ etc.}}{(n-1)(n-2)(n-3) \text{ etc.}} = n.$
7. $\frac{n(n-1)(n-2) \dots (n-r+2)}{(r-1)!}.$

If $n = 7$, and $r = 5$,

$$\frac{7 \cdot 6 \cdot 5 \cdot 4}{4 \cdot 3 \cdot 2} = 35.$$

8. If
- $n = 15$
- , and
- $r = 8$
- ,

$$\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 6435.$$

9. If
- $n = 21$
- , and
- $r = 12$
- ,

$$\frac{21 \cdot 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 352,716.$$

10. If
- $n = 20$
- , and
- $r = 15$
- ,

$$\frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 38,760.$$

11. If
- $n = 18$
- , and
- $r = 17$
- ,

$$\frac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 153.$$

12. If
- $n = 10$
- , and
- $r = 11$
- ,

$$\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 1.$$

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$$2. \text{ Sixth term of } (a + b)^9 = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{5 \cdot 4 \cdot 3 \cdot 2} a^4 b^5 = 126 a^4 b^5.$$

$$3. \text{ Fourth term of } (a + b)^{20} = \frac{20 \cdot 19 \cdot 18}{3 \cdot 2} a^{17} b^3 = 1140 a^{17} b^3.$$

$$4. \text{ Seventh term of } (a - b)^{10} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} a^4 b^6 = 210 a^4 b^6.$$

$$5. \text{ Eighth term of } (a - b)^{15} \\ = - \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} a^8 b^7 = - 6435 a^8 b^7.$$

$$6. \text{ Fourth term of } \left(a + \frac{1}{a}\right)^{30} = \frac{30 \cdot 29 \cdot 28}{3 \cdot 2} a^{27} \left(\frac{1}{a}\right)^3 = 4060 a^{24}.$$

$$7. \text{ Fifth term of } (a^2 - b)^{20} = \frac{20 \cdot 19 \cdot 18 \cdot 17}{4 \cdot 3 \cdot 2} (a^2)^{16} b^4 = 4845 a^{32} b^4.$$

$$8. \text{ Sixth term of } \left(\frac{a}{b} - \frac{b^2}{a}\right)^{18} = - \frac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14}{5 \cdot 4 \cdot 3 \cdot 2} \left(\frac{a}{b}\right)^{13} \left(\frac{b^2}{a}\right)^5 = - 8568 \frac{a^8}{b^3}.$$

$$9. \text{ Ninth term of } (x^2 - x)^{16} = \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} (x^2)^8 x^8 \\ = 12,870 x^{24}.$$

$$10. \text{ Seventh term of } \left(\frac{a^2}{b} - \frac{2b^2}{a}\right)^{14} = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} \left(\frac{a^2}{b}\right)^8 \left(\frac{2b^2}{a}\right)^6 \\ = 192,192 a^{10} b^4.$$

11. Fifth term of $\left(\sqrt{x} - \sqrt{\frac{y}{x}}\right)^{15} = \frac{15 \cdot 14 \cdot 13 \cdot 12}{4 \cdot 3 \cdot 2} (\sqrt{x})^{11} \left(\sqrt{\frac{y}{x}}\right)^4$
 $= 1365 x^{\frac{7}{2}} y^2.$
12. If $x^{r-1} = x^5$, $r = 6$. The coefficient of $x^5 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2} = 252.$
13. If $(x^2)^{r-1} = x^8$, $r = 5$. The coefficient of $x^8 = \frac{16 \cdot 15 \cdot 14 \cdot 13}{4 \cdot 3 \cdot 2} = 1820.$
14. If $(x^3)^{15-r+1} = x^{15}$, $r = 11$. The coefficient of x^{15}
 $= \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 3003.$
15. If $(x^2)^{14-r+1} (x^{-1})^{r-1} = x^{10}$, $r = 7$. The coefficient of x^{10}
 $= \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 3003.$

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1. $\frac{4a^2 + a - 3}{4 + \frac{a^2 - 1}{a^2 - a - 2}} = \frac{(4a - 3)(a + 1)}{(a - 1)(a + 1)} \times \frac{(a + 1)(a - 2)}{(2a - 3)(2a + 3)}$
 $= \frac{(4a - 3)(a + 1)(a - 2)}{(a - 1)(2a - 3)(2a + 3)}.$
2. $\frac{1 \text{ kilometer}}{1 \text{ mile}} = \frac{.62}{1} = .62.$
3. $\frac{1 \text{ liter}}{1 \text{ quart}} = \frac{(.001)(39.4)^3}{\frac{231}{4}} = 1.05+.$
4. $\frac{100 \times 160}{43560} = \frac{400}{1089}.$
5. $\frac{5\frac{1}{2} \times 3\frac{5}{8}}{7\frac{1}{4} \times 4\frac{3}{4}} = \frac{95}{16} \times \frac{29}{8} \times \frac{4}{29} \times \frac{4}{19} = \frac{5}{8}.$

6. Let $5x$ and $7x$ equal the required parts.

$$5x + 7x = 24,000.$$

$$12x = 24,000.$$

$$x = 2000, 5x = 10,000,$$

$$\text{and } 7x = 14,000.$$

7. Let $2x$, $5x$, $7x$, and $9x$ equal the required parts.

$$2x + 5x + 7x + x = 690.$$

$$x = 30.$$

$$2x = 60, 5x = 150,$$

$$\text{and } 7x = 210, 9x = 270.$$

CR

$$8. \frac{x}{x+2} = \frac{x^2 + 5x}{(x+2)(x+5)}.$$

$$\frac{x+3}{x+5} = \frac{x^2 + 5x + 6}{(x+2)(x+5)}.$$

$$\text{Since } x^2 + 5x < x^2 + 5x + 6,$$

$$\text{then } \frac{x}{x+2} < \frac{x+3}{x+5}.$$

$$9. \frac{3}{4} = \frac{27}{36}.$$

$$\frac{7}{9} = \frac{28}{36}.$$

$$\frac{28}{36} > \frac{27}{36},$$

$$\text{then } \frac{7}{9} > \frac{3}{4}.$$

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$$\begin{aligned} 1. (a) \quad \frac{3}{m} &= \frac{m}{27}. \\ m^2 &= 3 \cdot 27. \\ m &= \pm 9. \end{aligned}$$

$$\begin{aligned} (b) \quad \frac{\frac{4}{7}}{m} &= \frac{m}{\frac{7}{9}}. \\ m^2 &= \frac{4}{9}. \\ m &= \pm \frac{2}{3}. \end{aligned}$$

$$\begin{aligned} (c) \quad \frac{\frac{81}{28}}{m} &= \frac{m}{\frac{7}{9}}. \\ m^2 &= \frac{9}{4}. \\ m &= \pm \frac{3}{2}. \end{aligned}$$

$$\begin{aligned} (d) \quad \frac{\frac{x}{y}}{m} &= \frac{m}{xy}. \\ m^2 &= x^2. \\ m &= \pm x. \end{aligned}$$

$$\begin{aligned} 2. (a) \quad \frac{9}{6} &= \frac{6}{t}. \\ 9t &= 36. \\ t &= 4. \end{aligned}$$

$$\begin{aligned} (b) \quad \frac{180}{60} &= \frac{60}{t}. \\ 180t &= 3600. \\ t &= 20. \end{aligned}$$

$$\begin{aligned} (c) \quad \frac{216}{36} &= \frac{36}{t}. \\ 216t &= 36^2. \\ t &= 6. \end{aligned}$$

$$\begin{aligned} 3. (a) \quad \frac{14}{10} &= \frac{7}{f}. \\ 14f &= 70. \\ f &= 5. \end{aligned}$$

$$\begin{aligned} (b) \quad \frac{27}{3} &= \frac{36}{f}. \\ 27f &= 108. \\ f &= 4. \end{aligned}$$

$$\begin{aligned} (c) \quad \frac{96}{12} &= \frac{8}{f}. \\ 96f &= 96. \\ f &= 1. \end{aligned}$$

$$\begin{aligned} 4. (a) \quad 5x &= 9y. \\ 5 : 9 &= y : x. \\ 9 : 5 &= x : y. \\ 9 : x &= 5 : y. \end{aligned}$$

$$\begin{aligned} (b) \quad (a+3) \cdot 2 &= (a+1) \cdot 3. \\ (a+3) : (a+1) &= 3 : 2. \\ (a+3) : 3 &= (a+1) : 2. \\ (a+1) : (a+3) &= 2 : 3. \end{aligned}$$

$$\begin{aligned} (c) \quad x^2 - y^2 &= r^2 - s^2. \\ (x-y)(x+y) &= (r-s)(r+s). \\ (x-y) : (r-s) &= (r+s) : (x+y). \\ (x-y) : (r+s) &= (r-s) : (x+y). \\ (r+s) : (x-y) &= (x+y) : (r-s). \end{aligned}$$

$$\begin{aligned} 5. (a) \quad \frac{2}{10} &= \frac{3}{15}. \\ (b) \quad \frac{2}{20} &= \frac{9}{x}. \end{aligned}$$

$$\begin{aligned} 6. (a) \quad \frac{6}{5} &= \frac{12}{10}. \\ (b) \quad \frac{x}{2} &= \frac{30}{12}. \end{aligned}$$

$$7. \quad \frac{a}{b} = \frac{c}{d}.$$

By alternation, $\frac{a}{c} = \frac{b}{d}.$

$$8. \quad \frac{a}{b} = \frac{c}{d}.$$

By inversion, $\frac{b}{a} = \frac{d}{c}.$

$$9. \quad \frac{a}{b} = \frac{c}{d}.$$

Like powers of equal ratios are equal.

$$\frac{a^n}{b^n} = \frac{c^n}{d^n}.$$

10. $\frac{a}{b} = \frac{c}{d}.$

Like roots of equal ratios are equal.

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \frac{\sqrt[n]{c}}{\sqrt[n]{d}}.$$

11. $\frac{a}{b} = \frac{c}{d}.$

Adding 1 to each member,

$$\frac{a+b}{b} = \frac{c+d}{d}.$$

By addition.

12. $\frac{a}{b} = \frac{c}{d}.$

By inversion,

$$\frac{b}{a} = \frac{d}{c}.$$

Adding 1 to each member,

$$\frac{a+b}{a} = \frac{c+d}{c}.$$

By addition.

13. $\frac{a}{b} = \frac{c}{d}.$

Subtracting 1 from each member,

$$\frac{a-b}{b} = \frac{c-d}{d}.$$

By subtraction.

14. $\frac{a}{b} = \frac{c}{d}.$

By inversion,

$$\frac{b}{a} = \frac{d}{c}.$$

Subtracting each member from 1,

$$\frac{a-b}{a} = \frac{c-d}{c}.$$

By subtraction.

15. $\frac{a}{b} = \frac{c}{d}.$

Adding 1 to each member,

$$\frac{a+b}{b} = \frac{c+d}{d}. \quad (1)$$

Subtracting each member from 1,

$$\frac{a-b}{b} = \frac{c-d}{d}. \quad (2)$$

Dividing (1) by (2),

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}.$$

By addition and subtraction.

16. $\frac{a}{b} = \frac{c}{d}.$

Multiplying both members by 5,

$$\frac{5a}{b} = \frac{5c}{d}.$$

17. $\frac{a}{b} = \frac{c}{d}.$

Multiplying both members by $\frac{2}{7}$,

$$\frac{2a}{7b} = \frac{2c}{7d}.$$

18. $\frac{a}{b} = \frac{c}{d}.$

Multiplying both members by $\frac{a}{b}$,

$$\frac{a^2}{b^2} = \frac{ac}{bd}.$$

19. $\frac{a}{b} = \frac{c}{d}.$

Multiplying both members by 5,

$$\frac{5a}{b} = \frac{5c}{d}.$$

By addition and subtraction,

$$\frac{5a+b}{5a-b} = \frac{5c+d}{5c-d}.$$

20. $\frac{a}{b} = \frac{c}{d}.$

Squaring both members,

$$\frac{a^2}{b^2} = \frac{c^2}{d^2}.$$

By subtraction,

$$\frac{a^2-b^2}{b^2} = \frac{c^2-d^2}{d^2}.$$

21. $\frac{a}{b} = \frac{c}{d}.$

Squaring both members, $\frac{a^2}{b^2} = \frac{c^2}{d^2}.$

Dividing both members by 7, $\frac{a^2}{7b^2} = \frac{c^2}{7d^2}.$

By subtraction, $\frac{a^2 - 7b^2}{7b^2} = \frac{c^2 - 7d^2}{7d^2}.$

Multiplying both members by 7, $\frac{a^2 - 7b^2}{b^2} = \frac{c^2 - 7d^2}{d^2}.$

22. Now $\frac{a}{b} = \frac{c}{d}.$ (1)

By addition, $\frac{a+b}{a} = \frac{c+d}{c}.$ (2)

By subtraction, $\frac{a-b}{b} = \frac{c-d}{d}.$ (3)

(2) · (3), $\frac{a^2 - b^2}{ab} = \frac{c^2 - d^2}{cd}.$ (4)

(4) · $\frac{1}{2}$, $\frac{a^2 - b^2}{2ab} = \frac{c^2 - d^2}{2cd}.$

23. $\frac{a}{b} = \frac{c}{d}.$ (1)

Squaring, $\frac{a^2}{b^2} = \frac{c^2}{d^2}.$ (2)

Multiplying by $\frac{7}{3}$, $\frac{7a^2}{3b^2} = \frac{7c^2}{3d^2}.$ (3)

By subtraction, $\frac{7a^2 - 3b^2}{7a^2} = \frac{7c^2 - 3d^2}{7c^2}.$ (4)

By alternation, $\frac{7a^2 - 3b^2}{7c^2 - 3d^2} = \frac{7a^2}{7c^2} = \frac{a^2}{c^2}.$ (5)

Writing (1) by alternation and then multiplying by $\frac{a}{c}$, $\frac{a^2}{c^2} = \frac{ab}{cd}.$ (6)

From (6), $\frac{a^2}{c^2} = \frac{5ab}{5cd}.$ (7)

From (5) and (7), $\frac{7a^2 - 3b^2}{7c^2 - 3d^2} = \frac{5ab}{5cd},$

or $\frac{7a^2 - 3b^2}{5ab} = \frac{7c^2 - 3d^2}{5cd}.$

$$24. \quad \frac{a}{b} = \frac{c}{d}. \quad (1)$$

$$\text{Cubing both members,} \quad \frac{a^3}{b^3} = \frac{c^3}{d^3}. \quad (2)$$

$$\text{By addition and subtraction,} \quad \frac{a^3 + b^3}{a^3 - b^3} = \frac{c^3 + d^3}{c^3 - d^3}. \quad (3)$$

$$\text{By addition and subtraction in (1),} \quad \frac{a + b}{a - b} = \frac{c + d}{c - d}. \quad (4)$$

$$(3) \div (4), \quad \frac{a^2 - ab + b^2}{a^2 + ab + b^2} = \frac{c^2 - cd + d^2}{c^2 + cd + d^2}.$$

$$\text{By inversion and alternation,} \quad \frac{a^2 + ab + b^2}{c^2 + cd + d^2} = \frac{a^2 - ab + b^2}{c^2 - cd + d^2}.$$

25. In a series of equal ratios the sum of the antecedents is to the sum of the consequents as any antecedent is to its consequent.

$$26. \quad \frac{a}{b} = \frac{c}{d} = \frac{5}{6}. \quad (1)$$

$$\frac{a}{b} = \frac{5}{6}. \quad (2)$$

$$28. \quad 25 : x = x : 169.$$

$$x^2 = 169 \cdot 25.$$

$$x = \pm 5 \cdot 13 = \pm 65.$$

$$29. \quad 5 : 4 = (x - 3) : (x - 4).$$

$$5x - 20 = 4x - 12.$$

$$x = 8.$$

$$30. \quad (15 + x) : (15 - x) = 13 : 17.$$

By addition and subtraction,

$$\frac{30}{2x} = \frac{30}{-4}.$$

$$x = -2.$$

Applying the theorem of Exercise 25 to (2),

$$\frac{a + 5}{5} = \frac{b + 6}{6}. \quad (3)$$

Then from (3) and (1),

$$\frac{a + 5}{b + 6} = \frac{5}{6} = \frac{c}{d}.$$

$$27. \quad 2x : (x + 8) = 10 : 3.$$

$$6x = 10x + 80.$$

$$x = -20.$$

$$31. \quad (10 + x) : (20 + 3x) = (10 - x) : -3x.$$

$$200 + 10x - 3x^2 = -30x - 3x^2.$$

$$40x = -200.$$

$$x = -5.$$

32.

$$\frac{\sqrt{x} + 2}{\sqrt{x} - 2} = \frac{x + 1}{x - 7}.$$

By addition and subtraction,

$$\frac{2\sqrt{x}}{4} = \frac{2x - 6}{8}.$$

$$4\sqrt{x} = 2x - 6.$$

$$x - 2\sqrt{x} - 3 = 0.$$

$$(\sqrt{x} - 3)(\sqrt{x} + 1) = 0.$$

Then

$$x = 9 \text{ or } 1. \text{ (Root 1 rejected.)}$$

33. Let x and y be the first and third terms, respectively, of the progression.

Then

$$\begin{aligned}\frac{x}{m} &= \frac{m}{y} \\ m^2 &= xy. \\ m &= \pm \sqrt{xy}.\end{aligned}\tag{1}$$

Let a, ar, ar^2, \dots , be a geometrical progression.

If $a = x$ and $ar^2 = y$, then $r^2 = \frac{y}{x}$.

$$r = \pm \sqrt{\frac{y}{x}} = \pm \frac{1}{x} \sqrt{xy}.$$

Also

$$ar = x \cdot \pm \frac{1}{x} \sqrt{xy} = \pm \sqrt{xy}.\tag{2}$$

From (1) and (2),

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1. (a) $\frac{S_1}{S_2} = \frac{4\pi R_1^2}{4\pi R_2^2} = \frac{R_1^2}{R_2^2}.$
 (b) $\frac{S_1}{S_2} = \frac{4\pi (\frac{1}{2} D_1)^2}{4\pi (\frac{1}{2} D_2)^2} = \frac{D_1^2}{D_2^2} = \frac{(2R_1)^2}{(2R_2)^2} = \frac{R_1^2}{R_2^2}.$
2. $\frac{R_1}{R_2} = \frac{1}{10}.$
 $\frac{S_1}{S_2} = \frac{4\pi 1^2}{4\pi 10^2} = \frac{1}{100}.$
3. $\frac{S \text{ of earth}}{S \text{ of Mars}} = \frac{7920^2}{4230^2} = \frac{88^2}{47^2} = \frac{7744}{2209}.$
4. $\frac{S \text{ of moon}}{S \text{ of earth}} = \frac{2160^2}{7920^2} = \frac{3^2}{11^2} = \frac{9}{121}.$
5. $\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3} = \frac{\frac{4}{3}\pi (\frac{1}{2} D_1)^3}{\frac{4}{3}\pi (\frac{1}{2} D_2)^3} = \frac{R_1^3}{R_2^3} = \frac{D_1^3}{D_2^3}.$
6. $\frac{\text{Volume of sun}}{\text{Volume of earth}} = \frac{109^3}{1^3} = \frac{1295029}{1}.$
7. $\frac{\text{Volume of Mars}}{\text{Volume of earth}} = \frac{4230^3}{7920^3} = \frac{47^3}{88^3} = \frac{103823}{681472}.$
8. $\frac{\text{Volume of earth}}{\text{Volume of moon}} = \frac{7920^3}{2160^3} = \frac{11^3}{3^3} = \frac{1331}{27}.$
9. $\frac{12^2}{20^2} = \frac{90}{x}.$
 $x = 250.$

$$\begin{aligned}
 10. \quad \frac{10^2}{x^2} &= \frac{147}{300} \\
 147x^2 &= 30,000. \\
 49x^2 &= 10,000. \\
 x^2 &= \frac{10,000}{49} \\
 x &= \frac{100}{7} = 14\frac{2}{7}.
 \end{aligned}$$

$$\begin{aligned}
 11. \quad \frac{AB^2}{AK^2} &= \frac{\triangle ABC}{\triangle AKR} \\
 \frac{15^2}{AK^2} &= \frac{225}{81} \\
 AK^2 &= \frac{15^2 \cdot 81}{225} = 81. \\
 AK &= 9.
 \end{aligned}$$

$$\begin{aligned}
 12. \quad \frac{BC^2}{KR^2} &= \frac{\triangle ABC}{\triangle AKR} \\
 \frac{13^2}{7^2} &= \frac{845}{\triangle AKR} \\
 \triangle AKR &= \frac{845 \cdot 49}{169} = 5 \cdot 49 \\
 &= 245.
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \frac{\triangle AKR}{\triangle ABC} &= \frac{AR^2}{AC^2} \\
 \frac{4}{25} &= \frac{AR^2}{15^2} \\
 25AR^2 &= 15^2 \cdot 4. \\
 5AR &= 15 \cdot 2, \text{ or } AR = 6. \\
 AC - AR &= RC = 15 - 6 = 9.
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \frac{\triangle AKR}{\triangle ABC} &= \frac{AR^2}{AC^2} \\
 \frac{1}{6} &= \frac{AR^2}{15^2} \\
 AR^2 &= \frac{15^2}{6} \\
 AR &= \frac{15}{\sqrt{6}}\sqrt{6} \\
 &= 6.12 \text{ inches.}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad \frac{\triangle AKR}{\triangle ABC} &= \frac{AK^2}{AB^2} \\
 \frac{1}{2} &= \frac{10^2}{AB^2} \\
 AB &= 10\sqrt{2} = 14.14. \\
 KB &= AB - AK \\
 &= 14.14 - 10 \\
 &= 4.14.
 \end{aligned}$$

$$\begin{aligned}
 16. \quad \frac{45}{4\frac{1}{2}} &= \frac{p}{8} \\
 4.5p &= 360. \\
 p &= 80 \text{ feet.}
 \end{aligned}$$

17. Let x = the shorter segment.
Then $24 - x$ = the longer segment.

$$\begin{aligned}
 \frac{x}{24 - x} &= \frac{15}{25} = \frac{3}{5} \\
 5x &= 72 - 3x. \\
 x &= 9. \\
 24 - x &= 15.
 \end{aligned}$$

$$\begin{aligned}
 18. \quad \frac{D - KRL}{D - ABC} &= \frac{DS^3}{DH^3} \\
 \frac{8}{27} &= \frac{DS^3}{15^3} \\
 \frac{2}{3} &= \frac{DS}{15} \\
 DS &= 10 \text{ inches.}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad \frac{D - KRL}{D - ABC} &= \frac{DS^3}{DH^3} \\
 \frac{1}{3} &= \frac{DS^3}{18^3} \\
 DS^3 &= \frac{18^3}{3} \\
 DS &= \frac{1}{3} \cdot 18\sqrt[3]{9} \\
 &= 6 \cdot 2.080 \\
 &= 12.48 \text{ inches}
 \end{aligned}$$

and two imaginary roots, which are rejected.

20. (a)

$$\frac{D - KRL}{D - ABC} = \frac{DS^3}{DH^3}.$$

$$\frac{1}{8} = \frac{DS^3}{16^3}.$$

$$DS^3 = \frac{16^3}{8}.$$

$DS = 8$ and two imaginary roots, which are rejected.

(b) $\frac{1}{8} = \frac{DS^3}{x^3}.$

$$DS^3 = \frac{x^3}{8}.$$

$$DS = \frac{x}{2}.$$

21. Let x = the upper segment of DH .

Then $75 - x$ = the lower segment of DH .

$$\frac{D - KRL}{D - ABC} = \frac{x^3}{75^3}.$$

$$\frac{1}{2} = \frac{x^3}{75^3}.$$

$$x = \frac{75}{\sqrt{2}} = \frac{75}{2} \sqrt{4} = 59.51 +.$$

$75 - (59.51 +) = 15.48 +$ and two imaginary roots, which are rejected.

22. $\frac{V_1}{V_2} = \frac{e_1^3}{e_2^3} = \frac{2^3}{3^3} = \frac{8}{27}.$

23. $\frac{V_1}{V_2} = \frac{R_1^3}{R_2^3}.$

$$\frac{125}{27} = \frac{R_1^3}{R_2^3}.$$

$$\frac{5}{3} = \frac{R_1}{R_2}.$$

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2. $x \propto y.$

$$\frac{8}{10} = \frac{15}{y}.$$

$$y = 18.75.$$

3. $x \propto y.$

$$\frac{h}{r} = \frac{k}{y}.$$

$$y = \frac{rk}{h}.$$

4. $x \propto y.$

$$x = Ky.$$

$$2 = 5K.$$

$$K = \frac{2}{5}.$$

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2. $x \propto \frac{1}{y}.$

$$1 : x = \frac{1}{25} : \frac{1}{10}.$$

$$x = 2\frac{1}{2}.$$

4. $m \propto \frac{1}{n}.$

$$2 : m = 3 : \frac{1}{12}.$$

$$m = \frac{1}{18}.$$

5. $t \propto \frac{1}{n}.$

$$2 : 8 = \frac{1}{8} : \frac{1}{n}.$$

$$n = 2.$$

6. $w \propto \frac{1}{d}.$

$$\frac{100}{w} = \frac{\frac{1}{4000}}{\frac{1}{5000}} = \frac{5000}{4000}.$$

$$w = 80.$$

7. $t \propto \frac{1}{r}.$

$$t = K \cdot \frac{1}{r}.$$

$$4 = \frac{K}{25}.$$

$$K = 100.$$

3. $y \propto \frac{1}{z}.$

$$h : y = \frac{1}{k} : \frac{1}{r}.$$

$$y = \frac{hk}{r}.$$

8. $w \propto \frac{1}{d}.$

$$w = K \frac{1}{d}.$$

$$200 = K \frac{1}{40000}.$$

$$K = 800,000.$$

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2. $x \propto yz.$

$$\frac{10}{x} = \frac{15 \cdot 6}{9 \cdot 8}.$$

$$x = 8.$$

3. $A \propto hb.$

$$\frac{30}{A} = \frac{5 \cdot 12}{7 \cdot 10}.$$

$$A = 35.$$

4. $A \propto hb.$

$$A = Khb.$$

$$48 = K \cdot 8 \cdot 12.$$

$$K = \frac{1}{2}.$$

5. $x \propto \frac{y}{z}.$

$$\frac{10}{x} = \frac{\frac{5}{27}}{\frac{12}{36}}.$$

$$x = 18.$$

6. $V \propto \frac{T}{P}.$

$$\frac{80}{40} = \frac{\frac{300}{30}}{\frac{400}{P}}.$$

$$P = 80.$$

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1. $W \propto d.$

$$(a) \quad \frac{172}{W} = \frac{4000}{3000}.$$

$$W = 129.$$

$$(b) \quad \frac{172}{W} = \frac{4000}{1000}.$$

$$W = 43.$$

$$(c) \quad \frac{W}{172} = \frac{0}{4000}.$$

$$W = 0.$$

2. $d \propto t.$

$$\frac{1100}{d} = \frac{1}{5\frac{2}{5}}.$$

$$d = 5940.$$

4. $d \propto t^2.$

$$(a) \quad \frac{16}{d} = \frac{1^2}{3^2}.$$

$$d = 144.$$

$$(b) \quad \frac{16}{d} = \frac{1^2}{10^2}.$$

$$d = 1600.$$

5. $I \propto \frac{1}{d^2}.$

Since the intensity varies inversely as the square of the distance, the page appears brighter when nearer, or at 3 feet.

$$I_1 = K \frac{1}{3^2}.$$

$$I_2 = K \frac{1}{6^2}.$$

$$\frac{I_1}{I_2} = \frac{\frac{1}{9}}{\frac{1}{36}} = \frac{4}{1}.$$

The light is four times as bright at 3 feet.

3. The less the number of cubic inches, the greater the pressure; therefore the variation is inverse.

If $p \propto \frac{1}{v}$, then

$$24 = K \cdot \frac{1}{30},$$

and $20 = K \cdot \frac{1}{36}.$

In each case $K = 720.$

$$42 = K \frac{1}{v} = 720 \frac{1}{v}.$$

$$v = 17\frac{1}{7} \text{ cubic inches.}$$

6. $I \propto \frac{1}{d_1^2}$. $\frac{I}{2I} = \frac{\frac{1}{5^2}}{\frac{1}{d_1^2}}$.
 $d_1^2 = \frac{2 \cdot 5}{1}$.
 $d_1 = \pm \frac{5}{2} \sqrt{2} = \pm 3.54$ feet.
 (Root — 3.54 rejected.)
7. $A \propto r^2$. $\frac{154}{616} = \frac{7^2}{x^2}$.
 $x^2 = 196$.
 $x = \pm 14$.
 (Root — 14 rejected.)
8. $A \propto d^2$. $\frac{40^2}{15^2} = \frac{10^2}{x^2}$.
 $x = \pm 3\frac{3}{4}$ feet.
 (Root — $3\frac{3}{4}$ rejected.)
9. $W \propto \frac{1}{d^2}$.
 (a) $\frac{172}{W_1} = \frac{5000^2}{4000^2}$.
 $W_1 = 110.08$.
 (b) $\frac{172}{W_2} = \frac{7000^2}{4000^2}$.
 $W_2 = 56.16$.
 (c) $\frac{172}{W_3} = \frac{9000^2}{4000^2}$.
 $W_3 = 33\frac{7}{8} = 33.97$.
10. $W \propto \frac{1}{d^2}$.
 $\frac{150}{100} = \frac{x^2}{4000^2}$.
11. $w \propto R^3$. $\frac{8}{w} = \frac{3^3}{5^3}$.
 $w = 37$ pounds.
12. $t \propto \sqrt{l}$. $\frac{1}{t} = \frac{\sqrt{100}}{\sqrt{81}}$.
 $t = \frac{9}{10}$ seconds.
13. $t \propto \sqrt{l}$. $\frac{1}{2} = \frac{\sqrt{100}}{\sqrt{l_1}}$.
 $\sqrt{l_1} = 20$.
 $l_1 = 400$ centimeters.
14. $p \propto av^2$. $\frac{.9}{72.9} = \frac{1 \cdot 15^2}{9 \cdot x^2}$.
 $x = \pm 45$ miles per hour.
 (Root — 45 rejected.)
15. $p \propto a \cdot d$. $\frac{62.5}{x} = \frac{1 \cdot 1}{154 \cdot 10}$.
 $x = 96,250$ pounds.
16. $c \propto l \cdot \frac{1}{d}$. $\frac{1320}{c} = \frac{1 \cdot \frac{1}{2}}{20 \cdot \frac{1}{2\frac{1}{4}}}$.
 $c = \$23,466.67$.
- $x = \pm 4898.9$.
 (Root — 4898.9 rejected.)
 $4898.9 - 4000 = 898.9$, the number of miles above the surface of the earth.

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1. 3.5593.	5. 2.8108.	9. 2.8485.	13. 1.4782.
2. 2.6672.	6. 1.9167.	10. .4782.	14. .4971.
3. 1.7220.	7. 1.0043.	11. $\bar{3}.4564.$	15. .4343.
4. 1.6309.	8. 2.6993.	12. $\bar{4}.8908.$	16. $\bar{3}.3703.$

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1. 35.67.	4. 373.3.	7. .6803.	10. 5.928.
2. 241.1.	5. .1805.	8. .009148.	11. .001276.
3. 4.776.	6. .002276.	9. .1847.	12. .0001705.

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2.	$\log 37 = 1.5682$ $\log 28 = \underline{1.4472}$ $\log (28 \times 37) = \underline{3.0154}$ $\text{antilog } 3.0154 = 1036.$	8.	$\log 589 = 2.7701$ $\log 375 = \underline{2.5740}$ $\log (589 \times 375) = \underline{5.3441}$ $\text{antilog } 5.3441 = 220,900.$
3.	$\log 29 = 1.4624$ $\log 9 = \underline{.9542}$ $\log (29 \times 9) = \underline{2.4166}$ $\text{antilog } 2.4166 = 261.$	9.	$\log 4326 = 3.6361$ $\log 497 = \underline{2.6964}$ $\log (4326 \times 497) = \underline{6.3325}$ $\text{antilog } 6.3325 = 2,150,000.$
4.	$\log 9.8 = .9912$ $\log 6 = \underline{.7782}$ $\log (9.8 \times 6) = \underline{1.7694}$ $\text{antilog } 1.7694 = 58.8.$	10.	$\log 2870 = 3.4579$ $\log 3754 = \underline{3.5745}$ $\log (2870 \times 3754) = \underline{7.0324}$ $\text{antilog } 7.0324 = 10,780,000.$
5.	$\log 42 = 1.6232$ $\log 3.3 = \underline{.5185}$ $\log (42 \times 3.3) = \underline{2.1417}$ $\text{antilog } 2.1417 = 138.6.$	11.	$\log 286.7 = 2.4575$ $\log 2.391 = \underline{.3786}$ $\log (286.7 \times 2.391) = \underline{2.8361}$ $\text{antilog } 2.8361 = 685.7.$
6.	$\log 386 = 2.5866$ $\log 27 = \underline{1.4314}$ $\log (386 \times 27) = \underline{4.0180}$ $\text{antilog } 4.0180 = 10,420.$	12.	$\log 3.412 = .5330$ $\log 2.526 = \underline{.4024}$ $\log (3.412 \times 2.526) = \underline{.9354}$ $\text{antilog } .9354 = 8.618.$
7.	$\log 432 = 2.6355$ $\log 263 = \underline{2.4200}$ $\log (432 \times 263) = \underline{5.0555}$ $\text{antilog } 5.0555 = 113,600.$	14.	$\log 385 = 2.5855$ $\log .647 = \underline{1.8109}$ $\log (385 \times .647) = \underline{2.3964}$ $\text{antilog } 2.3964 = 249.1.$

15. $\log 571 = 2.7566$
 $\log .073 = \bar{2}.8633$
 $\log (571 \times .073) = 1.6199$
 $\text{antilog } 1.6199 = 41.68.$
16. $\log 37.6 = 1.5752$
 $\log .00865 = \bar{3}.9370$
 $\log (37.6 \times .00865) = \bar{1}.5122$
 $\text{antilog } \bar{1}.5122 = .3252.$
17. $\log .0476 = \bar{2}.6776$
 $\log 673 = 2.8280$
 $\log (.0476 \times 673) = 1.5056$
 $\text{antilog } 1.5056 = 32.04.$
18. $\log .07325 = \bar{2}.8648$
 $\log 6.354 = .8031$
 $\log (.07325 \times 6.354) = \bar{1}.6679$
 $\text{antilog } \bar{1}.6679 = .4654.$
19. $\log .6381 = \bar{1}.8049$
 $\log .01897 = \bar{2}.2781$
 $\log (.6381 \times .01897) = \bar{2}.0830$
 $\text{antilog } \bar{2}.0830 = .01211.$
 $.6381 \times (-.01897) = -.01211.$
20. $\log 675 = 2.8293$
 $\log .0286 = \bar{2}.4564$
 $\log (675 \times .0286) = 1.2857$
 $\text{antilog } 1.2857 = 19.30.$
 $675 \times (-.0286) = -19.30.$
21. $\log .437 = \bar{1}.6405$
 $\log .0046 = \bar{3}.6628$
 $\log (437 \times .0046) = \bar{3}.3033$
 $\text{antilog } \bar{3}.3033 = .002010.$

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2. $\log 96 = 1.9823$
 $\log 32 = 1.5051$
 $\log (96 \div 32) = .4772$
 $\text{antilog } .4772 = 3.001.$
3. $\log 888 = 2.9484$
 $\log 47 = 1.6721$
 $\log (888 \div 47) = 1.2763$
 $\text{antilog } 1.2763 = 18.89.$
4. $\log 976 = 2.9894$
 $\log 361 = 2.5575$
 $\log (976 \div 361) = .4319$
 $\text{antilog } .4319 = 2.703.$
5. $\log 439 = 2.6425$
 $\log 27.1 = 1.4330$
 $\log (439 \div 27.1) = 1.2095$
 $\text{antilog } 1.2095 = 16.20.$
6. $\log 3860 = 3.5866$
 $\log 4.32 = .6355$
 $\log (3860 \div 4.32) = 2.9511$
 $\text{antilog } 2.9511 = 893.5.$
7. $\log 4627 = 3.6653$
 $\log 281 = 2.4487$
 $\log (4627 \div 281) = 1.2166$
 $\text{antilog } 1.2166 = 16.47.$
8. $\log 9896 = 3.9954$
 $\log 52.78 = 1.7224$
 $\log (9896 \div 52.78) = 2.2730$
 $\text{antilog } 2.2730 = 187.5.$
9. $\log 6732 = 3.8281$
 $\log 7.81 = .8927$
 $\log (6732 \div 7.81) = 2.9354$
 $\text{antilog } 2.9354 = 861.8.$
11. $\log 2.35 = 10.3711 - 10$
 $\log .0683 = \bar{2}.8344$
 $\log (2.35 \div .0683) = 11.5367 - 10$
 $\text{antilog } 1.5367 = 34.41.$
12. $\log 4.86 = 10.6866 - 10$
 $\log .751 = \bar{1}.8756$
 $\log (4.86 \div .751) = 10.8110 - 10$
 $\text{antilog } .8110 = 6.471.$

13. $\log .0635 = \bar{2}.8028$
 $\log .277 = \bar{1}.4425$
 $\log (.0635 \div .277) = \bar{1}.3603$
 $\text{antilog } \bar{1}.3603 = .2293.$
14. $\log .2674 = \bar{1}.4271$
 $\log 3.66 = .5635$
 $\log (.2674 \div 3.66) = \bar{2}.8636$
 $\text{antilog } \bar{2}.8636 = .07305.$
15. $\log .07882 = \bar{2}.8966$
 $\log 68.72 = \bar{1}.8371$
 $\log (.07882 \div 68.72) = \bar{3}.0595$
 $\text{antilog } \bar{3}.0595 = .001147.$
16. $\log 356 = 2.5514$
 $\log 392 = 2.5933$
 $\log (356 \times 392) = 5.1447$
 $\log 128 = 2.1072$
 $\quad \quad \quad 3.0375$
 $\text{antilog } 3.0375 = 1090.$
17. $\log 347 = 2.5403$
 $\log 625 = 2.7959$
 $\log (347 \times 625) = 5.3362$
 $\log 346 = 2.5391$
 $\quad \quad \quad 2.7971$
 $\text{antilog } 2.7971 = 626.7.$
 $\quad \quad \quad - 626.7. \text{ Ans.}$
18. $\log 473.2 = 2.6751$
 $\log 4.78 = .6794$
 $\log (473.2 \times 4.78) = 3.3545$
 $\log 68.3 = 1.8344$
 $\quad \quad \quad 1.5201$
 $\text{antilog } 1.5201 = 33.12.$
 $\quad \quad \quad - 33.12. \text{ Ans.}$
19. $\log 9.63 = .9836$
 $\log .0892 = \bar{2}.9504$
 $\log (9.63 \times .0892) = \bar{1}.9340$
 $\log .00635 = \bar{3}.8028$
 $\quad \quad \quad 2.1312$
 $\text{antilog } 2.1312 = 135.3.$

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2. $\log 6.32 = .8007.$
 $\log (6.32)^4 = 4 (.8007)$
 $\quad \quad \quad = 3.2028.$
 $\text{antilog } 3.2028 = 1595.$
3. $\log 34.26 = 1.5348.$
 $\log (34.26)^2 = 3.0696.$
 $\text{antilog } 3.0696 = 1174.$
4. $\log 6.715 = .8271.$
 $\log (6.715)^3 = 2.4813.$
 $\text{antilog } 2.4813 = 302.9.$
6. $\log .362 = \bar{1}.5587.$
 $\log (.362)^4 = \bar{2}.2348.$
 $\text{antilog } \bar{2}.2348 = .01717.$
7. $\log .0972 = \bar{2}.9877.$
 $\log (.0972)^2 = \bar{3}.9754.$
 $\text{antilog } \bar{3}.9754 = .009450.$
8. $\log .003597 = \bar{3}.5559.$
 $\log (.003597)^5 = \bar{13}.7795.$
 $\text{antilog } \bar{13}.7795 = .000,000,000,000,601,9.$
9. $\log (486.2)^2 = 2 (2.6868) = 5.3736$
 $\log (3.85)^3 = 3 (.5855) = 1.7565$
 $\log (486.2)^2 (3.85)^3 = 7.1301$
 $\text{antilog } 7.1301 = 13,490,000.$
10. $\log (.375)^5 = 5 (\bar{1}.5740) = \bar{3}.8700$
 $\log (62.5)^4 = 4 (1.7959) = 7.1836$
 $\log (.375)^2 (62.5)^4 = 5.0536$
 $\text{antilog } 5.0536 = 113,100.$
11. $\log (2.25)^4 = 4 (.3522) = 1.4088$
 $\log (1.232)^3 = 3 (.0906) = .2718$
 $\log (2.25)^4 \div (1.232)^3 = 1.1370$
 $\text{antilog } 1.1370 = 13.71.$

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2. $\log \sqrt[3]{783} = \frac{1}{3} (2.8938)$
 $= .9646.$
 $\text{antilog } .9646 = 9.218.$
3. $\log \sqrt[5]{1435} = \frac{1}{5} (3.1569)$
 $= .6314.$
 $\text{antilog } .6314 = 4.280.$
4. $\log \sqrt[4]{3421} = \frac{1}{4} (3.5341)$
 $= .8835.$
 $\text{antilog } .8835 = 7.647.$
6. $\log \sqrt{.0756} = \frac{1}{2} (8.8785 - 10)$
 $= 4.4393 - 5.$
 $\text{antilog } \bar{1}.4393 = .2750.$
7. $\log \sqrt[5]{.0007624} = \frac{1}{5} (6.8822 - 10)$
 $= 1.3764 - 2.$
 $\text{antilog } \bar{1}.3764 = .2379.$
8. $\log \sqrt[4]{.005679} = \frac{1}{4} (1.7542 - 4)$
 $= .4386 - 1.$
 $\text{antilog } \bar{1}.4386 = .2745.$
9. $\log (38.4)^{\frac{2}{3}} = \frac{2}{3} (1.5843)$
 $= 1.0562.$
 $\text{antilog } 1.0562 = 11.38.$
10. $\log (4.925)^{\frac{3}{2}} = \frac{3}{2} (.6924)$
 $= 1.0386.$
 $\text{antilog } 1.0386 = 10.93.$
11. $\log (6.387)^{\frac{5}{3}} = \frac{5}{3} (.8053)$
 $= 1.3422.$
 $\text{antilog } 1.3422 = 21.99.$
 $- 21.99. \text{ Ans.}$
12. $\log 283 = 2.4518$
 $\log 7.627 = .8824$
 $\log (283) (7.627) = 3.3342$
 $\log (3.423)^3 = 1.6032$
 $\log \text{ of radicand} = 1.7310$
 $\log \text{ of radical} = .8655.$
 $\text{antilog } .8655 = 7.337.$
 $- 7.337. \text{ Ans.}$
13. $\log (43.56)^2 = 3.2782$
 $\log 7.984 = .9022$
 $\log \text{ of numerator} = 4.1804$
 $\log (7.623)^3 = 2.6466$
 $\log \text{ of quotient} = 1.5338$
 $\log \text{ of expression} = \frac{1}{2} \cdot 1.5338$
 $= .7669.$
 $\text{antilog } .7669 = 5.846.$
14. $\log \sqrt[11]{269} = \frac{1}{11} (2.4298)$
 $= .2209.$
 $\text{antilog } .2209 = 1.663.$
15. $\log \sqrt[4]{463} = .6664$
 $\log 87 = 1.9395$
 $\log \text{ of product} = 2.6059$
 $\log \text{ of expression} = 1.3030.$
 $\text{antilog } 1.3030 = 20.09.$
16. $\frac{2.5}{3.81} \sqrt[1.37]{\frac{1}{6.7}} \sqrt[3]{972}$
 $= \frac{25}{381} \cdot \frac{(137)^{\frac{1}{2}}}{(67)^{\frac{1}{2}}} \cdot (972)^{\frac{1}{6}}.$
 $\log 25 = 1.3979$
 $\log (137)^{\frac{1}{2}} = 1.0684$
 $\log (972)^{\frac{1}{6}} = .4980$
 $\log \text{ of numerator} = 2.9643$
 $\log 381 = 2.5809$
 $\log (67)^{\frac{1}{2}} = .9131$
 $\log \text{ of denominator} = 3.4940$
 $\log \text{ of expression} = \bar{1}.4703.$
 $\text{antilog } \bar{1}.4703 = .2953.$
17. $\log 31 = 1.4914$
 $\log \sqrt[3]{.07241} = \bar{1}.6199$
 $\log \text{ of numerator} = 1.1113$
 $\log 426 = 2.6294.$
 $\log \text{ of expression} = \bar{2}.4819.$
 $\text{antilog } \bar{2}.4819 = .03033.$

18. For any number n , if $\log_{10} n = x$,
 then $10^x = n$. (1)

For the same number n , if $\log_{100} n = y$,
 then $100^y = n$. (2)

From (1) and (2), $10^x = 100^y$,
 whence $10^x = 10^{2y}$ or $y = \frac{x}{2}$.

Therefore $\log_{100} n = \frac{1}{2} \log_{10} n$.

$$\log_{10} 5732 = 3.7583 \quad \text{and} \quad \log_{100} 5732 = 1.8792.$$

$$\log_{10} 573.2 = 2.7583 \quad \text{and} \quad \log_{100} 573.2 = 1.3792.$$

$$\log_{10} 57.32 = 1.7583 \quad \text{and} \quad \log_{100} 57.32 = .8792.$$

$$\log_{10} 5.732 = .7583 \quad \text{and} \quad \log_{100} 5.732 = .3792.$$

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1. No. No real power of a positive real number is a negative number.
 Hence no logarithm can be found for a negative number to a positive base.

2. Let $\log_3 9 = x$. 6. Let $5 \log_{27} 9 = x$.
 $3^x = 9 = 3^2$. Then $\log_{27} 9 = \frac{x}{5}$.
 $x = 2$.

$$(27)^{\frac{x}{5}} = 9.$$

3. Let $\log_2 8 = x$. $3^{\frac{3x}{5}} = 3^2$.
 $2^x = 8 = 2^3$. $\frac{3x}{5} = 2$.
 $x = 3$. $x = \frac{10}{3}$.

4. Let $\log_8 2 = x$. 7. Let $\log_4 8 = x$.
 $8^x = 2^{3x} = 2^1$. $4^x = 8$.
 $x = \frac{1}{3}$. $x = \frac{3}{2}$.

5. Let $4 \log_9 27 = x$. Now let $3 \log_8 4 = y$.
 $\log_9 27 = \frac{x}{4}$. $\log_8 4 = \frac{y}{3}$.
 $9^{\frac{x}{4}} = 3^{\frac{x}{2}} = 3^3$. $8^{\frac{y}{3}} = 4$.
 $\frac{x}{2} = 3$, $2^y = 2^2$.
 $x = 6$. $y = 2$.

or $\log_4 8 + 3 \log_8 4 = x + y$
 $= \frac{3}{2} + 2 = 3\frac{1}{2}$.

8. $2 \log_{27} 81 - 4 \log_{81} 27 = 2(\frac{4}{3}) - 4(\frac{3}{4}) = -\frac{1}{3}$.

9. $3 \log_{25} 125 + 2 \log_5 25 - 2 \log_{125} 5 = \frac{9}{2} + 4 - \frac{2}{3} = 7\frac{5}{6}$.

10. $4 \log_3 (\frac{1}{3}) - 5 \log_9 (\frac{1}{27}) + 2 \log_{27} 9 = 4(-1) - 5(-\frac{3}{2}) + 2(\frac{2}{3}) = 4\frac{5}{6}$.

11. $\log \frac{5}{8} + \log \frac{2}{5} = \log (\frac{5}{8} \cdot \frac{2}{5}) = \log \frac{1}{4}$.

$$12. \log \frac{7}{3^{\frac{1}{2}}} - \log \frac{3^{\frac{5}{6}}}{4} = \log \left(\frac{7}{3^{\frac{1}{2}}} \cdot \frac{6}{3^{\frac{5}{6}}} \right) = \log \frac{2}{5}.$$

$$13. \log \frac{2^{\frac{5}{4}}}{4} + \log \frac{3^{\frac{6}{5}}}{6} - \log \frac{3}{4} = \log \left(\frac{2^{\frac{5}{4}}}{4} \cdot \frac{3^{\frac{6}{5}}}{6} \cdot \frac{4}{3} \right) = \log 6.$$

$$14. 2 \log 3 + 3 \log 2 = \log (3^2 \cdot 2^3) = \log 72.$$

$$15. 3 \log 4 + 4 \log 3 - 2 \log 6 = \log (4^3 \cdot 3^4 \div 6^2) = \log 144.$$

$$16. \quad \log \left(a - \frac{x^2}{a} \right) = \log \frac{a^2 - x^2}{a}$$

$$= \log \frac{(a+x)(a-x)}{a}$$

$$= \log (a+x) + \log (a-x) - \log a.$$

$$17. \quad \log \sqrt{a^2 - x^2} = \log (a^2 - x^2)^{\frac{1}{2}} = \frac{1}{2} [\log (a+x) + \log (a-x)].$$

$$18. \quad \log \sqrt{s(s-a)} = \log [s(s-a)]^{\frac{1}{2}} = \frac{1}{2} [\log s + \log (s-a)].$$

$$19. \quad \log \sqrt{\frac{s(s-b)(s-c)}{s-a}} = \log \left(\frac{s(s-b)(s-c)}{s-a} \right)^{\frac{1}{2}}$$

$$= \frac{1}{2} [\log s + \log (s-b) + \log (s-c) - \log (s-a)].$$

$$20. (a) \quad 2\pi R = 2(85)(3.1416).$$

$$\log 2 = .3010$$

$$\log 85 = 1.9294$$

$$\log 3.1416 = .4971$$

$$\log \text{ of product} = 2.7275$$

$$\text{antilog } 2.7275 = 534 \text{ inches.}$$

$$(b) \quad 3281 = 2\pi r.$$

$$r = 3281 \div 2\pi.$$

$$\log 3281 = 3.5160$$

$$\log 2\pi = .7981$$

$$\log \text{ of quotient} = 2.7179$$

$$\text{antilog } 2.7179 = 522.3 \text{ centimeters.}$$

$$21. (a) \quad \pi R^2 = 3.1416(5.672)^2$$

$$\log 3.1416 = .4971$$

$$\log (5.672)^2 = 1.5074$$

$$\log \text{ of product} = 2.0045$$

$$\text{antilog } 2.0045 = 101.0 \text{ square feet.}$$

$$(b) \quad \pi R^2 = 67.37.$$

$$R = \sqrt{67.37 \div 3.1416}.$$

$$\log 67.37 = 1.8285$$

$$\log 3.1416 = .4971$$

$$\log \text{ of quotient} = 1.3314$$

$$\log \text{ of expression} = .6657.$$

$$\text{antilog } .6657 = 4.631 \text{ feet.}$$

22. (a)

$$4\pi R^2 = 4(3.1416)(3958.79)^2.$$

$$\log 4 = .6021$$

$$\log 3.1416 = .4971$$

$$\log (3958.79)^2 = \underline{7.1952}$$

$$\log \text{ of product} = \underline{8.2944}$$

$$\text{antilog } 8.2944 = 196,900,000 \text{ square miles.}$$

(b)

$$2\pi R = 2(3.1416)(3958.79).$$

$$\log 2 = .3010$$

$$\log 3.1416 = .4971$$

$$\log 3958.79 = \underline{3.5976}$$

$$\log \text{ of product} = \underline{4.3957}$$

$$\text{antilog } 4.3957 = 24,870 \text{ miles.}$$

23. (a)

$$\frac{4\pi R^3}{3} = \frac{4 \cdot 3.1416 R^3}{3} = 86.$$

$$R = \sqrt[3]{\frac{86 \cdot 3}{4 \cdot 3.1416}}.$$

$$\log 86 = 1.9345$$

$$\log 3 = \underline{.4771}$$

$$\log \text{ of numerator} = \underline{2.4116}$$

$$\log 4 = .6021$$

$$\log 3.1416 = \underline{.4971}$$

$$\log \text{ of denominator} = \underline{1.0992}$$

$$\log \text{ of fraction} = 1.3124.$$

$$\frac{1}{3} \log \text{ of fraction} = .4375.$$

$$\text{antilog } .4375 = 2.738 \text{ feet.}$$

(b) Here

$$47 = \frac{4}{3}\pi R^3 \text{ or } R = \sqrt[3]{\frac{47 \cdot 3}{4 \cdot 3.1416}}.$$

$$\log 47 = 1.6721$$

$$\log 3 = \underline{.4771}$$

$$\log \text{ of numerator} = \underline{2.1492}$$

$$\log 4\pi = \underline{1.0992}$$

$$\log \text{ of fraction} = \underline{1.0500}$$

$$\log R = \frac{1}{3} \log \text{ of fraction} = .3500.$$

$$\text{antilog } .3500 = 2.239.$$

The diameter is $2R$, or 4.478 inches.

24. (a)

$$c + a = 377 + 288 = 665.$$

$$c - a = 377 - 288 = 89.$$

$$\log 665 = 2.8228$$

$$\log 89 = 1.9494$$

$$\log \text{ of product} = 4.7722$$

$$\log \text{ of expression} = 2.3861$$

$$\text{antilog } 2.3861 = 243.3, \text{ the side.}$$

(b)

$$c + a = 1285 + 924 = 2209.$$

$$c - a = 1285 - 924 = 361.$$

$$\log 2209 = 3.3442$$

$$\log 361 = 2.5575$$

$$\log \text{ of product} = 5.9017$$

$$\log \text{ of expression} = 2.9509$$

$$\text{antilog } 2.9509 = 893.0, \text{ the side.}$$

25.

$$\frac{s^2}{4} \sqrt{3} = \frac{(34.23)^2}{4} \sqrt{3}.$$

$$\log (34.23)^2 = 3.0688$$

$$\log \sqrt{3} = .2386$$

$$\log \text{ of numerator} = 3.3074$$

$$\log 4 = .6021$$

$$\log \text{ of expression} = 2.7053$$

$$\text{antilog } 2.7053 = 507.3 \text{ square inches}$$

$$= 3.523 \text{ square feet.}$$

26.

$$3^x = 25.$$

$$x \log 3 = \log 25.$$

$$x = \frac{\log 25}{\log 3}$$

$$= \frac{1.3979}{.4771} = 2.92.$$

27.

$$64^x = 4.$$

$$4^{3x} = 4^1.$$

$$3x = 1.$$

$$x = \frac{1}{3}.$$

$$29. \quad (-2)^x = 64.$$

$$(-2)^x = (-2)^6.$$

$$x = 6.$$

$$30. \quad 3 = (1.04)^x.$$

$$x \log 1.04 = \log 3.$$

$$x = \frac{\log 3}{\log 1.04}$$

$$= \frac{.4771}{.0170} = 28.06.$$

28.

$$16^x = 1024.$$

$$2^{4x} = 2^{10}.$$

$$4x = 10.$$

$$x = 2.5.$$

$$31. \quad 2^x = 64 = 2^6.$$

$$x = 6.$$

$$32. \quad 4^{2x+1} = 84.$$

$$(2x+1)\log 4 = \log 84.$$

$$x = \frac{\log 84 - \log 4}{2 \log 4}$$

$$= \frac{1.9243 - .6021}{1.2041}$$

$$= \frac{1.3222}{1.2041}$$

$$= 1.09 +.$$

$$33. \quad 3^{x+3} = 6561.$$

$$(x+3)\log 3 = \log 6561.$$

$$x = \frac{\log 6561 - 3 \log 3}{\log 3}$$

$$= \frac{3.8170 - 1.4313}{.4771}$$

$$= 5.0.$$

$$34. \quad 10^{x-1} = 3.$$

$$(x-1)\log 10 = \log 3.$$

$$x-1 = \log 3.$$

$$x = 1 + \log 3 = 1.4771.$$

$$35. \quad 8^{x+2} = 6.$$

$$(x+2)\log 8 = \log 6.$$

$$x = \frac{\log 6 - 2 \log 8}{\log 8}$$

$$= \frac{.7782 - 1.8062}{.9031}$$

$$= -1.13.$$

$$36. \quad (.3)^{-x} = 5.$$

$$-x \log .3 = \log 5.$$

$$x = -\frac{\log 5}{\log .3} = -\frac{.6990}{1.4771}$$

$$= -\frac{.6990}{-.5229} = 1.33 +.$$

$$37. \quad (.07)^x = 9.$$

$$x = \frac{\log 9}{\log .07} = \frac{.9542}{2.8451}$$

$$= \frac{.9542}{-1.1549}$$

$$= -.82.$$

$$38. \quad (a) \log (3)^{52} = 52 \log 3$$

$$= 52(.4771)$$

$$= 24.8092.$$

25 digits. *Ans.*

$$(b) \quad \log 2^{340} = 340(.3010)$$

$$= 102.3400.$$

103 digits. *Ans.*

$$(c) \quad \log 2^9 = 2.7090$$

$$\log 3^8 = 3.8168$$

$$\log 5^7 = 4.8930$$

$$\log \text{ of product} = 11.4188$$

12 digits. *Ans.*

$$40. \quad (1.06)^x = 2.$$

$$x = \frac{\log 2}{\log 1.06}$$

$$= \frac{.3010}{.0253}$$

$$= 11.89 + \text{ years.}$$

$$41. \quad (1.04)^x = 3.$$

$$x = \frac{\log 3}{\log 1.04}$$

$$= \frac{.4771}{.0170}$$

$$= 28.06 + \text{ years.}$$

$$42. \quad (1.035)^x = 2.$$

$$x = \frac{\log 2}{\log 1.035}$$

$$= \frac{.3010}{.0149}$$

$$= 20.20 + \text{ years.}$$

$$\begin{aligned}
 43. \quad & 4000(1.05)^x = 7360.80. \\
 & x = \frac{\log 7360.80 - \log 4000}{\log 1.05} \\
 & = \frac{3.8669 - 3.6021}{.0212} \\
 & = 12.49 + \text{years.}
 \end{aligned}$$

$$\begin{aligned}
 44. \text{ Let } & x = \text{amount in dollars.} \\
 & 24(1.04)^{300} = x. \\
 & 300 \log 1.04 + \log 24 = \log x. \\
 & 5.1000 + 1.3802 = \log x. \\
 & 6.4802 = \log x. \\
 & x = \$3,021,000.
 \end{aligned}$$

$$\begin{aligned}
 45. \quad & (1.015)^x = 2. \\
 & x = \frac{\log 2}{\log 1.015} = \frac{.3010}{.0065} = 46.3. \\
 & \frac{46.3}{2} = 23.15 \text{ years.}
 \end{aligned}$$

46. The amount of P dollars in one year at $r\%$ is $P(1+r)$.
 The amount of P dollars in two years at $r\%$ is $P(1+r)^2$.
 Hence the amount of P dollars in t years at $r\%$ is $P(1+r)^t$.
 The amount of P dollars in one year at $r\%$ (semiannual interest) is
 $P\left(1 + \frac{r}{2}\right)\left(1 + \frac{r}{2}\right)$.
 The amount of P dollars in two years at $r\%$ (semiannual interest) is
 $P\left(1 + \frac{r}{2}\right)^4$.
 The amount of P dollars in t years at $r\%$ (semiannual interest) is
 $P\left(1 + \frac{r}{2}\right)^{2t}$.
 In like manner for the others.

$$\begin{aligned}
 47. (a) \quad & 5000(1.04)^4 = A. \\
 & \log A = \log 5000 + 4 \log 1.04 \\
 & = 3.6990 + .0680 \\
 & = 3.7670. \\
 & A = \$5848.
 \end{aligned}$$

$$\begin{aligned}(b) \quad 5000(1.02)^8 &= A. \\ \log A &= \log 5000 + 8 \log 1.02 \\ &= 3.6990 + .0688 = 3.7678. \\ A &= \$5859.\end{aligned}$$

$$\begin{aligned}(c) \quad 5000(1.01)^{16} &= A. \\ \log A &= \log 5000 + 16 \log 1.01 \\ &= 3.6990 + .0688 = 3.7678. \\ A &= \$5859.\end{aligned}$$

(Four-place tables do not show a difference here between semiannual and quarterly payment of interest. Seven-place tables show the amount for semiannual payments to be \$5858.30, and for quarterly payments to be \$5862.90.)

$$\begin{aligned}48. \quad A &= 4.12(1.01)^{20}. \\ \log A &= \log 4.12 + 20 \log 1.01 \\ &= .6149 + .0860 \\ &= .7009. \\ A &= 5.022.\end{aligned}$$

$$\begin{aligned}49. \quad 5 &= P(1.01)^{20}. \\ \log P &= \log 5 - 20 \log 1.01 \\ &= .6990 - .0860 \\ &= .6130. \\ P &= 4.102.\end{aligned}$$

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